Australian Earthquake Engineering Society
2023 National Conference, 23-25 November 2023
Brisbane, Queensland, Australia

Simplified Method of Modelling Tension Stiffening in Reinforced Concrete Walls

Prashidha Khatiwada[1], Elisa Lumantarna[1], Nelson Lam[1]

1. Department of Infrastructure Engineering, University of Melbourne, Parkville VIC 3010

Abstract

This study introduces a tension stiffening model designed to address the response of reinforced concrete (RC) walls when subjected to seismic loads. Under seismic loading, the wall experiences both axial compression and tension. Beyond the concrete's threshold for maximum tensile stress, cracks emerge in the concrete, transferring tensile resistance to the reinforcement. Consequently, the local strain in the reinforcement is significantly higher compared to the global tensile strain in the wall. The derived mathematical model establishes a direct relationship between the localized tensile strain encountered by vertical reinforcement and the overall tensile strain experienced by an RC wall. Principal factors influencing tension stiffening, including the ultimate tensile strength of both concrete and reinforcement, alongside the proportion of vertical reinforcement, are considered in the model. Validation is achieved by establishing a correspondence between the model's predictions and the observed outcomes from cyclically loaded specimens of RC walls. Furthermore, the model is seamlessly integrated into sectional analysis procedures, facilitating the incorporation of the tension stiffening effect within the evaluation of RC walls. By enhancing the understanding of tension stiffening phenomena within RC walls, this model significantly improves the precision of seismic analysis and design methodologies. Ultimately, it contributes to the enhanced performance and heightened reliability of RC structures under seismic loading scenarios.

Keywords: reinforced concrete walls; tension stiffening; sectional analysis; seismic performance of buildings.

1 Introduction

Reinforced concrete (RC) members subjected to uniaxial tension exhibit a mechanical response characterized by the development of elastic tensile stresses within both the concrete matrix and the longitudinal reinforcement. Beyond the point of surpassing the concrete's critical tensile stress, cracks form, leading to the transfer of tensile resistance solely to the reinforcement. This phenomenon is accompanied by the re-establishment of tensile forces into the concrete through interfacial mechanical interaction, commonly referred to as bond stress. Termed tension stiffening, this mechanism contributes to an increased effective tensile stiffness, distinct from the inherent tensile stiffness of the reinforcement alone. Of particular relevance is the influence of crack patterns within RC walls, influenced by bond properties, tensile strengths of both the concrete and reinforcement, and the longitudinal reinforcement ratio. Notably, limited ductile walls characterized by the formation of fewer tensile cracks are more susceptible to heightened tension stiffening effects compared to their ductile counterparts. Consequently, for assessments of structures involving limited ductile RC walls which is the prevalent element in the Australian building inventory (Menegon et al., 2018; Hoult et al., 2018), the consideration of tension stiffening becomes pivotal.

Within the realm of structural engineering, substantial attention has been devoted to investigating tension stiffening, encompassing the intricate interplay between concrete and reinforcing steel under tensile conditions. Previous inquiries have scrutinized diverse factors such as crack spacing, arrangement of reinforcement, material characteristics of concrete, and the history of imposed loading (Bazant and Cedolin, 2003; Collins and Mitchell, 1991; Lee et al., 2011). The comprehensive exploration of tension stiffening phenomena has been facilitated through analytical, experimental, and numerical methodologies, examining stiffness alterations, crack propagation mechanisms, and the overall structural response (Clark and Cranston, 1980; Prakhya and Morley, 1990; Hsu and Zhang, 1996; Bentz, 2000). In parallel, considerable efforts have been invested in devising predictive models capable of accurately predicting the intricate effects of tension stiffening (Massicotte et al., 1990; Choi and Cheung, 1996; Murray et al., 2018; Menegon et al., 2021).

However, the integration of complex tension stiffening models into sectional analysis procedures poses a formidable challenge due to their inherent computational demands. In response, this study presents a mathematical formulation that directly establishes the relationship between the localized tensile strain encountered by vertical reinforcement and the overall strain in the wall, hence reducing significant computation costs. The model incorporates key factors influencing tension stiffening, encompassing the ultimate tensile strength of both concrete and reinforcement, as well as the percentage of vertical reinforcement. Validation is achieved through a rigorous correlation between the model's predictions and the observed responses derived from cyclically loaded RC wall specimens. Notably, the model's integration into sectional analysis procedures facilitates a more profound comprehension of tension stiffening effects which refines seismic analysis methodologies and ultimately enhances the reliability of RC structures under seismic loading conditions. A detailed exposition of the model's formulation and validation is elaborated upon in the ensuing sections.

2 Simplified Tension Stiffening Model for RC Walls

A simple-to-use mathematical model is proposed here for calculating the local tensile strain in the longitudinal reinforcement of RC walls. It is assumed that the tension stiffening effect can be measured through the local tensile strain developed in the longitudinal reinforcement as a result of the bond mechanism between the reinforcement and the cracked concrete. Assuming relative material strengths of the concrete and longitudinal reinforcement, and the size and percentage of longitudinal reinforcement as the main parameters affecting tension stiffening, Eqs. (1)-(4) are proposed to establish the relationship between the local tensile strain in the reinforcement (e_{sl}) and the global average tensile strain (e_{sg}) in the wall. The relationship between these two strains in the linear and the nonlinear states is shown in Figure 1.

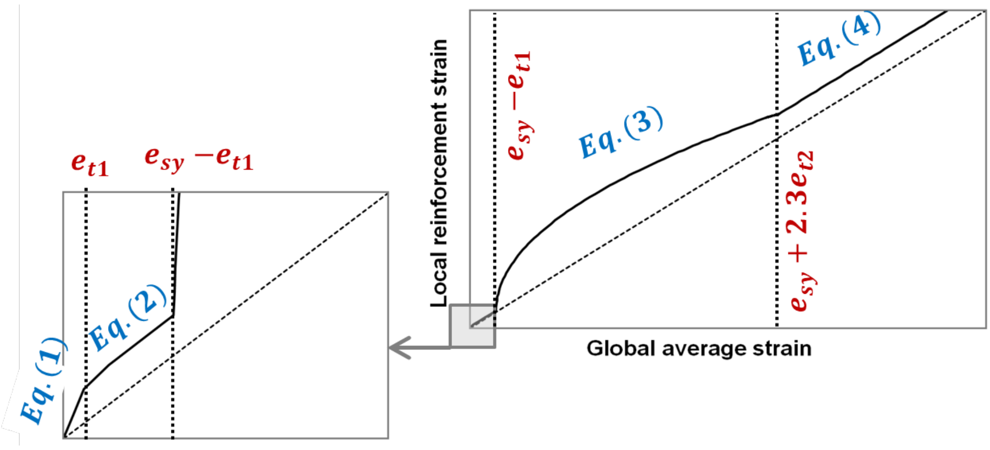


Figure 1. The relationship between the local tensile strain in the reinforcement and the average global tensile strain in the linear and nonlinear states.

When $e_{sg} \leq e_{t1}$,

$$e_{sl} = 2\sqrt{e_{sg}e_{t1}} \quad (1)$$

When $e_{t1} < e_{sg} < e_{sy} - e_{t1}$,

$$e_{sl} = e_{sg} + e_{t1} \quad (2)$$

When $e_{sg} \leq e_{sy} + 2.3e_{t2}$,

$$e_{sl} = e_{sy} + 1.75\sqrt{e_{t2}(e_{sg} + e_{t1} - e_{sy})} \quad (3)$$

When $e_s > e_{sy} + 2.3e_{t2}$,

$$e_{sl} = e_{sg} + e_{t2} - 40e_{t1} \quad (4)$$

In Eqs. (1)-(4), strains "e_{sl}" and "e_{sg}" are local tensile strain in the reinforcement and the global average tensile strain in the wall respectively; strain 'e_{sy}' is the yield strain of reinforcement, and strains "$e_{t1} = 0.28f_{ct}/p_vE_s$" and "$e_{t2} = 0.173f_{ct}/p_vE_s'd_v^{0.18}$ ". Here, "f_{ct}" is the tensile strength of concrete; "p_v" and "d_v" are the vertical reinforcement ratio and its diameter; and "E_s" and "$E_s' = (f_{su} - f_{sy})/(e_{su} - e_{sy})$" are the modulus of rigidity of reinforcement at the elastic and inelastic states, respectively.

The proposed method is validated in Section 3 by comparing it with the observed responses derived from cyclically loaded RC wall specimens.

3 Validation of the Proposed Method

The validation of the proposed simplified method is achieved by comparison of predicted and the experimental results of the local tensile strain in reinforcement vs global average tensile strain curve. Wall test specimens named: P08 and P10 (Menegon, 2018) whose cross-section and the reinforcement detailing are shown in Figures A1 and A2 respectively are used for the comparative study. The length, thickness, and height of both wall specimens are 450 mm, 130 mm, and 800 mm, respectively. The remaining information on the wall cross-section and reinforcement details are summarised in Table 1.

Table 1. Information on the wall specimen and its material properties.

Parameter	Specimen P08	Specimen P10
d_v	16 mm	16 mm
p_v	0.021	0.01
(f_{sy}, f_{su})	(550 MPa, 660 MPa)	(550 MPa, 660 MPa)
(f_c', f_{ct})	(45.4 MPa, 2.9 MPa)	(45.4 MPa, 2.9 MPa)

The comparison of the local tensile strain in longitudinal reinforcement vs the average global tensile strain in the wall cross-section predicted by the proposed simplified model and the results obtained from the experimental tests are presented in Figure 2, where we can see a close match between the predicted and the test results.

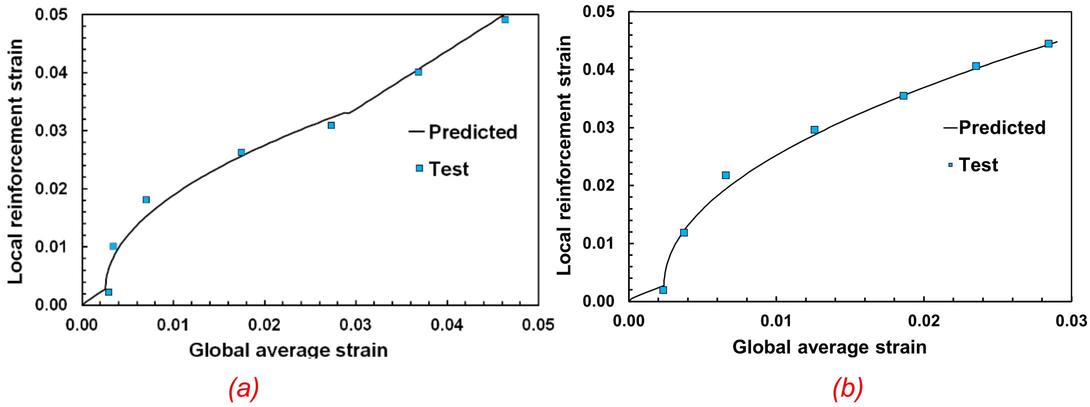


Figure 2. Comparison of the local vs global tensile strains predicted by the proposed model and the experimental results for the wall specimens P08 (a) and P10 (b).

The developed tension stiffening model was used in the MATLAB based "Nonlinear Sectional Analysis of Reinforced Concrete (NSARC)" tool developed by the authors to obtain the force-displacement plots of the two limited ductile rectangular wall specimens: C1 (Lu et al., 2017) and R1 (Oesterle et al.,1976). Wall specimen C1 has a uniform longitudinal reinforcement ratio of 0.0053 while wall specimen R1 has longitudinal reinforcement ratio of approximately 0.0147 and 0.0026 in the boundary element and web respectively. The force-displacement curve obtained with and without considering the effect of tension stiffening in NSARC is compared with the experimental results in Figure 3. The comparison shows that the "NSARC with Tension Stiffening" and the experimental results are closely matching with each other. "NSARC without Tension Stiffening" predicted a force-displacement curve consistent with the other two results before the state of the first crack. After the development of the first cracks, "NSARC without Tension Stiffening" predicts higher values of curvatures and lower values of forces compared to the other two results. This is expected as tension stiffening only occurs when crack formation initiates.

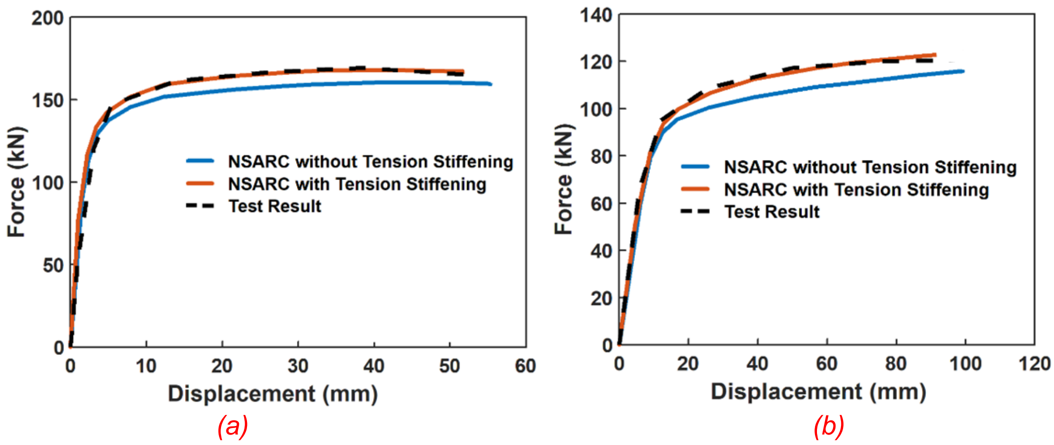
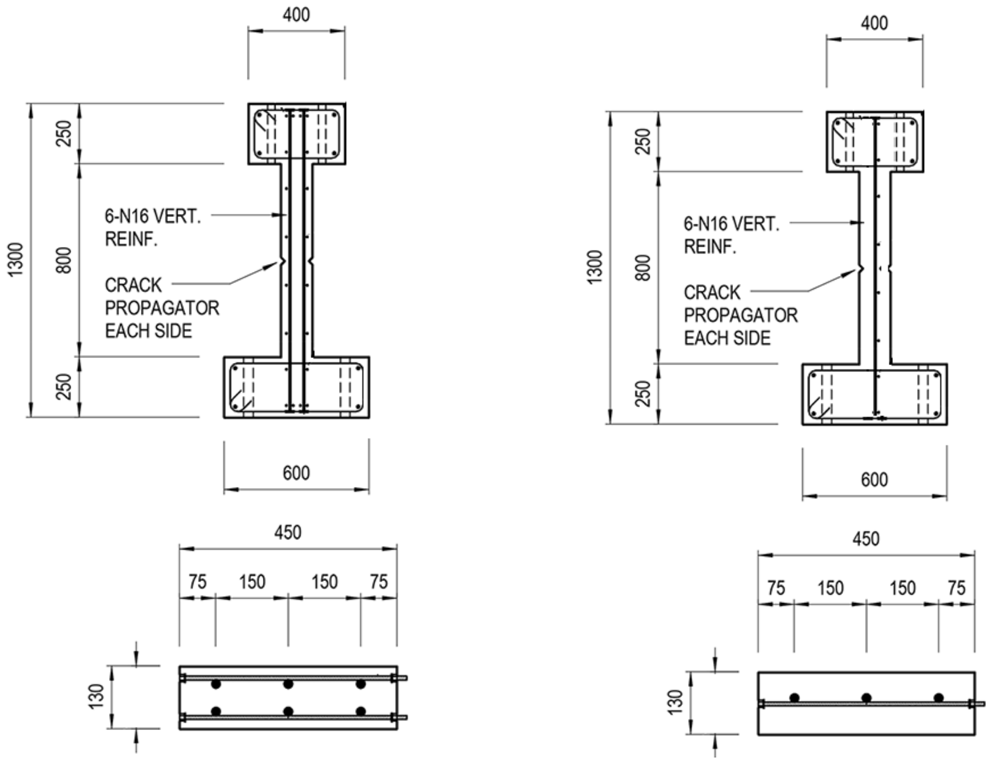


Figure 3. Comparison of the force-displacement capacity predicted by the NSARC tool (with and without tension stiffening) and the experimental results for the wall specimens C1 (a) and R1 (b).

4 Conclusions

In summary, this paper introduces a simplified method for modelling tension stiffening in reinforced concrete walls. The proposed approach's accuracy is validated through a rigorous comparison with cyclic test results from limited ductile RC wall specimens. The model's practical utility is further evidenced by its seamless integration into the NSARC MATLAB sectional analysis program. This integration, coupled with the consideration of tension stiffening effects, yields closely matched force-displacement predictions compared to experimental outcomes. This research enhances understanding, provides a valuable tool for structural analysis, and ensures precise predictions, thus contributing to the optimized design and performance of reinforced concrete structures.

5 Appendix



(a) P08 *(b) P10*

Figure A1. Cross-section and reinforcement detailing of P08(a) and P10(b). Source: Menegon (2018).

6 References

Bazant, Z. P., & Cedolin, L. (2003). Stability of Structures: Elastic, Inelastic, Fracture, and Damage Theories. Oxford University Press.

Bentz, E. C. (2000). Sectional analysis of reinforced concrete members (p. 310). Toronto: University of Toronto.

Choi, C. K., & Cheung, S. H. (1996). Tension stiffening model for planar reinforced concrete members. Computers & structures, 59(1), 179-190.

Clark, L. A., & Cranston, W. B. (1980). The influence of bar spacing on tension stiffening in reinforced concrete slabs. In Advances in concrete slab technology (pp. 118-128). Pergamon.

Collins, M. P., & Mitchell, D. (1991). Tension stiffening in concrete beams and slabs. Journal of Structural Engineering, 117(9), 2631-2650.

Hoult, R. (2017). Seismic assessment of reinforced concrete walls in Australia (Doctoral dissertation, Department of Infrastructure Engineering, The University of Melbourne.

Hsu, T. T., & Zhang, L. X. (1996). Tension stiffening in reinforced concrete membrane elements. Structural Journal, 93(1), 108-115.

Lee, S. C., Cho, J. Y., & Vecchio, F. J. (2011). Model for post-yield tension stiffening and rebar rupture in concrete members. Engineering structures, 33(5), 1723-1733.

Lu, Y., Henry, R. S., Gultom, R., & Ma, Q. T. (2017). Cyclic testing of reinforced concrete walls with distributed minimum vertical reinforcement. Journal of Structural Engineering, 143(5), 04016225.

Massicotte, B., Elwi, A. E., & MacGregor, J. G. (1990). Tension-stiffening model for planar reinforced concrete members. Journal of Structural Engineering, 116(11), 3039-3058.

Menegon, S. J. (2018). Displacement behaviour of reinforced concrete walls in regions of lower seismicity. Doctor of Philosophy thesis, Department of Civil and Construction Engineering, Swinburne University of Technology.

Menegon, S. J., Wilson, J. L., Lam, N. T., & Gad, E. F. (2021). Tension stiffening model for lightly confined reinforced concrete elements. Magazine of Concrete Research, 73(7), 366-378.

Murray, A., Gilbert, R. I., & Castel, A. (2018). A new approach to modeling tension stiffening in reinforced concrete. ACI Structural Journal, 115(1), 127-137.

Oesterle, R. G., Fiorato, A. E., Johal, L. S., Carpenter, J. E., Russell, H. G., & Corley, W. G. (1976). Earthquake resistant structural walls-tests of isolated walls. Research and Development Construction Technology Laboratories, Portland Cement Association.

Prakhya, G. K. V., & Morley, C. T. (1990). Tension-stiffening and moment-curvature relations of reinforced concrete elements. Structural Journal, 87(5), 597-605.