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Simplified Method of Modelling Inelastic Shear Deformation in Reinforced Concrete Walls

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Abstract

Reinforced concrete buildings that undergo large shear deformation fails in a brittle and catastrophic manner during an earthquake. To prevent such failure, buildings with RC walls are designed as flexural-dominated walls having a higher aspect ratio. To ensure flexural domination, seismic code requires design engineers to meet only the ultimate shear strength requirements but not the inelastic shear deformation. The shear deformation is mainly not checked for two reasons: (1) it is complex to predict compared to flexural deformation; and (2) inherently walls of high aspect ratio are assumed to have negligible shear deformation. However, experimental studies show that even walls of higher aspect ratios can have considerably high shear deformation due to shear-flexural interaction and it may result in rapid loss of strength and degradation in displacement ductility. It is therefore required that the shear deformation be calculated and checked for walls of all aspect ratios. As the current method of determining shear deformation of RC walls are either based on an overly simplified beam model or a highly complex finite element model, this research aims to propose a simple and accurate hand calculation method. The proposed simplified method has been validated to be accurate by use of the test results obtained from the cyclic test of RC wall specimens.

Keywords: shear strength; shear deformation; reinforced concrete walls.

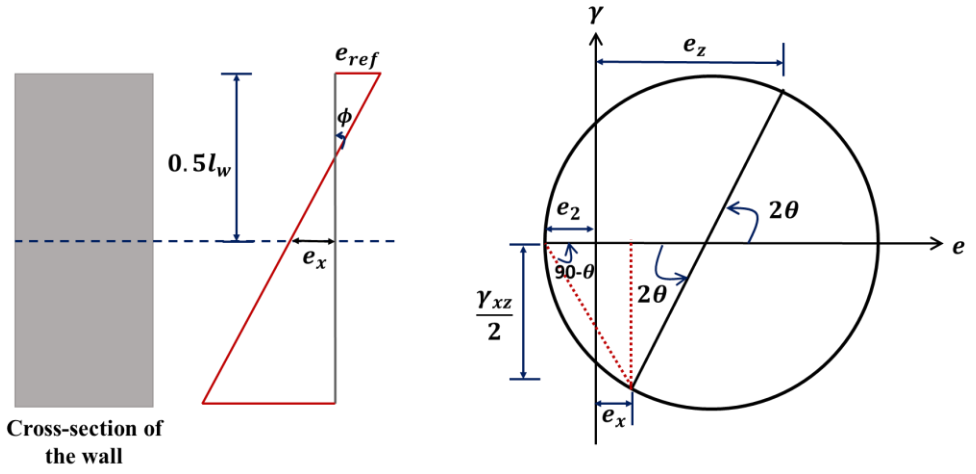
1 Introduction

Reinforced concrete (RC) building that undergoes large shear deformation fails in a brittle and catastrophic manner during an earthquake. To prevent such failure, design engineers inherently assume that RC buildings will have a flexural-dominated response (large shear strength than flexural strength) if the RC walls are designed to have a sufficient aspect ratio or shear span. However, experimental studies show that such RC walls can have considerably high shear deformation due to flexural-induced shear deformation or shear-flexural interaction (Oesterle et al., 1979; Massone and Wallace, 2004; and Tran and Wallace, 2012). For example, in a study by Menegon (2018), the shear deformation of up to 20% of the total displacement was observed in walls with an aspect ratio of about 2. As walls having larger shear deformation are found to have a rapid loss of strength and degradation in displacement ductility (Krolicki et al., 2011), it is therefore required that the shear deformation be calculated and checked for walls of all aspect ratios. The current method of determining shear deformation for RC walls is either based on an overly simplified beam model or a highly complex finite element model. This research aims to propose a simple hand calculation of inelastic shear deformation which is as accurate as the continuum models.

One of the most popular simplified models known as the “Truss Model” that is given by Ritter (1899) and Morsch (1909) determines the shear resistance because of the combination of diagonal compressive stresses in the concrete (concrete struts) and a network of longitudinal ties representing longitudinal reinforcement and transverse ties representing transverse reinforcement. The latter development of this model includes the refinement applied to the angle of inclination of the concrete strut. Similarly, another simplified approach, the "Tooth Model" that is initially developed by Kani (1964) and improved by Taylor (1974) and Reineck (1991) mainly determine shear resistance because of uncoupled shear. A more comprehensive simplified model that considers the shear-flexure interaction is the model given by Beyer et al. (2011). In this model, the shear-to-flexural deformation ratio is calculated from axial strains, cracking angle, and the dimension of the cross-section. However, this model has limited implications due to the overly simplified assumption of constant axial strain and cracking angle; and the complexity involved in the calculation of the cracking angle. Regarding, the continuum-mechanics model, Collins (1978) used the diagonal compression field theory to determine the shear force-deformation response of the two-dimensional cracked reinforced concrete elements subjected to in-plane shear and normal stresses. Vecchio and Collins (1986) further refined the model and named it a "Modified Compression Field Theory – MCFT" model. In this model, the shear resistance is determined considering the equilibrium of external and internal forces in a cross-section that is established using average stresses calculated from average strains and material constitutive relationships. The key assumptions of the MCFT model are: (1) concrete fibre and the reinforcing bars have a perfect bond and therefore displace by the same amount; (2) the reinforcements are uniformly distributed over the element; (3) the average stresses calculated from average strains consider the combined effects of the stresses and strains at and between cracks, interface shear on cracks, dowel action, and bond and crack slip mechanisms; and (4) the angle of inclination of principal stress coincide with principal strain. These assumptions were validated by Vecchio and Collins (1986) using experimental results of 30 RC panel elements. The MCFT procedure explained in Vecchio, and Collins (1986) involves an iterative procedure consisting of about 22 steps of calculation in each iteration performed to ensure that at cracks the average tensile stresses developed in the concrete do not exceed tensile stresses developed in reinforcements, and the reinforcement is capable of transmitting stresses at cracks. Despite accounting for the shear-flexural interaction and more realistic prediction of the shear deformation, MCFT is highly complex and costly due to the increased demand for the computation power and input information. The MCFT procedure is further simplified as detailed in Section 2 to achieve a level of computation that can be achieved without the use of a computer but at the same time does not compromise on the level of accuracy. The proposed simplified method has been validated using two RC wall specimens. The validation is presented in Section 3.

2 Proposed Method for Modelling Shear Deformation in RC Walls

A very simple to use hand calculation method is proposed in this section for the calculation of inelastic shear deformation of RC wall. In the proposed method, the shear deformation is determined from the product of shear strain and plastic hinge length as shown in Equation (1). The shear strain is determined from the average strain of "$2(e_x + e_2)cot\theta$" as given by Vecchio, and Collins (1986) which satisfies the condition of compatibility and equilibrium (refer to Figure 1b). The average axial strain at the mid-depth of the section 'e_x' (see Figure 1a) can be calculated from top fibre concrete compressive strain 'e_{ref}' and the corresponding curvature 'ϕ' as shown in Equation (2). The value of curvature is determined from either the moment-curvature analysis or using the simplified calculation (Menegon, 2018) using vertical reinforcement ratio (p_v), axial load ratio (n) and length of the wall (l_w) as $(0.15p_v - 2p_v^2 + 0.0031)/l_w$ at yield and $[(19.5p_v - 545p_v^2 - 0.066)(0.158 - \mathrm{n}) + 0.017]]/l_w$ at ultimate state (1.5% drift). Similarly, the diagonal compressive strain 'e_2' can be calculated from Equation (3) as concrete core compressive stress (sum of compressive stresses in concrete and stirrups) divided by the modulus of rigidity of concrete. The inclination of principal compressive strain in concrete 'θ' is determined from 'ϕ' and crack spacing parameter 'aS_h' as shown in Equation (4). The plastic hinge length (L_p) in Equation (1) can be calculated using appropriate plastic hinge models given in the literature, for example, Hoult (2022) and Priestley et al. (2007). The Priestley et al. (2007) model which is suitable for rectangular walls is given in Equation (5).



(a) Wall cross-section and its curvature profile (b) Average strains and their relationship

Figure 1. Curvature profile (a), and the relationship between average strains in RC wall (b).

$$\Delta_s = \gamma_{xz} \times L_p = 2(e_x + e_2)cot\theta \times L_p \tag{1}$$

$$e_x = e_{ref}\left(\frac{0.5l_w}{NA} - 1\right) = 0.5l_w\phi - e_{ref} \approx 0.5l_w\phi - 0.002 \tag{2}$$

$$e_2 = \frac{\left(\frac{0.2\sqrt{f_c'}cot\theta}{375l_w\phi - 1}\right) + p_s f_{sy}}{155f_c' + 27000} \tag{3}$$

$$\theta = (15 + 3500l_w\phi)\left(0.88 + \frac{aS_h}{2500}\right) \leq 70^o \tag{4}$$

$$L_p = Min[0.2(f_{su}/f_{sy} - 1), 0.08] \times H_e + 0.1L_w + 0.022f_{sy}d_v \tag{5}$$

'f_{sy}' and 'f_{su}' are yield and ultimate stress of reinforcement; 'H_e' is the effective height of the wall (approximately 0.7 of the total height); 'l_w' is the length of the wall; and 'd_v' is the diameter of vertical reinforcement; 'a' is a constant equal to 1.23 for $f_c' \leq 65\ MPa$ and 2 for $f_c' > 65\ MPa$; 'S_h' is the vertical spacing of shear reinforcement; 'p_s' is the stirrups reinforcement ratio; and 'f_c'' is the characteristic compressive strength of concrete.

The proposed method is validated with the shear deformation obtained from the experimental results of two test specimens in Section 3.

3 Validation of the Proposed Method

The validation of the proposed simplified method is achieved by comparison of the shear deformation predicted by the proposed method and experimental result for the case of wall specimens: TUA (Beyer et al., 2008) and RW2 (Thomsen and Wallace, 1995) whose cross-section and the reinforcement detailing are shown in Figures A1 and A2, respectively. The basic information of the cross-section and the reinforcement details are also summarised in Table 1. Similarly, the example calculation of the shear deformation for the TUA wall specimen at 2.5% top drift is shown below. The results for the two wall specimens at 2% and 2.5% top drifts are summarised and compared with test results in Table 2.

Table 1. Information on the cross-section and the reinforcement details of the test specimens.

Parameter	TUA (Beyer et al., 2008)	RW2 (Thomsen and Wallace, 1995)
Length of web	1300 mm	1219 mm
Length of flange	1050 mm	-
Height	3350 mm	3658 mm
d_v	6 mm	9.53 mm
p_s	0.003	0.003
S_h	125 mm	76 mm
f_{sy}	518 MPa	395 MPa
f_{su}	681 MPa	550 MPa
f_c'	77.9 MPa	42.8 MPa

Example calculation for wall specimen TUA: Using Equations (1-5) and the input information listed in Table 1, the shear deformation at 83.2 mm total displacement (≈ 2.5% roof drift) is calculated as shown below.

$$L_p = Min[0.2(f_{su}/f_{sy} - 1), 0.08] \times H_e + 0.1L_w + 0.022f_{sy}d_b = 409\ mm$$

$\phi = 7.1 \times 10^{-5}/mm$ (from Beyer et al., 2008).

$$e_x = 0.5l_w\phi - 0.002 = 0.044$$

$$e_2 = \frac{\left(\frac{0.2\sqrt{f_c'}cot\theta}{375l_w\phi - 1}\right) + p_s f_{sy}}{155f_c' + 27000} = 4 \times 10^{-5}$$

$$\theta = \min\left[(15 + 3500l_w\phi)\left(0.88 + \frac{2S_h}{2500}\right), 70\right] = 70^o$$

$$\Delta_s = 2(e_x + e_2)cot\theta \times L_p = 13.2\ mm$$

Table 2. Comparison of the shear deformation obtained from the proposed method and the test results.

Specimen		Δ_s at 2% top drift	Δ_s at 2.5% top drift
TUA (C-shaped)	test results	12.0 mm	14.9 mm
	proposed method	11.0 mm	13.2 mm
	difference	1 mm	1.7 mm
RW2 (rectangular)	test results	7.5 mm	9.2 mm
	proposed method	7.1 mm	9.1 mm
	difference	0.4 mm	0.1 mm

Table 2 shows that the analytical and the test results are matching for both wall specimens at 2% and 2.5% top drift (sum of flexural and shear displacement divided by wall height). Compared to TUA (non-rectangular wall), a better prediction is obtained for RW2 (rectangular). The lesser accuracy with TUA has mainly arises because of the plastic hinge model (Equation 5 which is mainly suitable for rectangular walls) that is used to estimate the plastic hinge length. Therefore, it is important that a more accurate plastic hinge model is used while predicting the shear deformation in non-rectangular RC walls. Further research is recommended for validating the developed method for walls of different shapes and to find out a general value of the shear deformation limit of RC walls.

4 Conclusions

A simplified method for determining the inelastic shear deformation in RC walls is proposed in this paper. The proposed method can be simply implemented by use of hand calculation. To demonstrate the implication of the proposed method, the hand calculation of the shear deformation for a wall specimen is also provided. The proposed method has been validated to be accurate by comparing the calculated results with test results obtained from the test of two RC wall specimens. Further research is recommended for validating the developed method for walls of different aspect ratios and shapes, for example, T-shaped walls, I-shaped walls, and core walls.

5 Appendix

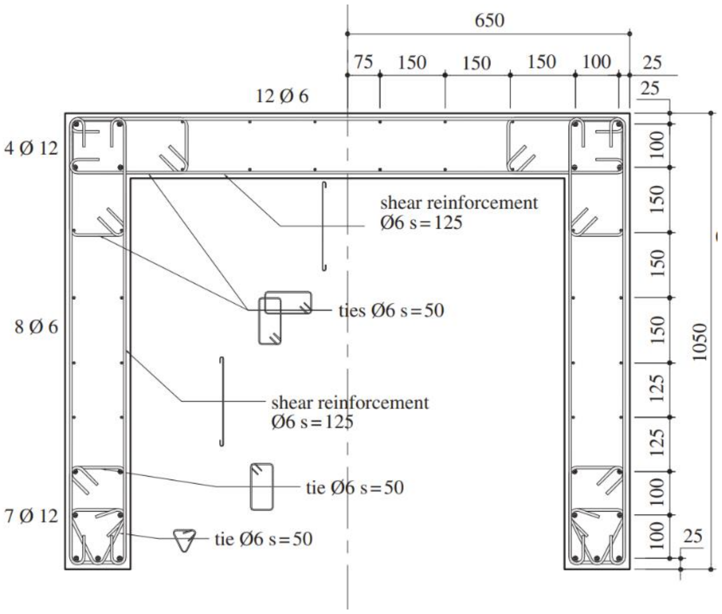


Figure A1. Cross-section and reinforcement detailing of TUA. Source: Beyer et al. (2008).

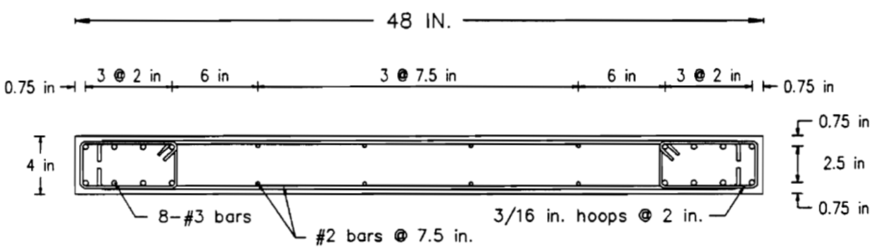


Figure A2. Cross-section and reinforcement detailing of RW2 (1 in = 25.4 mm). Source: Thomson and Wallace (1995).

6 References

Bentz, E. C., Vecchio, F. J., & Collins, M. P. (2006). Simplified modified compression field theory for calculating shear strength of reinforced concrete elements. *ACI structural journal*, 103(4), 614.

Beyer, K., Dazio, A., & Priestley, N. (2011). Shear deformations of slender reinforced concrete walls under seismic loading. *ACI Structural Journal*, *108*(ARTICLE), 167-177.

Beyer, K., Dazio, A., & Priestley, M. J. N. (2008). Quasi-static cyclic tests of two U-shaped reinforced concrete walls. Journal of earthquake engineering, *12*(7), 1023-1053.

Collins, Michael P. "Towards a rational theory for RC members in shear." *Journal of the Structural Division* 104, no. 4 (1978): 649-666.

Hoult, R. (2022). Universal Plastic Hinge Length for Reinforced Concrete Walls. *ACI Structural Journal*, 119(4).

Kani, G. N. J. (1964, April). The riddle of shear failure and its solution. In *Journal Proceedings* (Vol. 61, No. 4, pp. 441-468).

Krolicki, J., Maffei, J., & Calvi, G. M. (2011). Shear strength of reinforced concrete walls subjected to cyclic loading. *Journal of Earthquake Engineering*, *15*(S1), 30-71.

Massone, L. M., & Wallace, J. W. (2004). Load-deformation responses of slender reinforced concrete walls. *Structural Journal*, *101*(1), 103-113.

Menegon, S. J. (2018). Displacement behaviour of reinforced concrete walls in regions of lower seismicity. *Doctor of Philosophy thesis, Department of Civil and Construction Engineering, Swinburne University of Technology*.

Mörsch, E. (1909). *Concrete-steel construction (der Eisenbetonbau)*. Engineering news publishing Company.

Oesterle, R. G., J. D. Aristizabal-Ochoa, A. E. Fiorato, H. G. Russell, and W. G. Corley. "Earthquake resistant structural walls-tests of isolated walls-phase II." *Construction Technology Laboratories, Portland Cement Association* (1979).

Priestley, M. J. N., Calvi, G. M., & Kowalsky, M. J. (2007, March). Direct displacement-based seismic design of structures. In *NZSEE conference* (pp. 1-23).

Reineck, K. H. (1991). Ultimate shear force of structural concrete members without transverse reinforcement derived from a mechanical model (SP-885). *Structural Journal*, *88*(5), 592-602.

Ritter, W. E. (1899). *A contribution to the knowledge of the Tunicata of the Pribilof Islands*.

Taylor, H. P. (1974). The fundamental behavior of reinforced concrete beams in bending and shear. *Special Publication*, *42*, 43-78.

Thomsen, J. H., & Wallace, J. W. (1995). *Displacement-based design of RC structural walls: an experimental investigation of walls with rectangular and T-shaped cross-sections*. Clarkson University, Department of Civil Engineering.

Tran, T. A., & Wallace, J. W. (2012, September). Experimental study of nonlinear flexural and shear deformations of reinforced concrete structural walls. In *Proceedings, 15th World Conference on Earthquake Engineering, Lisbon, Portugal*.

Vecchio, F. J., & Collins, M. P. (1986). The modified compression-field theory for reinforced concrete elements subjected to shear. *ACI J.*, *83*(2), 219-231.