Torsional Rigidity of Asymmetrical Multi-storey Reinforced Concrete Buildings

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Abstract

Reinforced concrete (RC) buildings possessing plan irregularities can be subjected to significant torsional actions. The torsional behaviour of buildings depends on their torsional stability characterised by a parameter known as the elastic radius ratio (b_r). Previous studies by the authors have shown that buildings with a b_r value of less than 1.1 can be subjected to high displacement demands in an earthquake. Such buildings have been deemed to be torsionally unstable and should be avoided.

Establishing the b_r value of multi-storey buildings is challenging and requires a three-dimensional analysis of the structures. This paper presents a study aimed to develop a simple method to estimate the displacement demands of asymmetrical multi-storey buildings. The developed method includes a simple rule to visually identify the torsional stability of a building. Parametric studies were undertaken based on single-storey buildings. The results were verified by comparison with multi-storey buildings. It is expected that this study will provide a significant contribution to the seismic design and assessment of RC buildings.

Keywords: reinforced concrete buildings; elastic radius ratio (b_r); torsional effect; torsional stiff or flexible.

1 INTRODUCTION

The torsional effects on reinforced concrete buildings due to the excitation of an earthquake have been investigated in the past decades (Anagnostopoulos et al., 2015; Ayre, 1938; Dempsey & Irvine, 1979; Hart et al., 1975; Housner & Outinen, 1958; Humar & Kumar, 1998; Khatiwada & Lumantarna, 2021; Lam et al., 2016; Lumantarna et al., 2018; Lumantarna et al., 2019; Lumantarna et al., 2020; Makarios, 2008; Stathopoulos & Anagnostopoulos, 2003; Tabatabaei & Saffari, 2011; Tso, 1990; Xing et al., 2020; Zalka, 2013). However, applying their recommendations on asymmetrical multi-storey buildings is still challenging. This is because the majority of the researches about torsional effects is based on a single-storey building model idealisation to represent the behaviour multi-storey buildings (Anagnostopoulos et al., 2015).

The torsional stability of RC buildings is an important factor affecting the torsional response behaviour of asymmetrical buildings. It is related to structural parameters, including torsional stiffness and translational stiffness of the buildings. Different researchers used different terminologies and notations to represent and assess the torsional stability of buildings. For example, elastic radius ratio (b_r) was initially defined by Lam et al. (1997) to identify torsional vulnerability. Humar and Kumar (1998) used the torsional stiffness ratio (Ω_R) to classify if a structure is torsional flexible or stiff. Anagnostopoulos et al. (2015) used the torsional sensitivity (or flexibility) parameter (Ω) to address the torsional vulnerability of a structure. The notations of elastic radius ratio (b_r), torsional stiffness ratio (Ω_R), torsional sensitivity (or flexibility) (Ω) all indicate the torsional stability of buildings, which is a function of torsional stiffness and the translational stiffness of a building structure. In this paper, the authors use the term elastic radius ratio (b_r) to define the torsional stability of buildings.

A building can be considered torsionally stable if its b_r value is more than 1.1. For this building, the displacement response behaviour is less sensitive to the change in eccentricity. Previous studies by the authors have shown that buildings with a b_r value of less than 1.1 can be subjected to high displacement demands in an earthquake and should be avoided (Lam et al., 2016; Lumantarna et al., 2019; Xing et al., 2019, 2020).

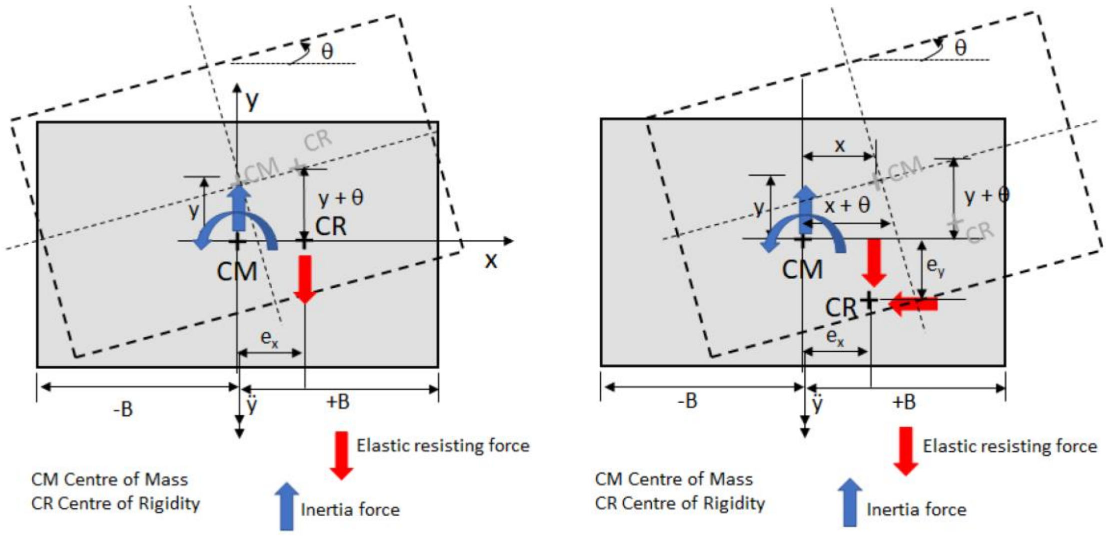
A static analysis method referred to as Generalised Force Method (GFM) of analysis has been developed to provide estimates of maximum displacement demands of asymmetrical buildings (Lam et al., 2016; Lumantarna et al., 2018). The developed method also introduces a procedure to evaluate the multi-storey buildings' elastic radius ratio b_r. However, establishing the b_r value of multi-storey buildings can be a challenge as it still requires a three-dimensional analysis of the structures. There is a scope to develop a simple rule to quickly identify the torsional stability of a building without the need for structural analysis.

The study presented in this paper extends the previous studies of Xing et al. (2019), Xing et al. (2020), Lumantarna et al. (2019) and Lumantarna et al. (2018). Section 2 presents the effect of torsional stability and maximum displacement demands of asymmetric buildings. Section 3 presents a study to define the requirement for building parameters to achieve torsional stability. Section 4 presents parametric studies on multi-storey buildings to validate the building parameter requirement set in Section 3. The study presented in this paper is based on rectangular buildings. However, the method proposed is applicable to buildings with non-rectangular plans.

2 TORSIONAL STABILITY AND MAXIMUM DISPLACEMENT DEMAND OF ASYMMETRICAL BUILDINGS

Buildings can be classified into uni- and bi-axial asymmetry based on the plan configuration. The building model with uni-axial asymmetry in Figure 1(a) has two degrees of freedom:

translation in the direction of the ground motion and rotation. The building model with bi-axial asymmetry in Figure 1(b) has three degrees of freedom: two translations in the direction and perpendicular direction of the ground motion and rotation. The combination of the translation and rotation results in a torsional response, causing amplification of displacement at the edge of the building.



(a) Uni-axial asymmetry *(b) Bi-axial asymmetry*

Figure 1 Asymmetric building model

The main parameters affecting the torsional stability of a building include eccentricity, torsional and translational stiffness, mass-radius gyration (r), and the dimension of the building's floor plan. Mass-radius gyration (r) of the structure represents the distributions of mass of the floor plan. Elastic radius ratio (b_r) represents how the lateral load resisting elements of a building are distributed from the center of rigidity CR of the building floor plan compared to the distribution of its mass (as represented by r).

The equations of edge displacement ratio for the building with uni- or bi-axial asymmetry have been developed by Lumantarna et al. (2018) to estimate the maximum displacement demand based on a single-storey building idealisation shown in Figure 1. The edge displacement ratio is the ratio of the maximum displacement demand at the edge of the asymmetrical building to the maximum displacement demand of the equivalent symmetrical building. A parametric study has been conducted by the authors to investigate the effect of torsional parameters on the edge displacement ratio of the building (Lumantarna et al., 2018; Xing et al., 2020). Some results are presented here for buildings with uni-axial and bi-axial asymmetry.

The edge displacement ratio of buildings with uni-axial asymmetry is presented in Figure 3 for the acceleration-, velocity-, and displacement-controlled condition (Figure 2). It is shown that the displacement demand of asymmetrical buildings is dependent on the eccentricity and elastic radius ratio b_r. However, it can be observed that when the b_r value is greater than 1.0, the displacement demand is less affected by the increasing eccentricity. Based on these results, it can be deduced that a building is torsionally stable when the b_r value is greater 1.1. The equations to estimate the edge displacement ratio of asymmetrical buildings with bi-axial asymmetry have also been proposed by the authors (Lumantarna et al., 2018). The edge displacement ratio presented in Figure 4 also demonstrates that the displacement demand of the buildings is less affected by the eccentricity when the b_r value of the building is larger than 1.1. Results for the analysis of bi-axial asymmetry are shown for the velocity-controlled condition; however, the trends have also been observed for bi-axial asymmetric buildings in the acceleration- and displacement-controlled condition.

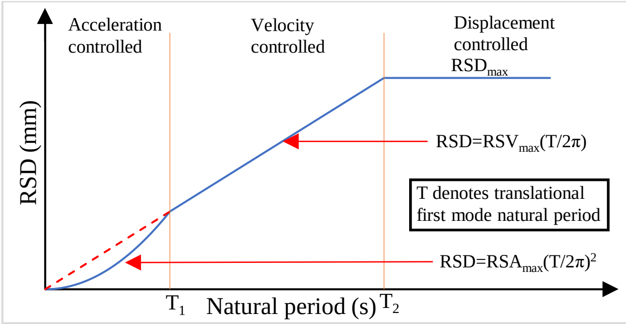
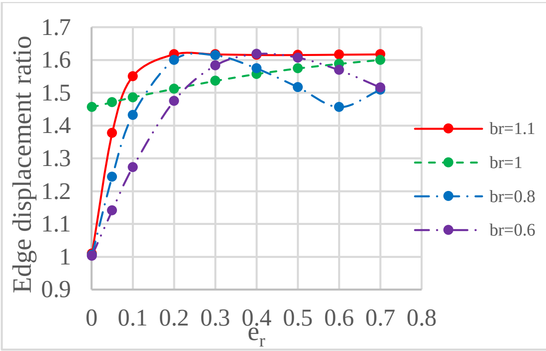
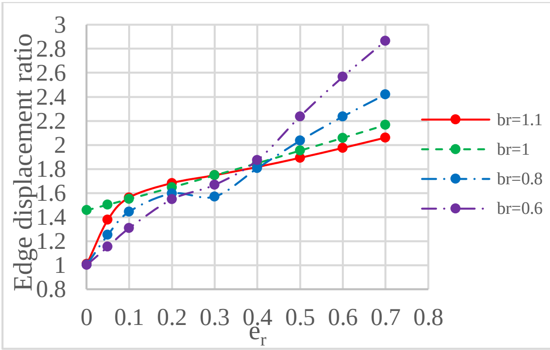


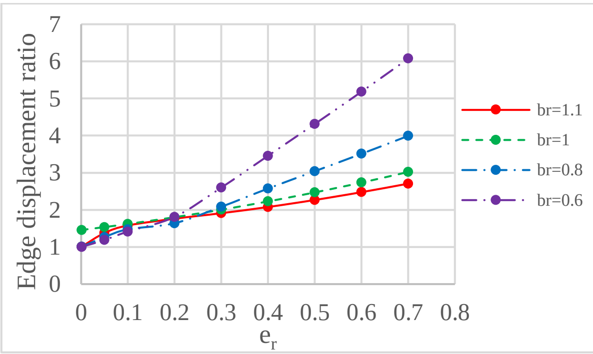
Figure 2 Displacement response spectrum featuring acceleration-, velocity-, and displacement-controlled region



(a) Displacement-controlled

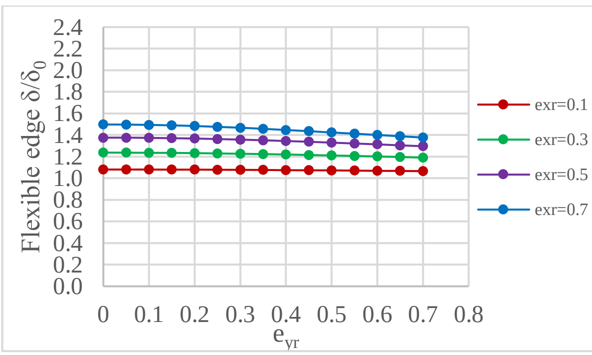


(b) Velocity-controlled

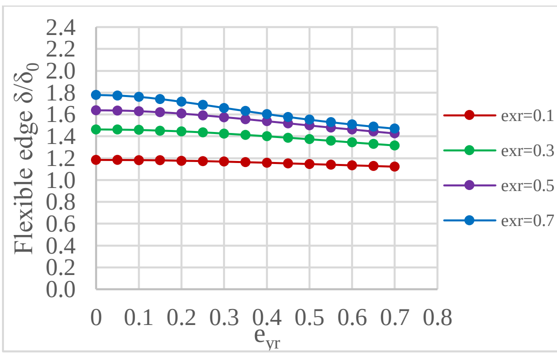


(c) Acceleration-controlled

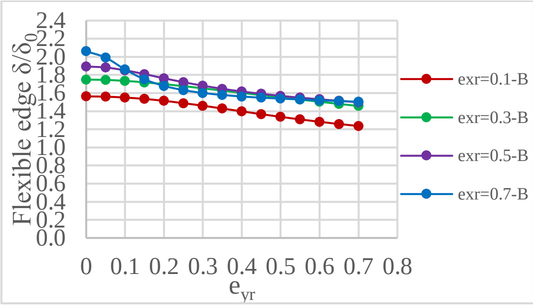
Figure 3 Edge displacement ratio based on the uni-axial asymmetry building model when B_r is 1.8 (B is the distance from the building CM to the edge of the building)



(a) b_r=1.8



(b) b_r=1.4

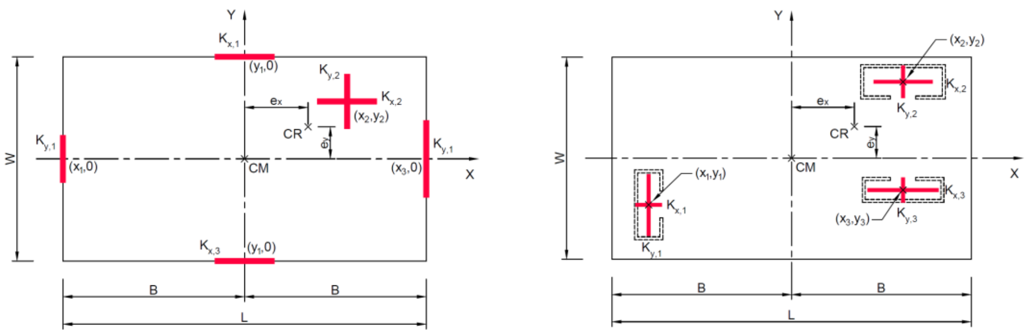


(c) b_r=1.1

Figure 4 Edge displacement ratio based on the bi-axial asymmetry building model for velocity-controlled condition, K_x/K_y is 0.5, and B_r is 1.8, where e_{xr} and e_{yr} are eccentricities along x- and y-axis of the building

3 SEPARATION DISTANCE AND TORSIONAL STABILITY OF ASYMMETRICAL BUILDINGS

The torsional stability of a building is directly affected by the torsional stiffness and translational stiffness of the buildings. The torsional stiffness of buildings is dependent on how far the lateral load resisting elements are located from the center of rigidity of the buildings. This, in turn, is dependent on the separation distance between the lateral load resisting elements of the building. This section presents a study aimed to define the separation distance between the lateral load resisting elements required to achieve torsional stability of the building (b_r value larger than 1.1). The study is based on two assumptions. The first is that each load resisting element can be decomposed into two components along the x- and y-axis of the building shown in Figure 5(a) (Anagnostopoulos et al., 2015). The second is that only core or shear walls contribute to torsional stiffness, and the moment-resting frame does not contribute to torsional stiffness in the building. Therefore, the shear and core walls can be simplified into two translational stiffnesses $K_{x,i}$ and $K_{y,i}$, respectively, along the x- and y-axis of the structural plan shown in Figure 5(b). It is to be noted that the torsional stability of a building has to be evaluated in two directions, with b_{ry} and b_{rx} representing the stability of the structure when subjected to ground motion along the y- and x-axis, respectively. This section presents results for the direction that results in the lower value of b_r.



(a) Idealisation of lateral resisting elements *(b) Idealisation of core wall systems*

Figure 5 The equivalent idealisation of RC building along x- and y-axis

3.1 ONE CORE WALL RC BUILDING WITH UNI-AXIAL ASYMMETRY

The RC building with uni-axial asymmetry and one core wall system is shown in Figure 6. The ground motion direction is assumed to act along the y-axis of the building, although the same principle applies to the ground motion that acts along the x-axis of the building. The building's elastic ratio (b_r) is calculated based on Equation 1 when ground motion is along the y-axis as shown in Figure 6. As the center of the rigidity of the building CR coincides with the center of the core wall ($x_1 = X_{CR}$), the b_r value of the building will always be equal to 0. It should be noted that when a building is laterally supported by a combination of moment-resisting frames and one core wall, the b_r value of the building will be larger than 0. However, the moment-resisting frames are unlikely to result in a torsionally stable building ($b_r > 1.1$). Hence, asymmetrical buildings with one core wall can be considered torsionally unstable irrespective of the size of the core wall.

$$b_r = \frac{\sqrt{\frac{K_\theta}{K_y}}}{r} = \frac{\sqrt{\frac{K_{y,1}(x_1 - x_{CR})^2}{K_{y,1}}}}{r} \qquad (1)$$

Where, x_{CR} is the coordinates of CR along the X-axis.

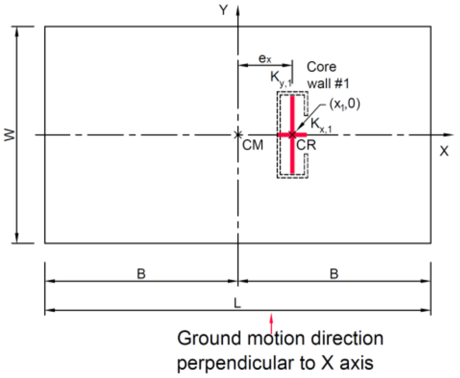


Figure 6 RC building with uni-axial asymmetry and one core wall system

3.2 TWO CORE WALLS RC BUILDING WITH UNI-AXIAL ASYMMETRY

The RC building with uni-axial asymmetry and two core wall systems is shown in Figure 7. Each core wall system can be idealised into two translational stiffnesses: K_y and K_x. The ground motion is assumed to act along the y-axis. The translational stiffnesses of $K_{y,1}$ and $K_{y,2}$ will contribute to the torsional stiffness of the building. In contrast, the translational stiffnesses of $K_{x,1}$ and $K_{x,2}$ will not contribute to the torsional stiffness of the structure. The minimum required separation distance ratio (d_r) between the two core wall systems centers to result in $b_r > 1.1$ is derived in Equations 2 and 3.

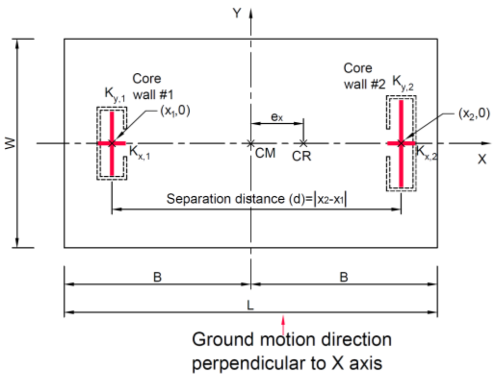


Figure 7 Building with uni-axial asymmetry and two core wall systems

$$K_y = K_{y,1} + K_{y,2} \qquad (2)$$

$$d_r = \frac{d}{r} = \frac{|x_2 - x_1|}{r} \geq 1.1\sqrt{\frac{K_y \times K_y}{K_{y,1} \times K_{y,2}}} \tag{3}$$

Letting $m = \frac{K_{y,1}}{K_{y,2}}$, Equation 3 can be rewritten into Equation 4.

$$d_r \geq \frac{1.1 \times (m+1)}{\sqrt{m}} \tag{4}$$

Where, d = $|x_2 - x_1|$ are the distance betwee the center of the 1st and 2nd core wall systems (the separation distance between the two core walls' centers), d_r is the separation ratio which is equal to the separation distance divided by the mass-radius gyration of the building r, $K_{y,1}$ and $K_{y,2}$ are the translational stiffnesses along the y-axis for the two cores, K_y is the total translational stiffness of the building along the y-axis.

The relationship between separation distance ratio (d_r) and m is shown in Figure 8 (based on Equation 4). The minimum separation ratio (d_r) is 2.2 when the two core wall translational stiffnesses (m = 1) are equal. It is also shown in the figure that a higher separation distance is required when the translational stiffnesses of the core wall systems are not equal. Figure 8 can be used to determine the separation distance required for such buildings.

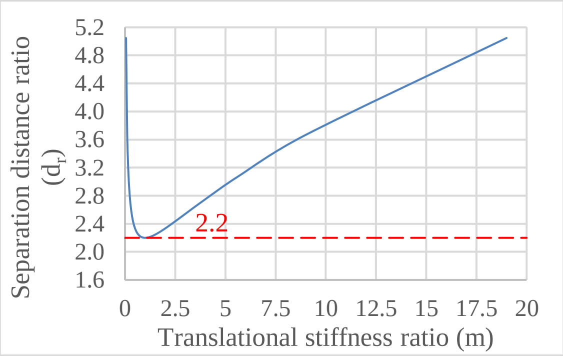
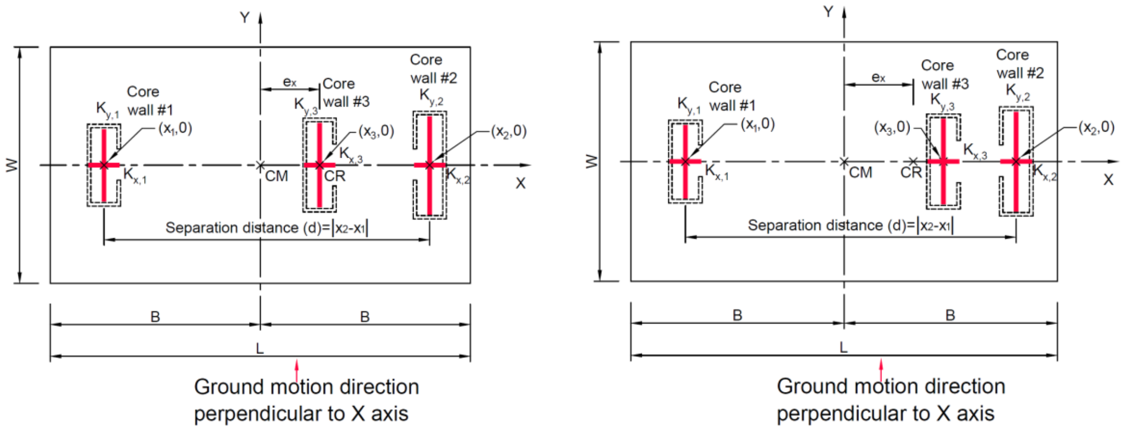


Figure 8 Relationship between d_r and m for RC building with two core wall systems

3.3 THREE CORE WALLS RC BUILDING WITH UNI-AXIAL ASYMMETRY

The RC building with uni-axial asymmetry and three core wall systems has two scenarios, as shown in Figure 9. The first scenario is that the central core wall system is located at the CR of the building (Figure 9(a)). The second scenario is that when the central core wall system does not coincide with the CR of the building, as shown in Figure 9(b).



(a) Central core is located at CR *(b) Central core is not located at CR*

Figure 9 Buildings with three core wall systems and uni-axial asymmetry

• Three core walls building with uni-axial asymmetry when the central core is located at the CR of the building

The required separation distance between the two edge core wall systems' centers is derived in Equations 5 to 7. It indicates that the addition of the central core wall system at the CR of the building reduces the torsional stability of the building. The required minimum d_r is dependent on the translational stiffnesses of the three core wall systems, as shown by Equation 7.

$$K_y = K_{y,1} + K_{y,2} + K_{y,3} \quad (5)$$

$$d_r = \frac{d}{r} = \frac{|x_2 - x_1|}{r} \geq 1.1\sqrt{\frac{(K_{y,1}+K_{y,2})\times(K_{y,1}+K_{y,2}+K_{y,3})}{K_{y,1}\times K_{y,2}}} \quad (6)$$

Let $K_{y,1} = K_{y,2} = K$ and $x = \frac{K_{y,3}}{K_{y,1\ or\ 2}} = \frac{K_{y,3}}{K}$

Equation 6 can be rearranged into Equation 7.

$$d_r \geq 1.1\sqrt{4 + 2x} \quad (7)$$

Where, x is the translational stiffness ratio between the central core and edge core wall system.

The minimum separation distance ratio (d_r) to achieve a torsionally stable building is shown in Figure 10 (based on Equation 7). Figure 10 shows that a larger separation distance between two edge core walls is required when the translational ratio x is higher. It is also shown that buildings with three core walls require a larger separation distance than buildings with two core walls (x = 0 in Figure 10).

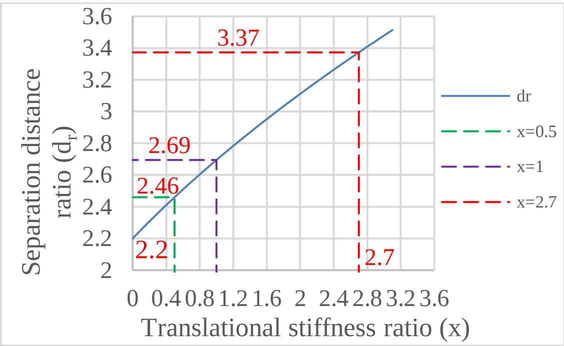


Figure 10 Relationship between d_r and x for RC building with three core wall systems with the central core wall system located at the CR of the building

• Three core walls RC building with uni-axial asymmetry when the central core is not located at the CR of the building

The second scenario is the case when a central core wall system is not located at the CR of the building. The central core wall system will contribute to the torsional stiffness of the building when it moves away from the CR of the building. The required distance shown in Figure 10 can be used as a conservative estimate for this type of buildings.

4 CASE STUDIES FOR VALIDATION OF THE REQUIREMENT FOR TORSIONAL STABILITY OF BUILDINGS

This section presents studies based on static analyses of three dimensional model of multi-storey buildings to determine their torsional stability (as defined by the b_r parameter). The analyses were conducted to verify the separation distance requirements further identified in Section 3. It includes RC buildings with one, two, and three core wall systems in the building plan. Sixty six-building models were created based on seven building layouts presented in Figures 11, 13, 15 and 17. The dimensions of the buildings are presented in Table 1. Only a typical building layout for each case study group is shown in Figures 11, 13, 15 and 17.

Table 1 Summary of the building geometries of case studies

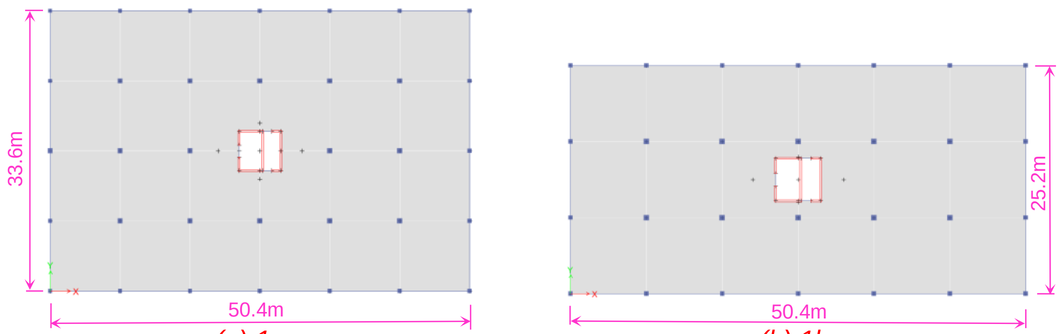
Case study	Length (m)	Width (m)	Height (m)	Ext./Int column (m)	Beam	Slab thickness (m)	No. of core wall	The thickness of wall (m)	Aspect ratio (length/ width)	Details for storey and floor height
1a	50.4	33.6	36	0.5x0.5/ 0.6x0.6	0.7Wx0.7D	0.25	1/2/3	0.2	1.5	L9 (4m)
1b	50.4	25.2	36	0.5x0.5/ 0.6x0.6	0.7Wx0.7D	0.25	1/2/3	0.2	2	L9 (4m)
1c	50.4	16.8	36	0.5x0.5/ 0.6x0.6	0.7Wx0.7D	0.25	1/2/3	0.2	3	L9 (4m)
1d	56.4	14.4	30.4	0.5x0.5/ 0.6x0.6	0.7Wx0.7D	0.25	1/2/3	0.2	3.9	L8 (3.8m)
1e	56.4	22.8	30.4	0.5x0.5/ 0.6x0.6	0.7Wx0.7D	0.25	2/3	0.2	2.5	L8 (3.8m)
1f	58.8	16.8	36	0.5x0.5/ 0.6x0.6	0.7Wx0.7D	0.25	2	0.2	3.5	L9 (4m)
1g	58.8	28	36.8	0.5x0.5/ 0.6x0.6	0.7Wx0.7D	0.25	3	0.2/0.25	2.1	L9 (4.8m for GF/4m)

For all case study buildings, the superimposed dead load and live load are 1.5kPa and 4kPa for a typical floor, 1.5kPa, and 0.25kPa for the roof. The buildings were assumed to be located on a site class D with a design return period of 500 years for earthquake actions in Melbourne (Z = 0.08g). The base connections for columns and walls were assumed to be fixed. The elastic radius ratio b_r for both principal directions was evaluated using the procedure proposed by Xing et al. (2020), which was based on static analyses. The elastic radius ratios of $b_{r,y}$, and $b_{r,x}$ denotes the torsional stability in the x- and y-axis of the building. The three-dimensional static analyses of the case study buildings were conducted using the program ETABS (CSI, 2015).

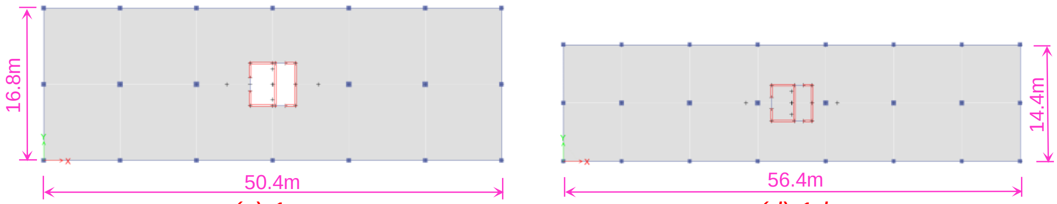
4.1 RESULTS OF BUILDINGS WITH ONE CORE WALL SYSTEM

The building layouts shown in Figure 11 are the case study models of 1a, 1b, 1c, and 1d used to investigate the torsional stability of one core wall system. The four building models have one core wall system that is symmetrically located in the building plans, as shown in Figure 11. The aspect ratios (a=L/W) of the buildings vary from 1.5 to 4.

The results from analyses in the form of b_r and aspect ratio values are presented in Figure 12. It is shown in Figure 12 that the building with one core wall system will have b_r value of less than 1.1 regardless of the buildings' aspect ratios. It indicates an RC building with only one core wall can be defined as a torsionally unstable building.

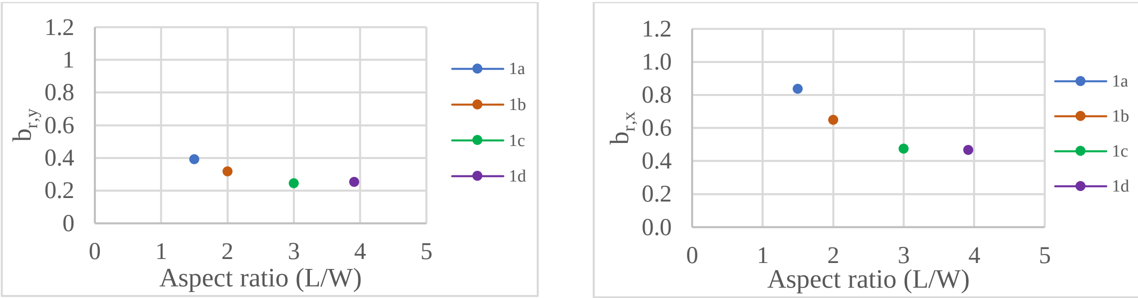


(a) 1a *(b) 1b*



(c) 1c *(d) 1d*

Figure 11 Case study buildings with one core wall system

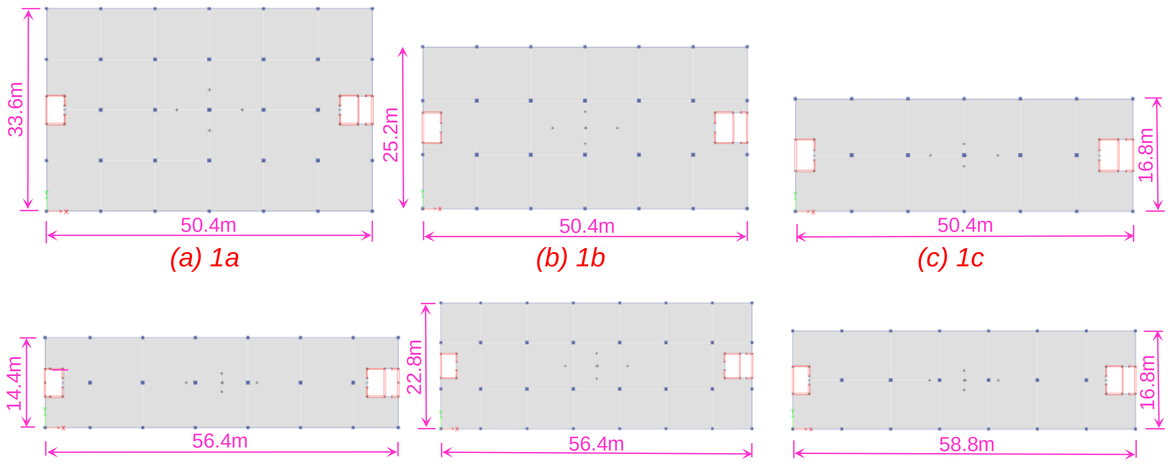


(a) Ground motion along the Y-axis *(b) Ground motion along the X-axis*

Figure 12 b_r values of case study buildings with one core wall

4.2 RESULTS OF BUILDINGS WITH TWO CORE WALL SYSTEMS

Figure 13 presents RC buildings with two core wall systems. For each building layout in Figure 13, the location of the left core is progressively shifted along the horizontal axis to introduce variations in the spacing between the two cores.

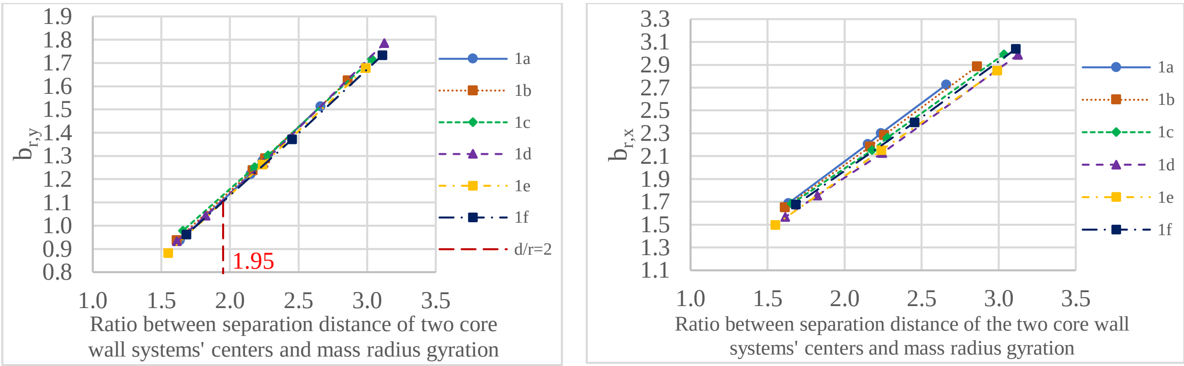


(a) 1a *(b) 1b* *(c) 1c*

(d) 1d *(e) 1e* *(f) 1f*

Figure 13 Case study buildings with two core wall systems

Results from the analyses in the form of b_r are plotted against the separation distance between the two core wall systems, normalised with respect to the mass-radius gyration of the building plan in Figure 14. It is shown in Figure 14 that the values of $b_{r,y}$, and $b_{r,x}$, increase as the spacing between the cores increases. It is shown in Figure 14 that the b_r value of the buildings is higher than 1.1 when the separation distance between the cores (normalised with respect to r) is greater than 2.0. The required separation distance defined in Section 3 (2.2 for core walls with similar translation stiffness) is shown to be conservative.

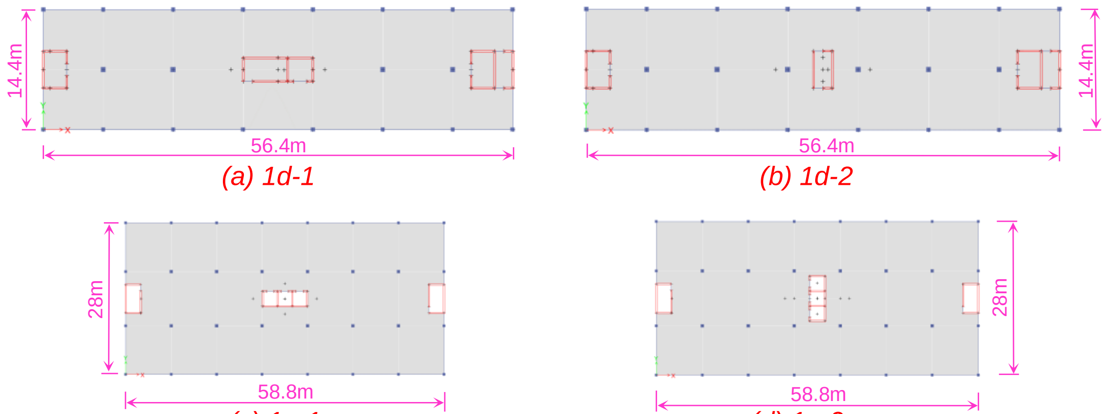


(a) Ground motion along the Y-axis *(b) Ground motion along the X-axis*

Figure 14 b_r values of case study buildings with two core walls

4.3 BUILDINGS WITH THREE CORE WALL SYSTEMS

The building models with three core wall systems are presented in Figure 15. Two building plans with three core wall systems (1d and 1g) were selected. For each of the building plans, the orientation of the central core was varied (compare Fig. 15 (a) to (d)) to investigate the effect of relative flexural stiffness between the core walls on the torsional stability of buildings. For the building models shown in Figures 15(a) and (b), the left and central core walls in Figures 15(a) and (b) were progressively shifted along the horizontal axis. For the building models shown in Figures 15 (c) and (d), the left and right cores were progressively shifted along the horizontal axis towards the central core.

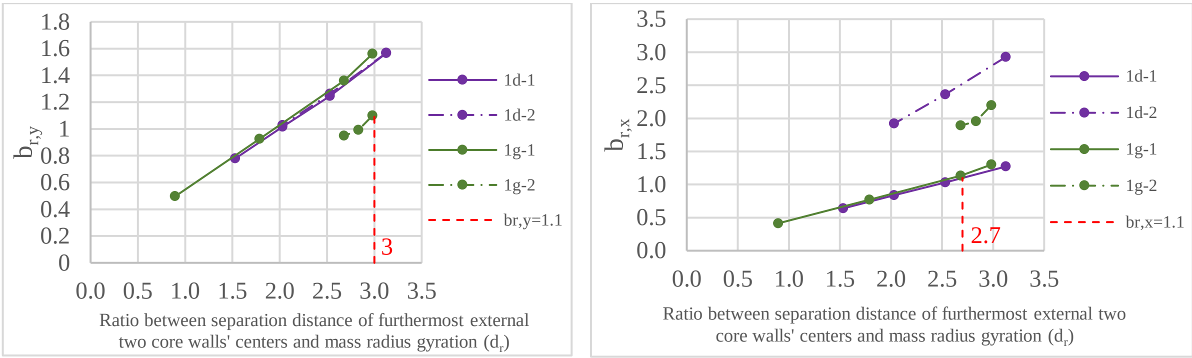


(a) 1d-1 *(b) 1d-2* *(c) 1g-1* *(d) 1g-2*

Figure 15 Case study buildings with three core walls systems – varying the separation distance between the outer walls

Results from the analyses are presented in Figure 16. The b_r values are plotted against the separation distance of the edge two core walls (normalised with respect to the mass radius of gyration r). Figure 16 shows that the increase in separation distance increases the value of b_r. Importantly, the results also show that buildings with a stiff central core (1d-2 and 1g-2) have lower b_r values compared to buildings with flexible central cores (1d-1 and 1g-1). This observed trend is expected as the central cores do not contribute to the torsional stiffness significantly, while they contribute to the translational stiffness. Building case 1g-2 was found to be the most critical out of the four building cases because the translational stiffness of its central core is large compared to the edge core (the translational stiffness ratio (x) is 2.9).

For case study building 1g-2, the separation distance required is 3r (r is the mass radius of gyration) (Figure 16). Based on Figure 10 (or Equation 7) in Section 3.3, the required separation distance is 3.4. This again demonstrates the robustness of the separation distance requirement set in Section 3.

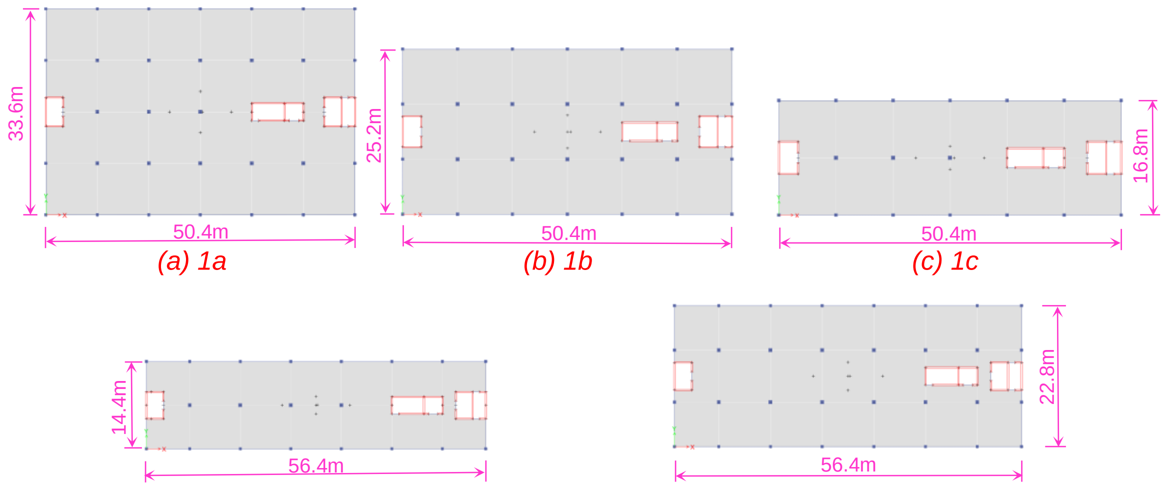


(a) Ground motion along the Y-axis *(b) Ground motion along the X-axis*

Figure 16 b_r values of case study buildings with three core wall systems

4.4 VALIDATION OF THE CRITERIA FOR RC BUILDING WITH THREE CORE WALL SYSTEMS (with two closely-spaced core walls)

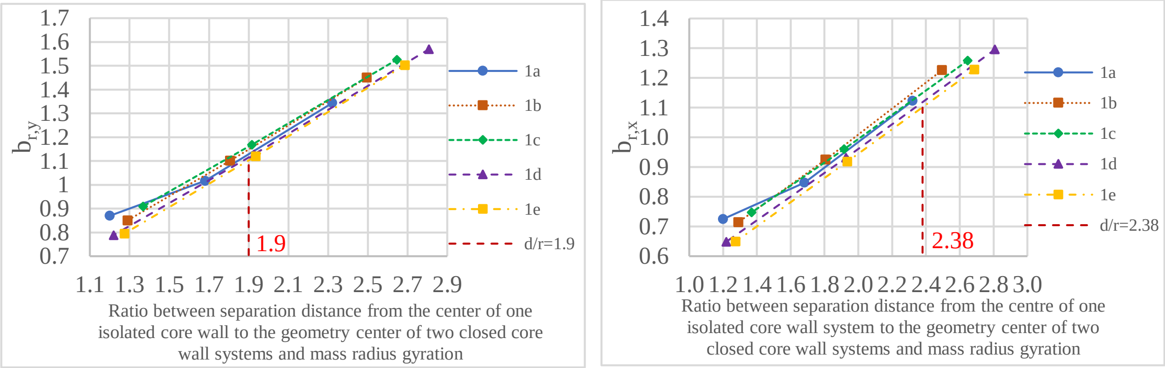
As the separation distance between the core wall systems is an essential parameter for the torsional rigidity of RC buildings, the layout of the core wall systems directly reflects the impact on the torsional response of the RC building. When the separation distance of closed two core wall systems is less than the limit (3m), the building with three core wall systems can be treated as two core wall systems for the purpose of the evaluation. Analyses were conducted on this type of buildings (buildings with three core walls but with two core walls that are closely spaced) to investigate the effects of the distance between the two core wall systems on the torsional stability of the buildings. The building models used in the analyses are shown in Figure 17. These building layouts have the same feature as the buildings with two core wall systems (shown in Figure 13). The left core wall system of each building presented in Figure 17 was progressively shifted along the horizontal axis towards the two closely spaced core wall systems, which were kept unchanged.



(a) 1a *(b) 1b* *(c) 1c* *(d) 1d* *(e) 1e*

Figure 17 Building layouts with three core wall systems (with closely spaced core walls)

Results from the analyses in the form of b_r against the separation distance (between the center of one core wall and the geometric center of two closely spaced core walls, normalised with respect to r) are presented in Figure 18. It is shown that when the ratio is higher than 1.9 and 2.4, for the y- and x-direction of motion, respectively, the b_r value of the building is larger than 1.1. The required separation distance is higher than that of a building with two core wall systems (shown in Section 4.2). This is because the two closely spaced core walls increase the difference in the translational stiffness between the two closely spaced core walls and the core wall at the opposite end. Based on Section 3.2, the ratio of the translational stiffness (m) is 2.2; hence, the required separation distance between the core walls is 2.4 based on Figure 8 (or Equation 4) in Section 3.2.



(a) Ground motion along the Y-axis *(b) Ground motion along the X-axis*

Figure 18 b_r values of case study buildings three core wall systems (and two closely spaced walls)

5 PROPOSED METHOD TO VISUALLY DETERMINE THE TORSIONAL STABILITY OF BUILDINGS

An RC building has an adequate torsional rigidity (b_r) when the elastic radius ratios ($b_{r,y}$ and $b_{r,x}$) along the building's two principal directions (y and x) are both more than 1.1. A proposed method is introduced in this section to visually identify if the value of b_r is larger than 1.1, which represents the RC building has adequate torsional stability. This method is a supplement to the three-tiered approach of tier 1.1 produced by Xing et al. (2020) to predict maximum displacement caused by an earthquake. The three-tiered approach was developed to satisfy the requirement of the different design stages. Tier 1 allows the torsional effect can be estimated conservatively without very detailed structural information. Tier 1 consists of two tiers, including tiers 1.1 and 1.2. Tier 1.1 can be used to identify if the b_r value of a building is larger than 1.1 without the need to do static analysis. When $b_r > 1.1$ can not be established using tier 1.1, tier 1.2 can be used, which involves applying static loading on the multi-storey building. A more accurate prediction can be obtained using Tiers 2 and 3 when more structural information is available.

The b_r value can be evaluated based on the spacing between the cores, the location of the cores, and the relative translational stiffness of the cores. This information can be obtained from structural floor plans without the need of conducting structural analysis. The example building models shown in Figure 21 illustrate how building's b_r value can be assessed. The b_r value of a building is considered to be greater than 1.1 (and hence the building can be considered to be torsionally stable) if one of the following criteria is met:

- One core wall system or closely spaced two core wall systems

Parametric studies conducted by the authors show that the building with only one core wall system will have a value of b_r that is less than 1.1, which has been investigated in sections 3.1 and 4.1. The example building with one core wall system is shown in Figure 21(a).

- Two core wall systems

The b_r value of a building with two core wall systems of a similar dimension is not less than 1.1 when the minimum separation distance between two core wall systems centers is larger than

2.2r (where r is mass-radius of gyration). Figure 19 (or Equation 8) can be used to determine the required separation distance when the building has two cores of different dimensions. They are simply replicated from Figure 8 (or Equation 4) and shown here for ease of reference. The limit of 2.2r has been investigated in sections 3.2 and 4.2. The typical example of a two-core wall system building is shown in Figure 21(b).

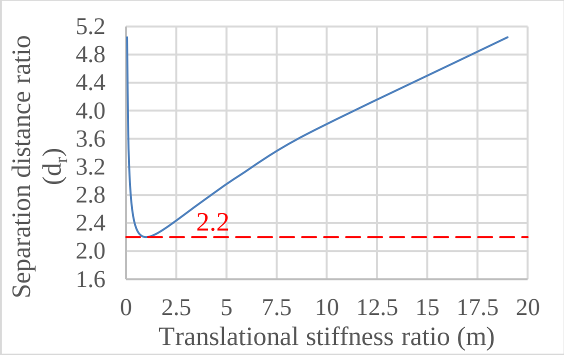


Figure 19 Relationship between d_r and m for RC building with two core wall systems

$$d_r \geq \frac{1.1\times(m+1)}{\sqrt{m}} \tag{8}$$

- For three core wall systems

For buildings with three core wall systems, Figure 20 (or Equation 9) can be used to identify the separation distance based on the ratio of the second moment of area (of the central core to one edge core wall systems). They are simply replicated from Figure 10 (or Equation 7) and shown here for ease of reference. The example building with three core wall systems is shown in Figure 21(c).

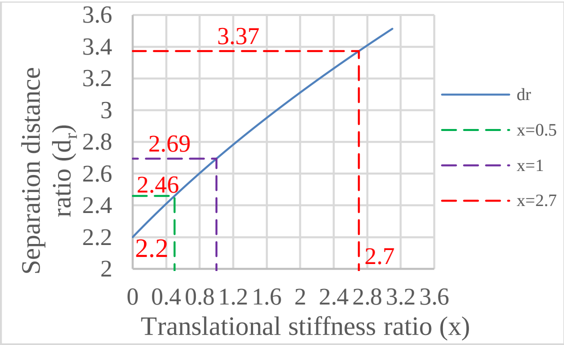


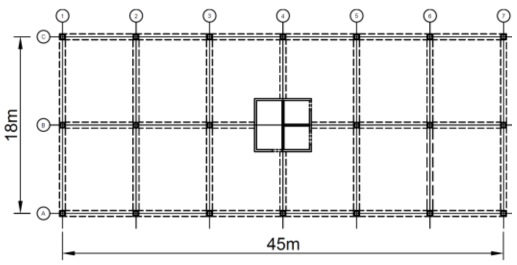
Figure 20 Relationship between d_r and x for RC building with three core wall systems with the central core wall system located at the CR of the building

$$d_r \geq 1.1\sqrt{4+2x} \tag{9}$$

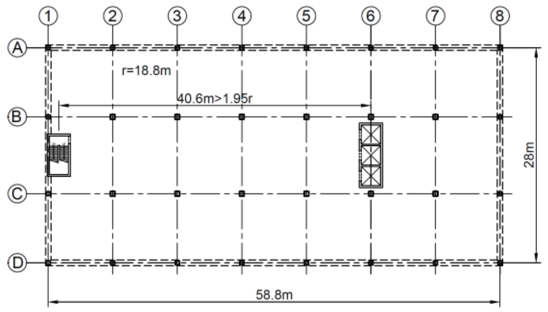
- For three core wall systems with two closely spaced core wall systems

When buildings with three core walls have two cores close to another, the three core wall buildings can be considered as two core wall buildings when the clear separation distance between the two cores is less than 3 m. This building is illustrated in Figure 21 (d). The building can be considered to have b_r value larger than 1.1 when the distance between the center of the core walls is greater than 2.4r as illustrated in Figure 21(d).

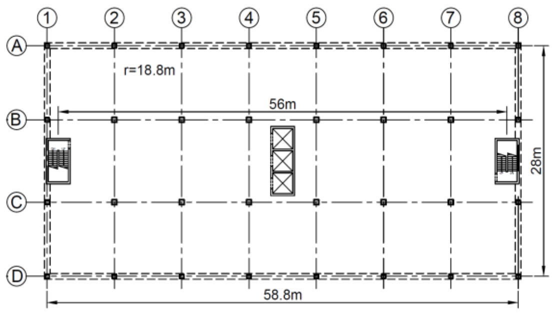
If a building has not met any of the above criteria, further analysis is required to evaluate the torsional stability of the building. A simple method based on static analyses and simple calculations has been proposed (tier 1.2 of the three-tiered approach developed by the authors (Xing et al., 2020)).



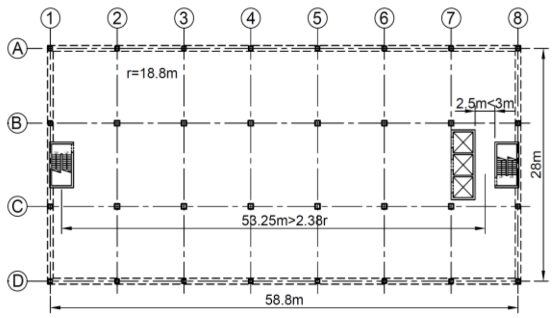
(a) One core wall system



(b) Two core wall systems



(c) Three core wall systems



(d) Three core wall systems to be classified into two core wall systems

Figure 21 Identification of the value of b_r for the buildings with three, two, and one core wall systems

6 CONCLUDING REMARKS

This paper presents a study of multi-storey buildings to investigate the effects of core wall systems and their parameters on the torsional stability of structures. The torsional stability is represented by b_r value, which indicates the spread of lateral load resisting elements in relation to the spread of mass of the floor plans of the building. The building is considered torsionally stable when the value of b_r is larger than 1.1.

This paper presents a method to visually determine if a multi-storey building is torsionally stable. This proposed method is a supplement to the three-tiered approach to estimate the maximum edge displacement demand of asymmetrical RC buildings previously introduced by the authors.

The proposed method introduced includes the assessment of the torsional stability for both principal directions of the RC building with one, two, and three core wall systems. The torsional stability depends on the distance between the core wall systems, the orientation of the core wall systems and the mass-radius gyration of the building.

The limits of the parameters defined for the proposed method have been validated by three-dimensional analyses of multi-storey buildings. The validation demonstrates the robustness of the proposed method.

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8 APPENDIX

8.1 Appendix A

For completeness, this section presents details of how the separation distance ratio (d_r) for two core walls RC building with uni-axial asymmetry as presented in the earlier section 3.2 of the paper were derived.

$$b^2 = \frac{K_\theta}{K_y} = \frac{K_{y,1}(x_1-e_x)^2+K_{y,2}(x_2-e_x)^2}{K_y} \tag{A1}$$

$$e_x = \frac{\Sigma(K_{y,i}x_i)}{K_y} = \frac{K_{y,1}x_1+K_{y,2}x_2}{K_y} \tag{A2}$$

$$K_y = K_{y,1} + K_{y,2} \tag{A3}$$

$$b^2 = \frac{K_{y,1}\left(x_1-\frac{K_{y,1}}{K_y}x_1-\frac{K_{y,2}}{K_y}x_2\right)^2+K_{y,2}\left(x_2-\frac{K_{y,1}}{K_y}x_1-\frac{K_{y,2}}{K_y}x_2\right)^2}{K_y} \tag{A4}$$

$$x_1 - \frac{K_{y,1}}{K_y}x_1 - \frac{K_{y,2}}{K_y}x_2 = \frac{x_1(K_{y,1}+K_{y,2})}{K_y} - \frac{K_{y,1}}{K_y}x_1 - \frac{K_{y,2}}{K_y}x_2 = \frac{K_{y,2}}{K_y}(x_1-x_2) \tag{A5}$$

$$x_2 - \frac{K_{y,1}}{K_y}x_1 - \frac{K_{y,2}}{K_y}x_2 = \frac{x_2(K_{y,1}+K_{y,2})}{K_y} - \frac{K_{y,1}}{K_y}x_1 - \frac{K_{y,2}}{K_y}x_2 = \frac{K_{y,1}}{K_y}(x_2-x_1) \tag{A6}$$

Let $d = |x_2 - x_1|$ and $d_r = \frac{d}{r}$

$$b^2 = \frac{K_{y,1}\left(\frac{K_{y,2}}{K_y}(x_1-x_2)\right)^2+K_{y,2}\left(\frac{K_{y,1}}{K_y}(x_2-x_1)\right)^2}{K_y} = \frac{K_{y,1}K_{y,2}}{{K_y}^2}d^2 \tag{A7}$$

$$b_r = \frac{b}{r} = \frac{\sqrt{\frac{K_{y,1}K_{y,2}}{{K_y}^2}d^2}}{r} = \sqrt{\frac{K_{y,1}K_{y,2}}{{K_y}^2}}\,d_r \geq 1.1 \tag{A8}$$

Let $m = \frac{K_{y,1}}{K_{y,2}}$

$$d_r \geq 1.1\sqrt{\frac{{K_y}^2}{K_{y,1}K_{y,2}}} = 1.1\sqrt{\frac{(K_{y,1}+K_{y,2})^2}{K_{y,1}K_{y,2}}} = \frac{1.1\times(m+1)}{\sqrt{m}} \tag{A9}$$

8.2 Appendix B

For completeness, this section presents details of how the separation distance ratio (d_r) for three core walls RC building with uni-axial asymmetry as presented in the earlier section 3.3 of the paper were derived.

$$x_3 = e_x \textit{ due to central core located at CR} \tag{B1}$$

$$K_y = K_{y,1} + K_{y,2} + K_{y,3} \tag{B2}$$

$$b^2 = \frac{K_\theta}{K_y} = \frac{K_{y,1}(x_1-e_x)^2+K_{y,2}(x_2-e_x)^2+K_{y,3}(x_3-e_x)^2}{K_y} = \frac{K_{y,1}(x_1-e_x)^2+K_{y,2}(x_2-e_x)^2}{K_y} \tag{B3}$$

$$e_x = \frac{\Sigma(K_{y,i}x_i)}{K_y} = \frac{K_{y,1}x_1+K_{y,2}x_2+K_{y,3}x_3}{K_y} = \frac{K_{y,1}x_1+K_{y,2}x_2+K_{y,3}e_x}{K_y} \tag{B4}$$

$$e_x = \frac{K_{y,1}x_1+K_{y,2}x_2+K_{y,3}e_3}{K_{y,1}+K_{y,2}+K_{y,3}} \tag{B5}$$

$$e_x(K_{y,1}+K_{y,2}+K_{y,3}) = K_{y,1}x_1 + K_{y,2}x_2 + K_{y,3}e_x \tag{B6}$$

$$e_x = \frac{K_{y,1}x_1 + K_{y,2}x_2}{K_{y,1} + K_{y,2}} \quad \text{(B7)}$$

$$K_\theta = K_{y,1}\left(x_1 - \frac{x_1K_{y,1} + x_2K_{y,2}}{K_{y,1} + K_{y,2}}\right)^2 + K_{y,2}\left(x_2 - \frac{x_1K_{y,1} + x_2K_{y,2}}{K_{y,1} + K_{y,2}}\right)^2 = \frac{(x_2 - x_1)^2 K_{y,1}K_{y,2}}{K_{y,1} + K_{y,2}} \quad \text{(B8)}$$

Let $d = |x_2 - x_1|$ which is the distance between the centroid of the outer two cores

$$b^2 = \frac{K_\theta}{K_y} = \frac{\frac{(x_2 - x_1)^2 K_{y,1}K_{y,2}}{K_{y,1} + K_{y,2}}}{K_{y,1} + K_{y,2} + K_{y,3}} = \frac{d^2 K_{y,1}K_{y,2}}{(K_{y,1} + K_{y,2})(K_{y,1} + K_{y,2} + K_{y,3})} \quad \text{(B9)}$$

$$b = \sqrt{\frac{K_{y,1}K_{y,2}}{(K_{y,1} + K_{y,2})(K_{y,1} + K_{y,2} + K_{y,3})}}\, d \quad \text{(B10)}$$

$$b_r = \sqrt{\frac{K_{y,1}K_{y,2}}{(K_{y,1} + K_{y,2})(K_{y,1} + K_{y,2} + K_{y,3})}}\, \frac{d}{r} \geq 1.1 \quad \text{(B11)}$$

$$d_r = \frac{d}{r} \geq 1.1\sqrt{\frac{(K_{y,1} + K_{y,2})(K_{y,1} + K_{y,2} + K_{y,3})}{K_{y,1}K_{y,2}}} \quad \text{(B12)}$$

Assume $K_{y,1} = K_{y,2} = K\ for\ two\ edge\ cores, K_3 = xK_{y,1\ or\ 2} = xK$

$$d_r \geq 1.1\sqrt{\frac{(K+K)(K+K+xK)}{K \times K}} = 1.1\sqrt{2x+4} \quad \text{(B13)}$$