# Predicting Maximum Displacement Demand of Asymmetric Reinforced Concrete Buildings

E. Lumantarna[1,3], N. Lam[1,3] and J. Wilson[2,3]

[1] Department of Infrastructure Engineering, The University of Melbourne, Parkville, Victoria, Australia

[2] Swinburne University of Technology, Sarawak Campus, Kuching, Sarawak, Malaysia

[3] Bushfire and Natural Hazards Cooperative Research Centre (BNHCRC)

**ABSTRACT:**

Reinforced concrete buildings make up the majority of Australian building stocks. The buildings generally consist of core walls and/or shear walls as lateral load carrying elements and moment resisting frames as gravitational load carrying elements. The core and/or shear walls are often eccentrically located in the buildings resulting in a large displacement demand on the moment resisting frames that are located at the edge of the buildings.

Seismic assessment methods for asymmetrical buildings commonly involve three-dimensional dynamic analyses that can be computationally expensive. This paper presents a simplified analysis method that has been developed to provide estimates for the maximum displacement demand of multi-storey buildings featuring plan asymmetry. The studies form a part of a collaborative research under the Bushfire and Natural Hazards Cooperative Research Centre (BNHCRC) on “cost-effective mitigation strategy development for building related earthquake risk”.

Keywords: torsion, asymmetrical buildings, simplified analysis method, reinforced concrete

## 1 Introduction

Many reinforced concrete buildings in regions of low to moderate seismicity such as Australia feature plan irregularities. The plan irregularities are commonly caused by the lateral load resisting elements such as cores/shear walls that are eccentrically located in the buildings. The eccentricity results in the buildings to translate and rotate causing amplification of displacement demand at the edges of the buildings.

The torsional response behaviour of asymmetrical buildings have been extensively researched in the past decades. The most comprehensive and detailed review conducted by Anagnostopoulos et al. (2015) revealed that around 700 articles have been published on this topic. Many of the studies focused on ensuring the displacement demands on the structural elements within the buildings under dynamic conditions can be simulated by static method (e.g., Dempsey and Tso, 1982; Chandler and Hutchinson, 1988; Rutenberg and Pekau, 1987; Chopra and Goel, 1991; Tso and Zhu, 1992; Chandler and Duan, 1997). These studies often reached contradictory findings making it difficult to withdraw definite conclusions to guide design. The contradictory findings were caused by the differences in the modelling approach, assumptions made and parameters used in defining torsional behaviour.

Contemporary seismic design and assessment guidelines (e.g., Eurocode 8 (EN 1998-1 2004); FEMA 450-1 (Building Seismic Safety Council 2003); FEMA 356 (ASCE 2000)) generally require three-dimensional dynamic analysis to be performed. Three-dimensional dynamic analysis is complex and the process often becomes a "black box" in structural design.

A simplified method has been introduced by the authors in recent years to allow designers to independently evaluate results from the dynamic analysis of a structure. The method referred to as Generalised Force Method (GFM) of analysis was first introduced as a static analysis procedure to provide estimates of maximum displacement in regular low-, or medium-rise, buildings (Lam et al., 2016). The method has been extended to the analysis of high-rise buildings incorporating higher mode effects (Lumantarna et al., 2017) and asymmetrical buildings with uni-axial (Lam et al., 2016) and bi-axial asymmetry (Lumantarna et al., 2018). In this paper, the GFM for asymmetrical buildings were further simplified by defining the peak displacement demands that can be imposed on structural elements at the edges of the buildings. The expressions that have been previously derived are summarised in Section 2. Section 3 presents a study to establish a range of building dimensions and torsional parameters that define the torsional response behaviour of asymmetrical buildings. Parametric studies were undertaken to investigate the effects of the torsional parameters on maximum displacement demands of asymmetrical buildings with uni-axial and bi-axial asymmetry (Section 3). The peak edge displacement ratios established from the parametric studies were evaluated by comparison with results of dynamic analyses of multi-storey buildings (Section 4).

## 2 Expressions defining maximum edge displacement ratio of asymmetrical buildings

Buildings with the center of rigidity that are offset from the center of mass will translate and rotate causing amplification of displacement demand at the edges of the buildings. This section presents expressions that have been derived by the authors to provide estimates of the amplification of displacement demands of asymmetrical buildings. The details of the derivation can be found in Lam et al. (2016) and Lumantarna et al. (2018).

The amplification of displacement demand $\Delta/\Delta_0$ of buildings can be defined based on a single-storey building idealisation by solving the dynamic equations of equilibrium. The $\Delta/\Delta_0$ is expressed by Equation (1) for the acceleration, velocity and displacement controlled conditions (as presented schematically in Fig. 1).

$$\frac{\Delta}{\Delta_0} = \sqrt{\sum_{j=1}^{n} \left[ \left(1 + \theta_j(\pm B_r)\right) PF_j \times \frac{1}{\lambda_j^2} \right]^2} \tag{1a}$$

$$\frac{\Delta}{\Delta_0} = \sqrt{\sum_{j=1}^{n} \left[ \left(1 + \theta_j(\pm B_r)\right) PF_j \times \frac{1}{\lambda_j} \right]^2} \tag{1b}$$

$$\frac{\Delta}{\Delta_0} = \sqrt{\sum_{j=1}^{n} \left[ \left(1 + \theta_j(\pm B_r)\right) PF_j \right]^2} \tag{1c}$$

where, n is the total number degree of freedoms. Buildings with uni-axial asymmetry have floor plans with the center of rigidity (CR) that is offset from the center of mass (CM) along one axis only ($e_{xr}$), as shown in Figure 2a. A uni-axial asymmetric building model has two degree of freedoms (2DOFs, n = 2): translation in the direction of motion $y$ and rotation $\theta$. Buildings with bi-axial asymmetry have their lateral load resisting elements located such that eccentricities occur along both orthogonal axes ($e_{xr}$ and $e_{yr}$), as shown in Figure 2b. A bi-axial asymmetric building model possesses three degree of freedoms (3DOFs, n= 3): translation in the direction which is orthogonal to the direction of motion $x$, translation in the direction of motion $y$, and rotation $\theta$ (Fig. 1b). A uni-axial ground motion has been assumed to act along the stronger direction of the bi-axial building model $y$ (as indicated in Fig. 2b). $B_r$ is the distance from the CM to the edge of the building, normalised with respect to radius of gyration $r$. The flexible edge of the building is defined as the edge that is furthest from the building CR whereas the stiff edge is the edge that is the closest to the CR of the building. $\theta_j$ is the rotational component of the eigenvector solutions to the dynamic equations of equilibrium defined by:

$$\theta_j = \frac{\lambda_j^2 - 1}{e_{xr}} \tag{2}$$

where, $\lambda_j^2$ are the eigenvalue solutions. For buildings with uni-axial asymmetry, the eigenvalue solutions are given by:

$$\lambda_j^2 = \frac{1+(b_r^2+e_{xr}^2)}{2} \pm \sqrt{\left[\frac{1-(b_r^2+e_{xr}^2)}{2}\right]^2 + e_{xr}^2} \tag{3a}$$

For buildings with bi-axial asymmetry, the eigenvalue solutions can be found by solving equation 3(b):

$$\det \begin{vmatrix} a-\lambda_j^2 & 0 & ae_{yr} \\ 0 & 1-\lambda_j^2 & e_{xr} \\ ae_{yr} & e_{xr} & \left(ae_{yr}^2+e_{xr}^2+b_r^2\right)-\lambda_j^2 \end{vmatrix} = 0 \tag{3b}$$

where, $e_{xr}$ is the eccentricity along the x-axis direction (perpendicular to the direction of the ground motion), normalised with respect to $r$, $e_{yr}$ is the eccentricity along the y-axis direction (parallel to the direction of the ground motion), $a$ is the ratio of the translational stiffness in the x-direction to the translational stiffness in the y-direction ($a = \frac{K_x}{K_y}$), $b_r = {}^1/_r\sqrt{{}^{K_\theta}/_{K_y}}$, is a parameter defining the torsional rigidity of the buildings, where, $K_\theta$ is the torsional stiffness of the buildings.

$PF_j$ in equation (1) is the participation factor for mode $j$, which is defined as:

$$PF_j == \frac{1}{1+\left(\frac{\lambda_j^2-1}{e_{xr}}\right)^2} \tag{4a}$$

for buildings with uni-axial asymmetry, and:

$$PF_j = \frac{1}{1+\left(\frac{e_{yr}}{e_{xr}}\left(\frac{a-a\lambda_j^2}{a-\lambda_j^2}\right)\right)^2+\theta_j^2} \tag{4b}$$

for a buildings with bi-axial asymmetry.

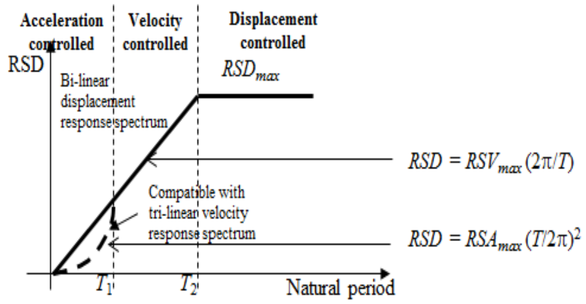
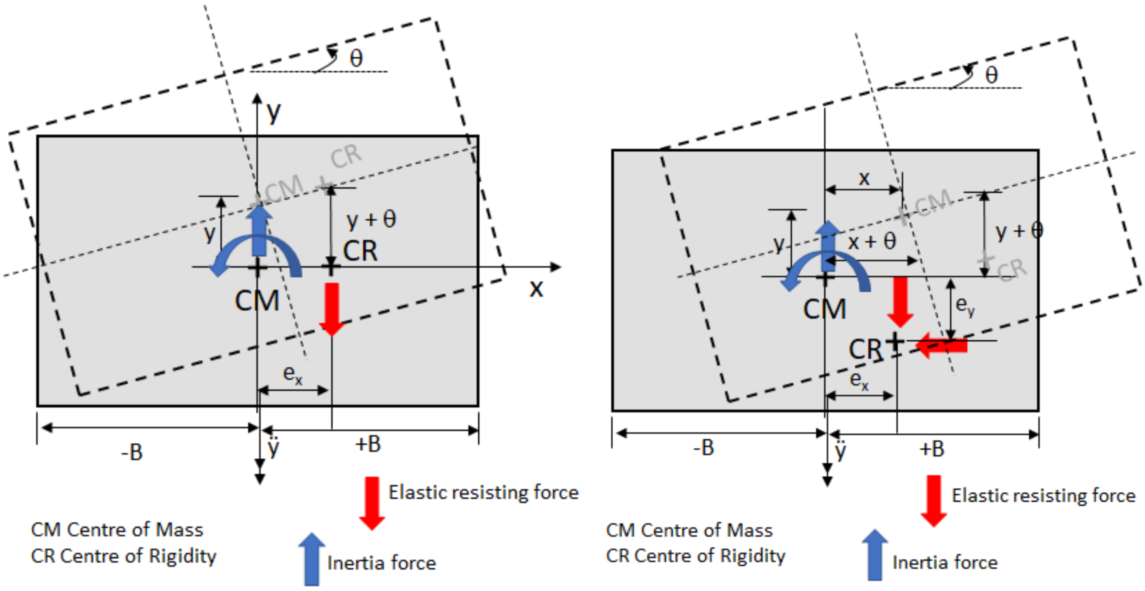



Figure 1 Displacement response spectrum showing acceleration-, velocity- and displacement-controlled regions




(a) uni-axial asymmetry (b) bi-axial asymmetry

Figure 2 Single-storey building models

Parametric studies have been undertaken by the authors on buildings with uni-axial asymmetry (Lam et al., 2016) and buildings with bi-axial asymmetry (Lumantarna et al., 2018). It was shown that asymmetrical buildings with $b_r \leq 1.0$ results in high values of $\Delta/\Delta_0$. The use of such torsionally flexible buildings ($b_r \leq 1.0$) is discouraged in practice.

## 3 Values of $e_r$ and $b_r$ in multi-storey buildings

To obtain a realistic range for $e_r$ and $b_r$ values in multi-storey buildings, a desktop study using Google Earth (https://earth.google.com/web/) were conducted to collate a range of dimensions (widths and lengths) of buildings in Melbourne CBD. The width (taken as the longest dimension) of each building is presented with respect to its aspect ratio (width/length) in Figure 3. It is shown that the width of the buildings typically ranges between 10 to 50 m. The aspect ratio ranges from 1 to 4.

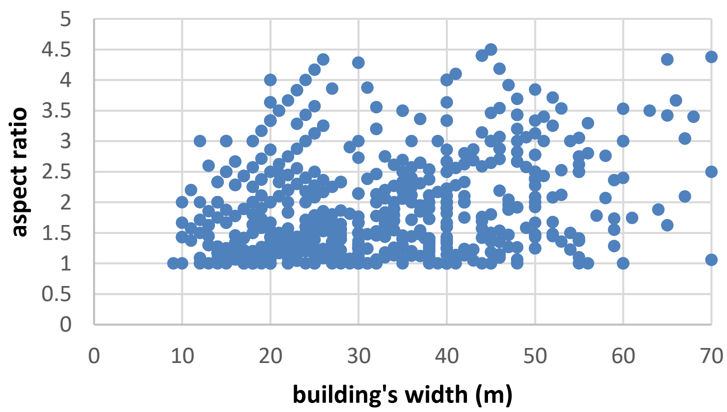
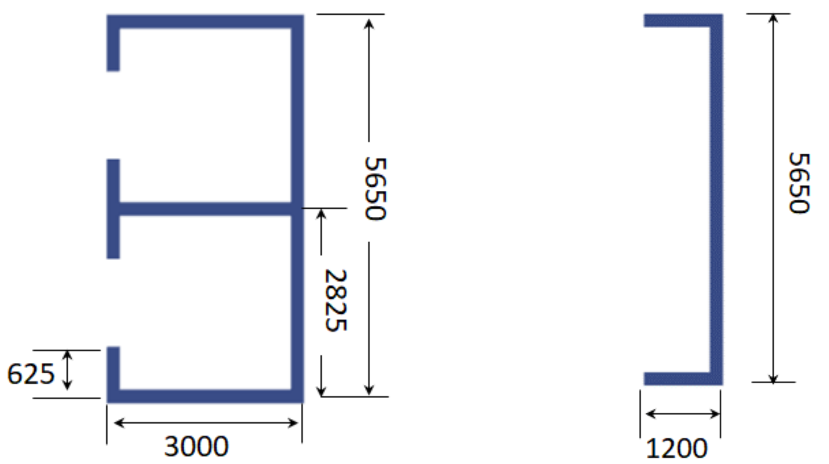



Figure 3 Dimensions of buildings in Melbourne CBD

A parametric study was undertaken based on linear elastic analyses to obtain ranges of values for the parameters defining the torsional behaviour of buildings, $e_r$, $b_r$ and $B_r$. Building models with varying core and shear wall layouts were investigated. The width of the buildings was varied from 10 to 50 m, the aspect ratio of the buildings was varied from 1 to 4. The location of the centre of mass CM was kept constant at the centre of the building. This results in the values of $B_r$ that range between 1.2 and 1.65.

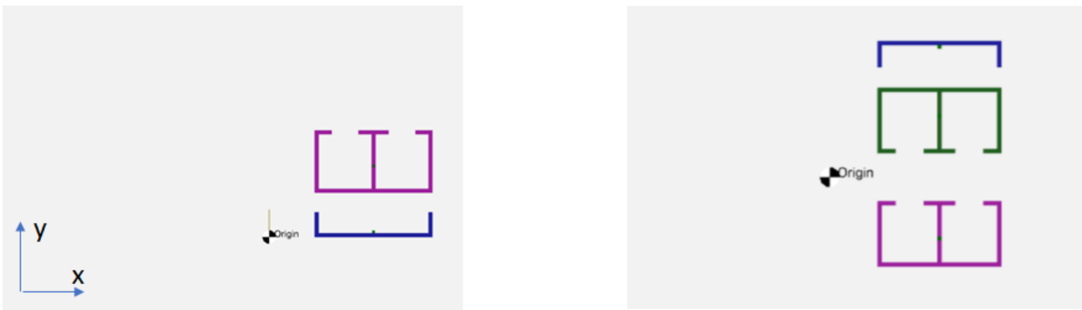
The core and shear wall dimensions are shown in Figure 4 and are based on the minimum requirements for emergency stairs and lifts as prescribed in the National Construction Codes (NCC, 2016). The number of lifts varies depending on the dimensions of the building, hence the length of the cores was varied in the study. The layouts for the building cores and shear walls investigated are presented in Figure 5. For each layout, the position of the cores was shifted along the horizontal axis whilst the position of the shear walls was fixed at the perimeter of the buildings as shown in Figure 5.




**Dimensions are in mm**
**All wall thickness is 200 mm**

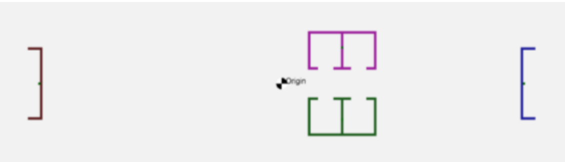
(a) core for lifts (b) shear wall for emergency stairs

Figure 4 Core and shear wall dimensions



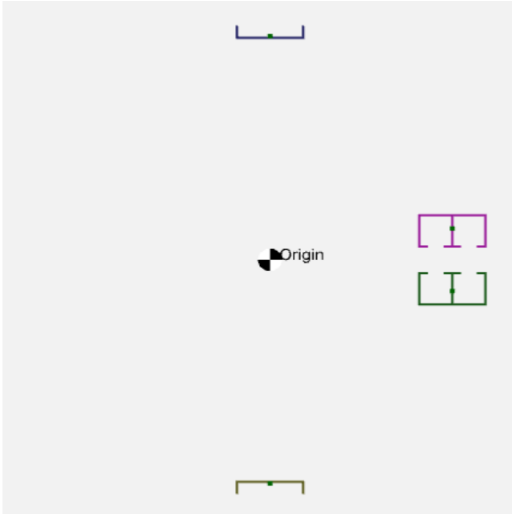

$b_r = 0.3\ to\ 0.55$
(a) for width 20 m

$b_r = 0.5\ to\ 0.8$
(b) for width 20 m and 30 m



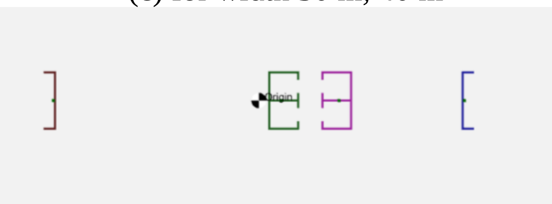
$b_r = 1.0\ to\ 1.6$

(c) for width 30 m, 40 m



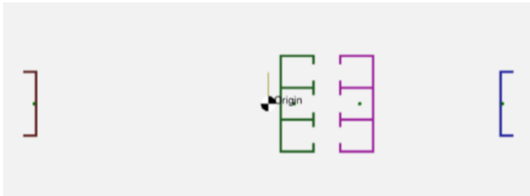

$b_r = 1.2\ to\ 1.4$

(d) for width 40 m



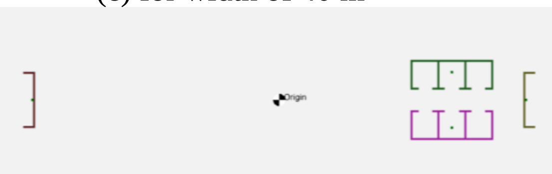
$b_r = 0.7\ to\ 0.9$

(e) for width of 40 m



$b_r = 0.5\ to\ 0.7$

(f) for width 40 m



$b_r = 1.0\ to\ 1.6$

(g) for width of 40, 50 m

$b_r = 1.0\ to\ 1.4$

(h) for width of 50 m

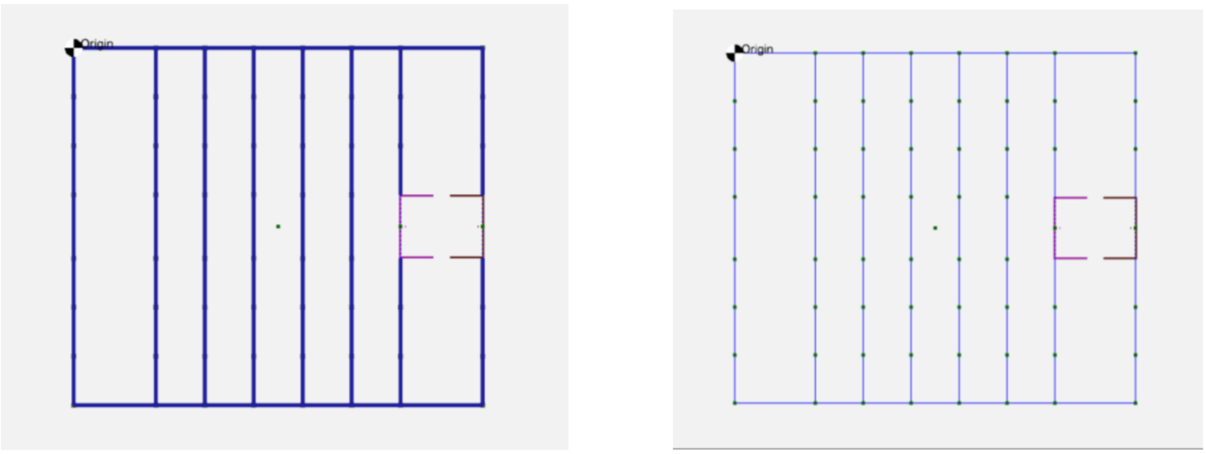
Figure 5 Layout of building cores and shear walls

Linear analyses were performed using program SPACE GASS). The building models were subjected to an arbitrary load at the center of mass in the direction of y-axis (as shown in Fig. 5a) to obtain the translational and rotational displacements of the buildings. The displacement values were consequently used to determine the eccentricity, lateral stiffness and torsional stiffness of the buildings. The values of $b_r$ for each building models were then calculated from the lateral and torsional stiffness by $b_r = {}^{1}\!/_{r}\sqrt{{}^{K_\theta}\!/_{K_y}}$. The ranges of $b_r$ values are included in Figure 5. From the analyses, it was found that buildings with closely spaced cores (Figs. 5a and 5b) have $b_r$ values that are less than 1.0. Additional cores or shear walls at the perimeter of the buildings (Figs. 5c, 5g, 5h) were found to increase the torsional stiffness of the building and generally increase the value of $b_r$ to greater than 1.0. The addition of perpendicular elements was also found to increase the value of $b_r$ to greater than 1.0. However when the cores are orientated such that their major axis is perpendicular to the direction of earthquake ground motion (Figs. 5e and 5f), the buildings' $b_r$ reduces to a value that is lower than 1.0. The observed reduction in $b_r$ is caused by the increase in lateral stiffness of the buildings that negate the beneficial effect of the orientation of the cores on the torsional stiffness of the buildings.

Existing multi-storey reinforced concrete buildings in Australia are often supported by a combination of moment resisting frames and cores/shear walls. It is a common practice in

Australia to ignore the contribution of the moment resisting frames to the lateral strength and stiffness of the buildings and design them as gravity load carrying elements. Building models combining shear cores and moment resisting frames were created to investigate the contribution of the moment resisting frames to the torsional stiffness of the buildings. For the investigated building model shown in Figure 6, the $b_r$ value was found to increase from 0.2 to 0.8 when the moment resisting frames parallel to the direction of the ground motion are incorporated in the analyses.

The eccentricity values of the building models with $b_r > 1.0$ (Figs. 5c, 5d, 5g and 5h) are plotted against the buildings' width in Figure 7. Figure 7 shows that despite a significant range of $e_r$ values, the values of $e_r$ is generally lower than 0.7. The range of $e_r$ and $b_r$ values obtained from the study is used as the basis of parametric studies presented in Section 4.



(a) with moment resisting frames (b) without moment resisting frames[1]

[1]the cross-sectional area of the beams and columns making up the moment resisting frames were reduced to almost zero

Figure 6 Building models – effects of moment resisting frames

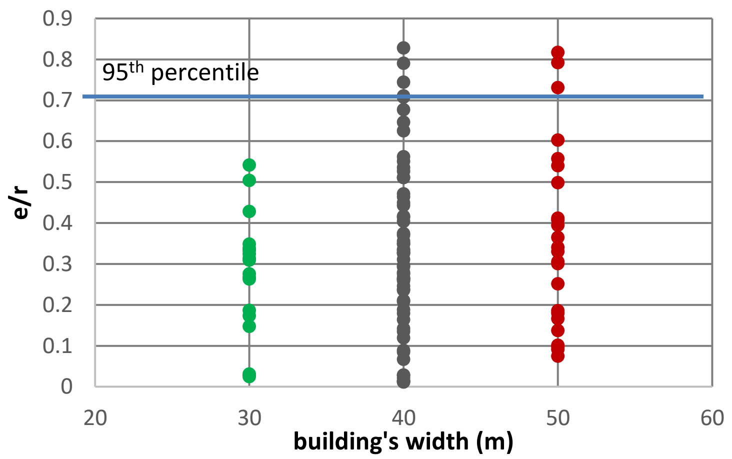
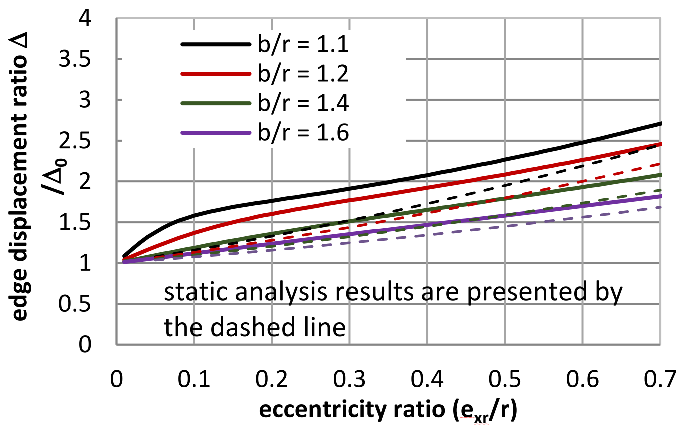



Figure 7 Values of eccentricity for buildings with $b_r > 1.0$

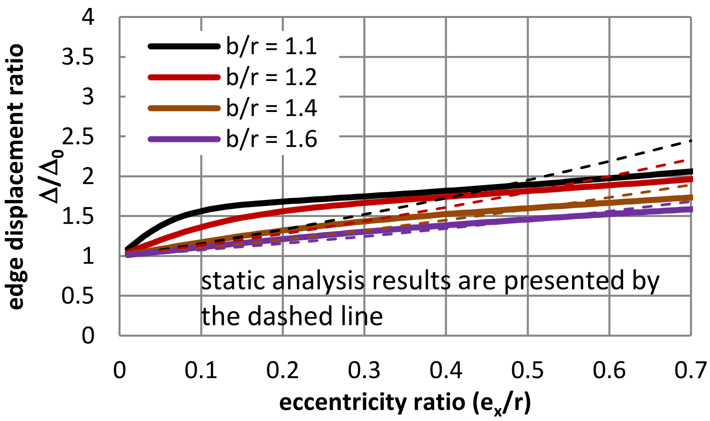
## 4 Maximum displacement demand of buildings with asymmetry

Equations (1), (2), (3a) and (4a) were used to calculate the edge displacement ratio $\Delta/\Delta_0$ for buildings with uni-axial eccentricity. The values of $e_{xr}$ (the eccentricity along the axis perpendicular to the direction of ground motion) were varied from 0 to 0.7 based on the findings from the parametric study presented in Section 3. The values of $b_r$ were varied from 1.1 to 1.6 as buildings with $b_r$ values lower than 1.0 are discouraged in practice. The value of $B_r$ was kept constant at 1.8 representing a plan with a high aspect ratio. The edge displacement ratio $\Delta/\Delta_0$ are presented along with the $\Delta/\Delta_0$ based on static analysis (presented by dashed lines) in Figure 8 for the acceleration-, velocity- and displacement-controlled region. Equation (1) provides estimates of the displacement ratios for stiff and flexible edges. However, only the maximum displacement ratio for the flexible edge (as it is higher than the displacement ratio for the stiff edge) is presented.

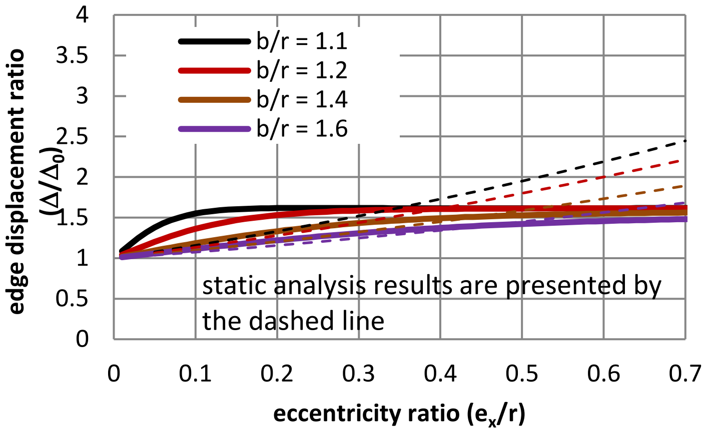
It is shown that the maximum displacement of asymmetrical buildings under dynamic conditions are sensitive to both $b_r$ and $e_{xr}$ values when the buildings are in the acceleration-controlled region. However, the maximum displacement is less sensitive to the changes in $e_{xr}$ for buildings in the velocity-controlled region and even less so for the buildings in the displacement-controlled region. This is in contrast to the trend of maximum displacement for buildings under static loading that continue to increase with the increase in eccentricity. It is shown in Figure 8 that the displacement response behaviour of asymmetrical buildings in velocity- and displacement-controlled regions are more significantly affected by the torsional rigidity of the buildings (represented by the values of $b_r$) than the eccentricity. Importantly, the peak edge displacement ratio can be conservatively defined as 2.7, 2.1 and 1.6 for buildings in acceleration-, velocity- and displacement-controlled regions, respectively. The peak edge displacement ratio can be multiplied with the maximum displacement demands of the equivalent two-dimensional buildings to obtain the maximum displacement demands at the edges of asymmetrical buildings.




(a) Acceleration-controlled region




(b) Velocity-controlled region

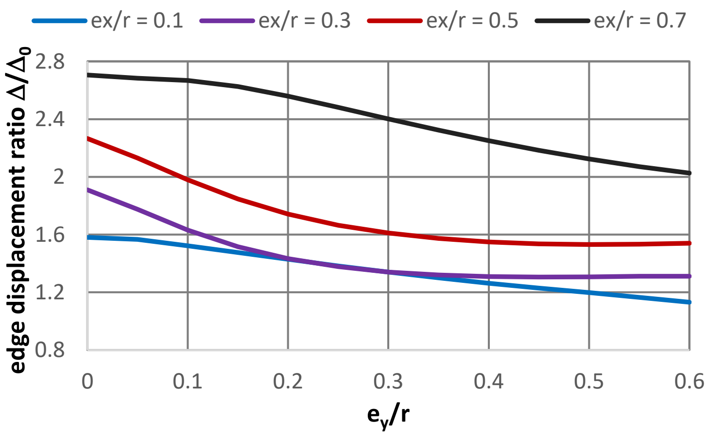



(c) Displacement-controlled region

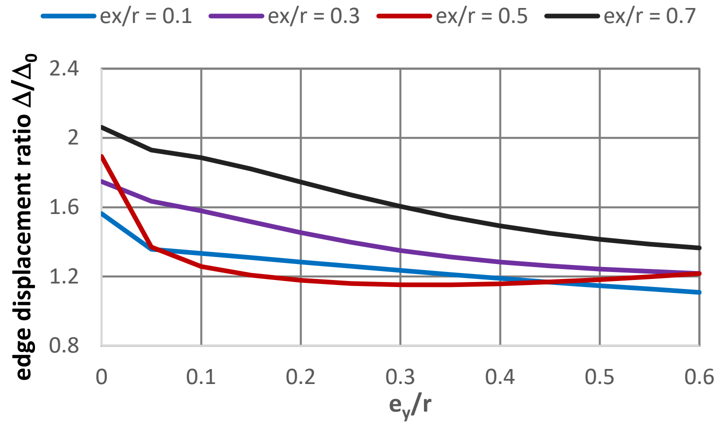
Figure 8 Edge displacement ratios $\Delta/\Delta_0$ for building models with uni-axial asymmetry

To investigate the effect of torsional parameters on the displacement response behaviour of asymmetrical buildings with bi-axial asymmetry. Equations (1), (2), (3b) and (4b) were used to obtain the edge displacement ratios $\Delta/\Delta_0$. In view of the aspect ratio of the buildings shown in Figure 3, the eccentricity along the axis parallel to the direction of ground motion ($e_{yr}$) were varied from 0.1 to 0.6. The value of $B_r$ was kept constant at 1.8. The value of $a$ was varied from 0.5 to 2. Figures 9 and 10 present the edge displacement ratio of buildings $\Delta/\Delta_0$ plotted against the eccentricity ratio along the axis parallel to the direction of ground motions ($e_{yr}$). Hence the values intersecting the y-axis (for $e_{yr}$= 0) represents buildings with uni-axial asymmetry. Only results for $b_r$of 1.1 and $a$ of 0.75 and 1.5 are presented in Figures 9 and 10. However similar trends were observed with building models with different values of $b_r$ and $a$.

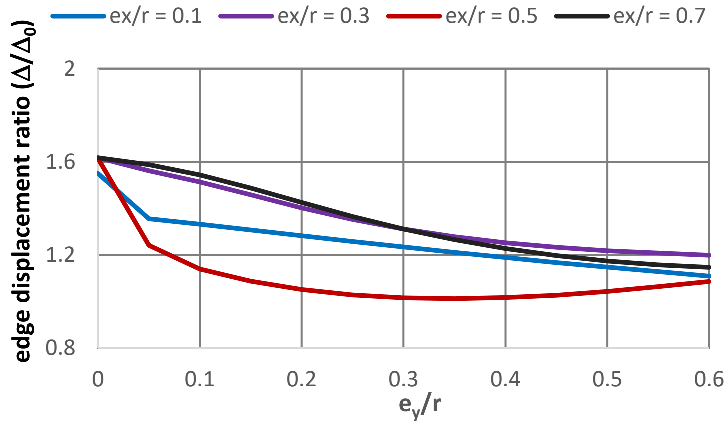
It is shown that the maximum displacement of buildings with bi-axial asymmetry are generally more sensitive to the values of eccentricity along the x- and y-axis for buildings in the acceleration regions. However, the asymmetrical building models with uni-axial asymmetry have been found to provide conservative estimates of the displacement demands of asymmetrical buildings. The peak edge displacement ratio previously defined for the uni-axial building models (2.7, 2.1 and 1.6 for buildings in acceleration-, velocity- and displacement-controlled regions, respectively) can also provide conservative estimates of asymmetrical buildings with bi-axial asymmetry.




(a) Acceleration-controlled region

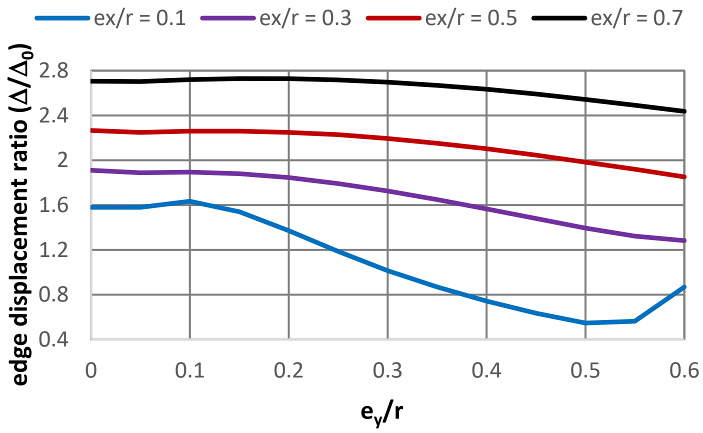



(b) Velocity-controlled region

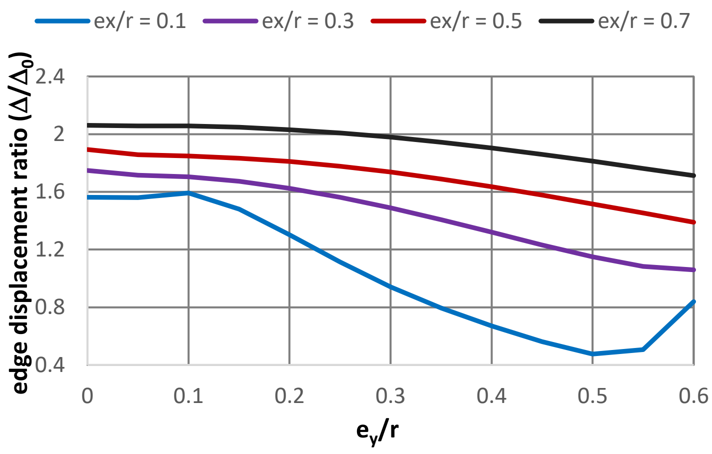



(c) Displacement-controlled region

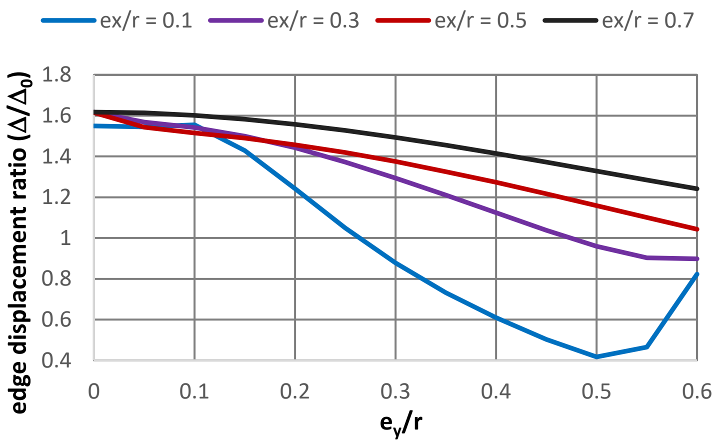
Figure 9 Displacement amplification factor $\Delta/\Delta_0$ for building models with bi-axial asymmetry, $b_r = 1.1$, $a = 0.75$




(a) Acceleration-controlled region




(b) Velocity-controlled region




(c) Displacement-controlled region

Figure 10 Displacement amplification factor $\Delta/\Delta_0$ for building models with bi-axial asymmetry, $b_r = 1.1$, $a = 1.5$

## 5 Comparison with dynamic analysis of multi-storey buildings

This section presents results from dynamic analyses of multi-storey buildings conducted by Master of Engineering students in the University of Melbourne as a part of their capstone research projects. The values of edge displacement ratio from dynamic analyses of the multi-storey buildings are compared with the maximum displacement ratios defined in Section 4.

Dynamic response spectrum analyses have been conducted on multi-storey building models. The plan view of the building models are presented in Figure 11. The plan views are presented only to represent the variability in the layouts, dimensions and the positions of the cores within the buildings. Hence details of the structural elements and their material properties have been omitted. Building model 1 is 6-storey high and building models 2 to 4 are 8-storey high, resulting in all buildings being in the velocity-controlled region. The contribution from the moment resisting frames has been included in the analyses. All buildings have $b_r$ value that is larger than 1.0.

The edge displacement ratio $\Delta/\Delta_0$ was obtained from dynamic analyses by taking the ratio between the maximum edge displacement demand and the maximum displacement demand of the corresponding two-dimensional building models. The values of $\Delta/\Delta_0$ are presented in

Figure 12 and compared with the peak $\Delta/\Delta_0$ for the velocity-controlled region. The peak $\Delta/\Delta_0$ of 2.1 for the displacement-controlled region is shown to provide conservative estimates of the maximum displacement demand of the multi-storey asymmetrical buildings.

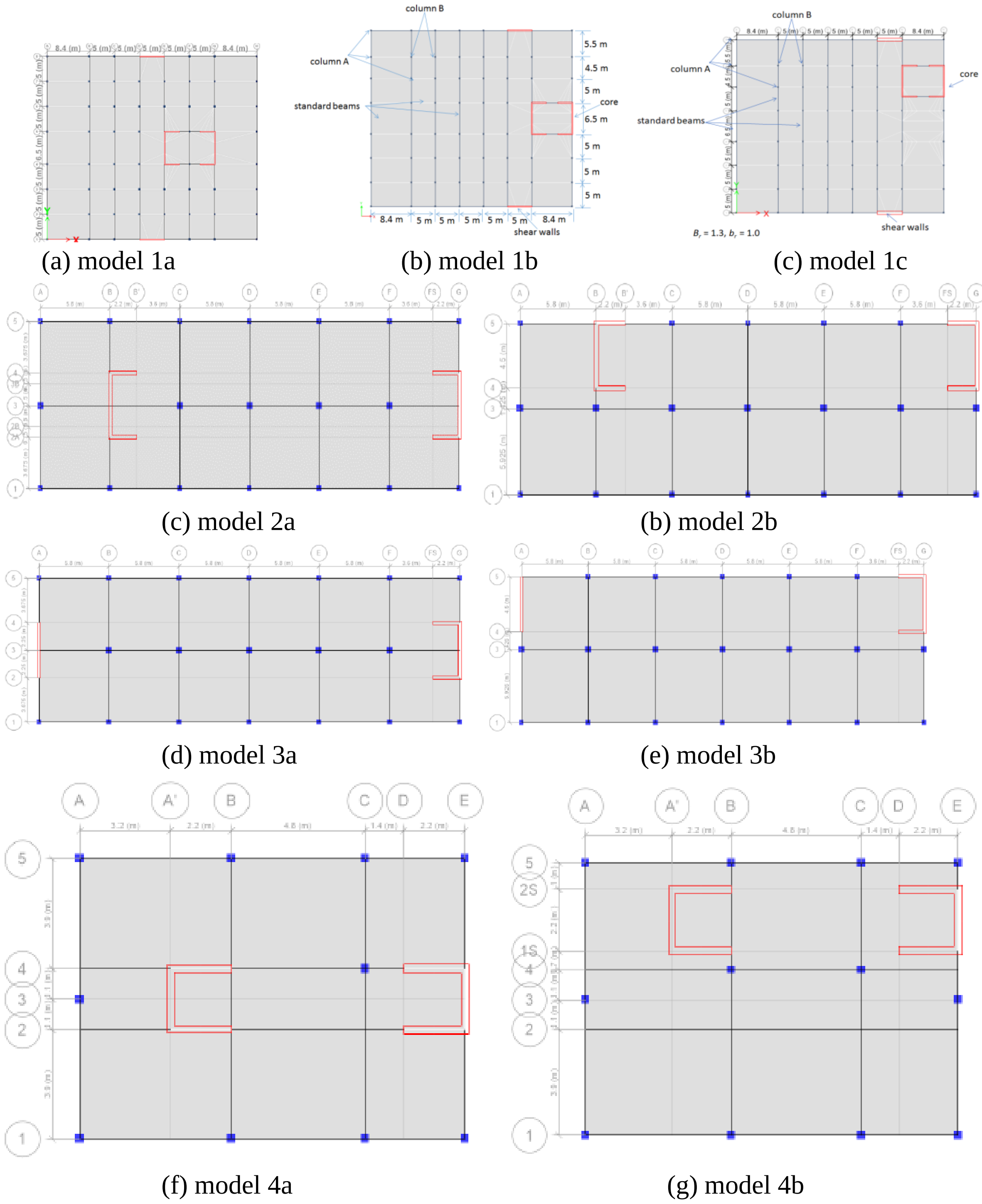



Figure 11 Layouts of building models subjected to dynamic analyses

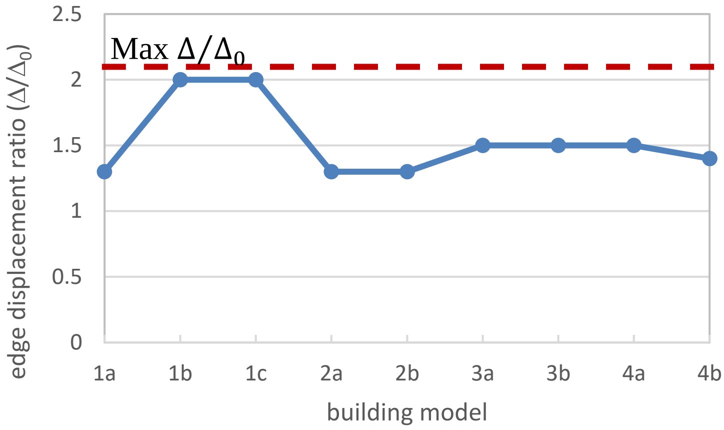



Figure 12 Edge displacement ratio from dynamic analyses

## 6 Concluding remarks

Reinforced concrete buildings in Australia are commonly laterally supported by cores/shear walls that are positioned such that the center of rigidity is offset from the center of mass of the buildings. Such asymmetrical buildings will be subjected to translational as well as rotational displacements under earthquake ground motions resulting in an amplification of displacement demand at the edges of the buildings.

A simplified method referred to Generalised Force Method of Analysis has been introduced by the authors to obtain estimates of the maximum displacement demands of asymmetrical buildings. The expressions developed to estimate the edge displacement ratio for buildings with uni-axial and bi-axial asymmetry are presented in this paper. The developed expressions were further simplified by defining the peak edge displacement ratios that can provide conservative estimates of the peak displacement demands of asymmetrical buildings. To do this, typical building dimensions were first collated to obtain realistic ranges of parameters which define the torsional response behaviour such as eccentricity and torsional rigidity. Based on these ranges, parametric studies were conducted on building models with uni-axial and bi-axial asymmetry. Results from the parametric studies show that the peak edge displacement ratio can be defined as 2.7, 2.1 and 1.6 for buildings in the acceleration-, velocity- and displacement-controlled regions, respectively. The robustness of the established edge displacement ratio was demonstrated by comparison with results from dynamic analyses of multi-storey buildings.

### Acknowledgement

The financial contribution of the Commonwealth Australia through the Cooperative Research Centre program is gratefully acknowledged. The analyses of multi-storey buildings were conducted by various Master of Engineering students. Their contribution is gratefully acknowledged.

### References

American Society of Civil Engineers (2000), Prestandard and commentary for the seismic rehabilitation of buildings FEMA 356, Washington, D.C.

Anagnostopoulos, S.A., Kyrkos, M.T. and Stathopoulos, K.G. (2015) “Earthquake induced torsion in buildings: critical review and state of the art”, *Earthquake and Structures*, **8**(2): 305-377.

Building Seismic Safety Council (2003), *NEHRP Recommended Provisions for Seismic Regulations for New Buildings and Other Structures Part I : Provisions (FEMA 450-1),* Washington.

Chandler, A.M., Duan, X.N. (1997), “Performance of asymmetric code-designed buildings for serviceability and ultimate limit states”, *Earthquake Engineering and Structural Dynamics* **26**: 717-735.

Chandler, A.M, Hutchinson, G.L. (1988), "A modified approach to earthquake resistant design of torsionally coupled buildings", *Bulletin of the New Zealand National Society of Earthquake Engineering* **21**: 140-152.

Chopra, A.K., Goel, R.K. (1991), "Evaluation of torsional provisions in seismic code", *Journal of Structural Engineering* **117**: 3762-3782.

Dempsey, K.M., Tso, W.K. (1982), "An alternative path to seismic torsional provisions", *Soil Dynamics and Earthquake Engineering* **1**: 3-10.

EN 1998-1 (2004), *Eurocode 8: Design of structures for earthquake resistance – Part 1: General rules, seismic actions and rules for buildings,* BSI.

Lam, N.T.K., Wilson, J.L. and Lumantarna, E. (2016), "Simplified elastic design checks for torsionally balanced and unbalanced low-medium rise buildings in lower seismicity regions", *Earthquakes and Structures* **11**(5): 741-777.

Lumantarna, E., Lam, N., & Wilson, J. (2018). Methods of analysis for buildings with uni-axial and bi-axial asymmetry in regions of lower seismicity. *Earthquakes and Structures*, **15**(1), 81-95.

Lumantarna, E., Mehdipanah, A., Lam, N., & Wilson, J. (2017), "Methods of structural analysis of buildings in regions of low to moderate seismicity", *The 2017 World Congress on Advances in Structural Engineering and Mechanics (ASEM17)*, Ilsan (Seoul), Korea, 28 August – 1 September .

Rutenberg, A., Pekau O.A. (1987), "Seismic code provisions for asymmetric structures: a re-evaluation", *Engineering Structures* **9**: 255-264.

Standards Australia (2007), AS 1170.4-2007 *Structural Design Actions – Part 4 Earthquake Actions commentary*. Sydney: Standards Australia.

Tso, W.K., Zhu, T.J. (1992), "Design of torsionally unbalanced structural systems based on code provisions I: ductility demand", *Earthquake Engineering and Structural Dynamics* **21**: 609-627.