Seismic source models for building code seismic hazard maps

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Abstract

The seismic hazard map produced for the 2018 update of the building code will include multiple weighted spatial seismic source models as well as fault source model(s). To my knowledge, this will be the first national map, for inclusion in the building code, anywhere in the world to include multiple seismic source models, though at least one consultant has used multiple source models in their hazard models of Australia. It has become standard to capture epistemic uncertainty in ground motion by including multiple GMPEs. Multiple source models extends this to sampling the epistemic uncertainty in the seismicity model. This paper discusses some of the assumptions, implicit and explicit, underlying the models as well as some of their limitations, particularly concerning the accurate estimate of the magnitude frequency distribution for each zone or grid point. Along with an understanding of the role of logic trees in PSHA, this should assist in the development of sound weightings of these alternative models.

Keywords: probabilistic seismic hazard assessment, area source model, logic tree weighting

1 - INTRODUCTION

From a building code perspective we are interested in the expected ground motions in the next 50 years (e.g. 10% in 50 years and 2% in 50 years); though I expect others have a different perspective. For example insurance is perhaps more interested in 0.5% or 0.05% in 1 year (which give return periods of 200 and 2100 years). Owners of a tailing dam of toxic waste might be interested in 0.5% in 100 years (20000 years return) and of a repository for long-lived dangerous waste perhaps 0.5% in 2000 years (400000 years return). Despite these potential other uses, when we are assessing seismic source models for inclusion in the national seismic hazard map, which primarily for the building code, we are most interested in those that are likely to be good estimators of the seismicity over the next 50 years.

The final Seismic Source Characterisation (SSC) model will include multiple weighted spatial source models and at least one fault source model. To rationally weight each model requires the models to be explicitly ranked in order to build the logic tree. In this paper I discuss some of the requirements of the spatial source models to be used in the Australian National Seismic Hazard Assessment 2018 (NSHA18) and the assumption underlying various seismic source models proposed. I will also discuss the requirements for obtaining accurate recurrence statistics and discuss PSHA logic trees.

2 - LOGIC TREES IN PSHA

There have been at least seven opinion papers related to the use of logic trees in PSHA to appear in Earthquake Spectra since 2005. With none since November 2012 the community appears to come to a degree of agreement on what the logic trees are and what traps should be avoided in building them.

One area of agreement is the Scherbaum and Keuhn (2011) proposal that as the weights are treated as probabilities in the PSHA process, they should be treated as probabilities by the hazard analyst; the weights should not represent expressions of relative merit of each alternative. Scherbaum and Kuehn (2011, p. 1238) go on to state that: “We therefore see the branch weights as subjective estimates for the degree-of-certainty or degree-of-belief—expressed within the framework of probability theory—that the corresponding model is the one that should be used.” Musson (2012) offers the view that the weights are the probability of each model being “the best model available” (p. 1293) carefully pointing out that this does not mean the probability of the model being true. Bommer (2012) endorses both definitions though has some concern that the Musson (2012) definition implicitly assumes that we are limited to those models available, so if only one source (or GMPE) model is available then it is acceptable to use just one.

To assist hazard modellers in developing sufficiently but not excessively complex logic trees Bommer (2012) reminded us that “what is being represented in the SSC logic tree is the underlying possible spatial distributions and rates of future earthquakes of different magnitudes”. So the purpose of the logic tree is not to sample the full range of possible methods and models. What matters is the resulting distribution of earthquakes in the PSHA calculation not the number of modelling techniques that are represented in the formulation of the logic tree. A consequence of these ideas and definitions is that “Selecting a model as useful (in the sense that it should be considered at all) should not automatically invite non-zero weight.” (Scherbaum and Keuhn 2011). Bommer (2012) gives the example: “ a decision to include alternative branches for both area source zones and smoothed seismicity in an SSC logic tree should be taken on the basis of uncertainty about the spatial stationarity of seismicity, not simply because there are two different approaches available for modelling seismicity not explicitly associated to fault sources.”.

There is general agreement that while in strict PSHA the logic tree must be mutually exclusive and collectively exhaustive (Bommer and Scherbaum, 2008) in practice the logic trees are rarely collectively exhaustive (and, except perhaps for SSHAC L4 and 5 projects, they do not need to be) and often not strictly mutually exclusive (e.g. GMPEs). In my assessment of the models I am attempting to understand the implicit and explicit assumptions behind each model to assist us in the process of assessing the degree-of-certainty or degree-of-belief that as part of the ensemble of models it reflects the likely future distribution of the seismicity of Australia.

3 - STATISTICAL PROPERTIES OF SEISMICITY

While the actual procedure varies, all PSHA modelling software can be seen as in effect producing a very large synthetic catalogue that is uniformly distributed across each source, in the seismic source model, and follows the magnitude frequency distribution (MFD) given for that source. At each site of interest the value and probability of the ground motion from every earthquake is calculated and used to estimate the hazard at the return period and spectral amplitude of interest (e.g. 475 years & PGA). While the actual methodologies are diverse the final results are very similar. Classical PSHA skips the synthetic catalogue approach and integrates the hazard integral directly, which is computationally much more efficient.

One of the fundamental assumptions in PSHA modelling is that within an area of uniform seismicity earthquakes are Poisson distributed in time and space. Except for sequences of associated events (fore-shocks, after-shocks and swarms) this holds for earthquakes in time and the primary aim of declustering the earthquake catalogue is to remove just enough earthquakes such that the catalogue remains approximately Poissonian in time. Spatially, at least at the scale of most source zones, seismicity is observed to be spatially clustered not randomly distributed (Bender 1984, Ma et al. 2008, Leonard 2012) and requires multiple Poisson models to spatially model it (Leonard 2010). This clustering is not well understood, for example we don't know on what time frames it is stationary. Liu et al. (2011) suggest that in China the distribution is stationary on the scale of several decades but highly variable on the scale of centuries, with the contemporary seismicity being a good predictor of moderate earthquakes but not of the largest earthquakes.

I think most seismologists working on Australian seismicity would agree that the contemporary M3.0-4.0 activity reflects the expected activity of most future M3.0-4.5, perhaps most M3.0-5.5, earthquakes. Kafka et al. (2007) demonstrated that small earthquakes are a good predictor of most moderate earthquakes in central and eastern US as did Williams and Leonard (2001) for Australia. However, history shows that even moderate (M>=4.8) earthquakes do occur in previously seismically quiet areas (e.g. Petermann Ranges 2016, Bowen 2011, Swan Hill 2001) as well as active areas (e.g. Eidsvold 2015, Moe 2012, Katanning 2007, Adelaide 1954) and larger (M>=6.0) often occur in areas that had been previously seismically quiet. (e.g. Tennant Creek 1989, Petermann Ranges 2016).

To capture this variation in earthquake density at a range of scale lengths for the NSHM 2013 we adopted three source models, a Background layer with source zones of ≈1000000 km^2, a regional layer with source zones of ≈10000 km^2 and a hotspot layer with source zones of ≈100 km^2 (and a lower Mmax) and with the final map being the maximum of the three hazards at each point. This lead to a map that did not strictly follow the PSHA methodology and probably overestimated the hazard due to the hotspots for return period maps of more than 20% in 50 years. The NSHA18 will include multiple source models that should capture the epistemic uncertainty inherent in estimating the future distribution of earthquakes. I think we should ensure that the final ensemble of models cover a range of scales so capture any potential differences in the future pattern of large earthquakes verses small earthquakes.

Temporally we should not expect earthquakes to be regular in time but expect periods of high activity and periods of no activity. While this does not feed directly into source zone selection it does affect estimates of the magnitude of completeness (Mcomp) and can lead to inappropriate labelling of some areas as being episodic (i.e. having active and quiescent phases). I suspect, though have not tested, that in all areas of Australia earthquakes are random in time and those areas that appear at first sight to be episodic (e.g. SE Queensland) cannot be excluded from being random at the 95% level.

In typical Australian source zones a decrease in "*b*" of 0.2 results in an almost doubling of the 500 year hazard and an increase of 0.2 an almost halving of the hazard (Leonard et al. 2014). A decrease in *b* of 0.1 will result in a 40% increase in hazard. If we want a confidence level of 95% that we have constrained *b* to within ±0.1 we require the σ to be ≤ 0.05. Theoretically to estimate b-values so $\sigma \leq 0.05$ requires 360 earthquakes using the Maximum Likelihood method, though in practice as catalogues are not ideal the number required is probably more like 500 – see Appendix A for a detailed discussion of the estimation of *b*-values. I suspect, but have not tested, that the *b*-value is uniform over areas of continuous activity (e.g. The SE highlands of Victoria north of -38S and eastern NSW) with the apparent fluctuation in *b* being just random statistical variation.

For Australia the catalogue of declustered earthquakes that includes only earthquakes above the magnitude(s) of completeness (Mcomp) contains about 2800 earthquakes. If only the post 1990 earthquakes, for which magnitudes are more reliable, are used this reduces to 1300 earthquakes. Approximately 1/2 of the earthquakes are in the relatively small areas of dense seismicity and networks (e.g. East Gippsland, Mt Lofty and Flinders Ranges and SW WA).

Outside these areas, constraining the *b* to ±0.1, at the 95% level, will only be possible for a few geographically large areas. Then, assuming we have a well constrained MFD b-value, at least 30 earthquakes are required to robustly estimate the activity rate of an area (i.e. *a*). For much of the continent large spatial areas (e.g. radius of 500 km) are required to obtain 30 earthquakes, with the average for Cratonic and Non Cratonic Australia being 350km and 500km respectively. In areas of denser seismicity 75km is a typical radius required to obtain 30 earthquakes.

It has become common in PSHA to sample the uncertainty in model parameters such as the uncertainty in *a* or *b* or fault slip rate or Mmax (e.g. Petersen et al. 2014). This is considered good practice as it is sampling the epistemic uncertainty within the models. In practice, for most of the key parameters, including uncertainty in the model parameters has very little effect on the hazard. As the hazard monotonically increases with increasing *a*, or decreasing *b* or increasing slip rate, the ± σ hazard values mostly cancel each other out resulting in the final hazard being close to that estimated from just the mean value of the parameter. It does have a dramatic effect on the computational load required to run the models. This emphasises the importance of ensuring our estimates of the key hazard parameters are the most robust we can reasonably achieve. The sensitivity of the hazard to changes in various parameters is discussed in Griffin et al. (2016)

In evaluating individual seismic source models, I suggest, we should consider how it takes into account:

1. The varying (sometimes random sometimes clustered) spatial distribution of earthquakes.
2. The uncertainty in how stationary the contemporary pattern is.
3. The balance between the potentially fine scale variation in seismicity with the need for large numbers of earthquakes to accurately estimate the MFD.
4. The spatially and temporally variable magnitude of completeness of the catalogue.

In evaluating an ensemble of source models we also should consider whether we have a set of models that are diverse and so hopefully capturing the inherently high epistemic uncertainty involved in estimating the seismicity over the next 50 years.

4 - THE MODELS.

4.1 Smoothed seismicity

Smoothed seismicity, essentially takes a catalogue and transfers the earthquake density onto a grid. Originally it used a fixed radius but later versions use variable or adaptive radii, which makes it more stable in areas of low seismicity. It assumes a fixed *b* (e.g. USGS uses 0.95 for the whole continent east of California). While one of the apparent advantages of smoothed seismicity is that there is less expert judgement, it is sensitive to the setting of the various tuning factors (e.g. grid spacing, radii and smoothing). It can also migrate seismicity into aseismic areas that are surrounded by areas of seismicity. The earthquake density is based on the number of small (M3.0-4.0) earthquakes.

It has a strong implicit assumption that seismicity is highly stationary, on the scale of the radius, and that the variations in seismicity observed in the catalogue (e.g. the last 50 years) is a good predictor of earthquakes of interest over the next 50 years. The corollary is that any other factors, such as geology, are fully captured in the catalogue. As normally less than 30 earthquakes are within the radius of interest, the variability of the *a* value is high. It is likely to be a good estimator of the moderate (e.g. M4.5-5.0) earthquakes but might not be so good for a estimating the occurrence of larger (M>=5.5) earthquakes. As can be seen from both the Griffin et al. (2016) and Cuthbertson (2016) smoothed seismicity models the resulting seismicity rate and hazard maps are spatially highly variable, with areas of relatively uniform geology having highly variable seismicity (e.g. Gawler Craton of the Nullarbor plains and western Eyre Peninsular).

The Nearest Neighbour interpolation method instead of a fixed grid has similar assumptions to smoothed seismicity, particularly that of highly stationary seismicity. It too will be sensitive to assumptions, such as the lower magnitude cut-off. In areas of sparse seismicity it produces large polygons, so it should avoid the problem of either extrapolating seismicity or having zero seismicity that fixed grid smoothed seismicity does.

4.2 Regional source zones

When PSHA tools use source zones they assume that the rate of seismicity is uniform within each zone. So any variations in observed seismicity within a zone are assumed to be the result of the under sampling of the long-term seismicity. Long-term for building code PSHA being 50 years, perhaps 200 years, but not thousands of years. Several seismic source zones have been developed for Australia.

4.3 Leonard (2008)

This model was developed for estimating large scale differences in seismicity and strain-rates across the country and consists of only six large regional source zones and a single background zone. As it was derived from maps of seismic density it is based almost entirely on seismicity so includes little geological information and implicitly assumes that variability in seismicity within the large zones is solely an under sampling issue. One advantage of the large source zones is that there are large numbers of earthquakes in each zone so the MFD is well constrained. One disadvantage is it perhaps doesn't capture the likely variation in occurrence of moderate (e.g. M4.5-5.5) earthquakes within the zones but might be a good choice for a estimating the occurrence of larger (M>=5.5) earthquakes. As can be seen from Griffin et al. (2016) this model gives large areas of uniform hazard with very steep gradients along zone boundaries.

These zones, or something similar, are likely to be useful for estimating regional *b* values. The very high gradients at the boundaries are not realistic, suggesting it might need significant modification before being useful for code purposes (e.g. multiple models with slightly perturbed zone boundaries) though as part of an ensemble of models it could be useful.

4.4 AUS6 (Dimas et al. 2016)

The original AUS5 (Brown and Gibson 2004) was a significant step forward in PSHA of Australia being the first to provide a complete mosaic of Australia (rather than zones and a default background), to use geology as the primary control over source zonation, use a modern GMPE and apply the Cornell (1968) methodology. The source zones were derived using a hierarchy of inputs starting with a large scale crustal elements layer and then using increasingly small scale geological and geophysical data sets. Very little seismological data was used in its development. AUS6 is a significant update to the model, incorporating many incremental changes over the last decade and entirely new offshore sources. With 125 individual source zones it is very detailed but the relevance of the 25 zones with areas less than 6000km^2 to a national scale map could be questioned.

Its strong assumption is that seismicity is very strongly controlled by geology and that the geology used, which is derived primarily from surface geological mapping, accurately maps the geology at seismogenic depths (i.e. 5–15 km). This assumption has been neither proved nor disproved and I would suggest that there is insufficient data to do this. Almost all cells will contain insufficient earthquakes to accurately calculate the MFD *b*-value, so requiring regional *b*-values to be estimated and only the *a*-value estimated for each zones. Determining an accurate a-value for the 25 smallest zones, from the national (publicly available) catalogue, will prove challenging.

4.5 DIM-AUS (Dimas and Venkatesan 2016)

This model uses a similar approach to the AUS5 and AUS6 methodology but its final set of zones are very different to the AUS5 and AUS6 models. At 69 zones it is quite detailed. Regional *b*-values will need to be calculated and I suspect it might prove challenging to estimate an accurate *a*-value, from the national catalogue, for the smallest 9 zones. This SSM includes the faults identified in the Geoscience Australia Neotectonic featurs databases (2011). The authors do not discuss how the faults are to be included (e.g. G-R vs CE).

4.6 GA 2013 (Leonard et al. 2014).

This source zone model used a seismic density map at various grid densities to guide the defining of source zones. Major geological boundaries were used to refine the polygons. The aim was to create a method that minimised the need for subjective source zone boundaries. This was only partially successful, with the specific boundaries of many zones being somewhat subjective and the boundaries of a few being highly subjective. To minimize the impact of this subjectivity the hazard map was smoothed with a Gaussian filter, the effect of this is similar to having many multiple source zones with perturbed boundaries. It also removed the very steep gradients in the hazard. It assumes that major geological boundaries demarcate areas of differing seismicity. With 23 zones, the smallest of which is 35000 km^2, accurately estimating the a-value will be straight forward although regional b-values will likely be needed in some areas.

It also includes a background layer consisting of just three zones (Craton, non-Craton and extended continental crust). This layer was used to set a floor and used to guide GMPE selection and Mmax. With the implicit assumption that seismicity depends only on the major crustal elements and is uniform across areas of 1M km^2 it is probably not particularly valid for a 50 year map, except perhaps as a placeholder onto which regional models could be grafted.

This model did not include a fault source model. In a study of the major capital cities most exposed to faults, Clark and Leonard (2014) combined this seismic source model with the Geoscience Australia fault model (Clark et al. 2011). They concluded that a Characteristic Fault model was more appropriate to Australian faults than a Gutenberg- Richter model. To reduce the problem of double counting earthquakes Leonard et al. (2014) demonstrated that by using an Mmin of 4.5 for the seismic source zone model but a higher Mmin (e.g. M5.3 or Mmax – 1.5) and a CE MFD for the fault source model the faults could be included without necessarily dramatically increasing the hazard.

5 - CONCLUSION

The proposal to include multiple seismic source models in the seismic source characterisation model introduces several issues that have not previously needed to be considered or at least were less important. In this paper we have discussed the purpose of logic trees and how they should be constructed, the assumptions behind the proposed models and how this can inform the weighting task, and the challenges of accurately estimating magnitude frequency distributions (MFD) of models with many small zones.

It is necessary to weight the models in a logic tree. Modern thinking is that this weighting should reflect the degree-of-certainty or degree-of-belief that as part of the ensemble of models it reflects the likely future distribution of the seismicity of Australia. For NSHA18, an expert elicitation process, similar to that described by Gerstenberger (2016), has been proposed.

There is general agreement that while in strict PSHA the logic tree must be mutually exclusive and collectively exhaustive (Bommer and Scherbaum, 2008) in practice the logic trees are rarely collectively exhaustive. The range of assumptions behind the various proposed seismic source models suggest that several will be mutually exclusive and they are diverse enough that they could approximate collective exhaustiveness. Several models, for example the multiple gridded (e.g. smoothed seismicity) source models, might not be mutually exclusive and it might be that not all the proposed models will be required to fulfil the aim of mutually exclusiveness and collective exhaustiveness.

In Australia the hazard is sensitive to the value of *b* in the MFD, with a change of 0.1 in *b* changing the final hazard by a factor of two. To constrain *b* to within ±0.1 at the 95% level requires 360 earthquakes in an ideal catalogue. As catalogues are not ideal, in practice the number required is probably more like 500. With perhaps as few as 2000 earthquakes available (above Mcomp and declustered), for most source zones in most area source models it will be necessary to calculate a regional *b* and calculate *a* for each zone.

How the fault source model(s) is(are) to be combined with the area source models remains an open question. Clark and Leonard (2015) concluded that a Characteristic Earthquake model (Young and Coppersmith 1985) better explained the observed behaviour of faults in Australia. A CE model is also consistent with the observation that very few, if any, of the small background earthquakes in Australia can be associated with a fault, even in areas where very accurate earthquake location is possible such as in the Flinders Ranges (Love et al. 2006; Balfour et al 2015) or East Gippsland (Brown et al. 2001). A CE model is consistent with the observation by Sandiford et al. (2012) that, based on the early aftershocks, the 2012 ML 5.4 Moe earthquakes was probably on one of the previously identified faults. The best method of how to combine them remains an open question. The AUS6 model choosing to reduce the Mmax of the zones to account for the presence of faults. Leonard et al. (2014) and Leonard and Clark (2015) preferred to reduce the Mmin of the fault and leave Mmax for both the area and fault sources unchanged. The method(s) to be adopted to combine the area and fault source models is yet to be determined. Whether just one or multiple fault source models are to be included will need to be decided first.

REFERENCES:

Abrahamson, N. A., and Bommer, J. J. (2005) Probability and uncertainty in seismic hazard analysis, Earthquake Spectra 21, 603–607.

Aki, K. (1965) Maximum likelihood estimate of *b* in the formula logN= a-bM and its confidence limits. Bulletin of the Earthquake Research Institute, 43, pp.237–239

Bender, B. (1984) A two state poisson model for seismic hazard estimation. Bull. Seism. Soc. Am. 70, 323-347.

Bommer, J. J. (2012) Challenges of Building Logic Trees for Probabilistic Seismic Hazard Analysis. Earthquake Spectra 28 1723-1735

Bommer, J. J. and Scherbaum, F. (2008) The use and misuse of logic-trees in probabilistic seismic hazard analysis, Earthquake Spectra 24, 997–1009.

Brown, A. and G. Gibson (2004) A multi-tiered earthquake hazard model for Australia. Tectonophysics 390, 25-43

Clark, D., McPherson, A. and Collins, C.D.N. (2011) Australia's seismogenic neotectonic record: a case for heterogeneous intraplate deformation. Record 2011/11. Geoscience Australia, Canberra.

Clark, D. et al. (2016) Incorporating fault sources into the Australian National Seismic Hazard Assessment (NSHA) 2018. In Proc. AEES Conference.

Clark, D. & Leonard, M. (2015) Do Australian intraplate faults generate characteristic earthquakes? In Proceedings of the Tenth Pacific Conference on Earthquake Engineering - Building an Earthquake-Resilient Pacific, Sydney, 6-8 November.

Clark, D. & Leonard, M. (2014) Regional variations in neotectonic fault behaviour in Australia, as they pertain to the seismic hazard in capital cities. In Proc. AEES Conference

Cuthbertson R. J. (2016) Automatic determination of seismicity rates in Australia. In Proc. AEES Conference

Dimas, V-A., Gibson, G. and Cuthbertson, R. (2016) Revised AUS6 model: Significant changes and approaches to the seismotectonic model. In Proc. AEES Conference

Dimas, V-A. and Venkatesan S. (2016) Seismotectonic model for the Australian Plate – Beyond Borders. In Proc. AEES Conference.

Gerstenberger, M.C. & McVerry, G. (2016) A hybrid time-dependent probabilistic seismic hazard model for Canterbury, New Zealand. Seismological Research Letters.

Ghasemi, H., Griffin, J. Heimann, S., Leonard, M. & Allen, T., (2016) Towards a homogeneous earthquake catalogue for Australia. In Proc. AEES Conference

Kafka, A. L. (2007). Does seismicity delineate zones where future large earthquakes are likely to occur in intraplate environments?, Special Paper 425, in Continental Intraplate Earthquakes: Science, Hazard, and Policy Issues, S. Stein and S. Mazzotti (Editors), Geological Society of America

Leonard, M., (2008) One Hundred Years of Earthquake Recording in Australia. Bull. Seism. Soc. Am., 98(3), pp.1458–1470.

Leonard, M. R. Hoult, P. Somerville, G. Gibson, D. Sandiford, H. Goldsworthy, E. Lumantarna and S Spiliopoulos (2014) The Challenges of Probabilistic Seismic-Hazard Assessment in Stable Continental Interiors: An Australian Example. Bull. Seism. Soc. Am., 104(6), pp.3008–3028.

Leonard, M., Burbidge, D.R. & Edwards, M. (2013) Atlas of Seismic Hazard Maps of Australia, Geoscience Australia Record 2013/41

Leonard, M. & Clark, D. (2011) A record of stable continental region earthquakes from Western Australia spanning the late Pleistocene: Insights for contemporary seismicity. Earth and Planetary Science Letters, 309(3-4), pp.207–212

Liu, M., Stein, S. & Wang, H. 2011. 2000 years of migrating earthquakes in North China: how earthquakes in mid-continents differ from those at plate boundaries. Lithosphere, 3, 128–132

McGuire, R. K., Cornell, C. A., and Toro, G. R., 2005. The case for the mean hazard curve, Earthquake Spectra 21, 879–886.

Musson, R., 2012. On the nature of logic trees in probabilistic seismic hazard assessment, Earthquake Spectra 28, 1291–1296.

Petersen, M.D., Moschetti, M.P., Powers, P.M., Mueller, C.S., Haller, K.M., Frankel, A.D., Zeng, Y., Rezaeian, S., Harmsen, S.C., Boyd, O.S., Field, E.H., Chen, R., Rukstales, K.S., Luco, N., Wheeler, R.L., Williams, R.A., and Olsen, A.H. (2014) Documentation for the 2014 update of the United States national seismic hazard maps: U.S. Geological Survey Open-File Report 2014–1091, 243 p.

Scherbaum, F., and Kuehn, N. M. (2011) Logic tree branch weights and probabilities: Summing up to one is not enough, Earthquake Spectra 27, 1237–1251.

Utsu, T. (1966) A statistical sigmficance test of the difference m b-value between two earthquake groups,J Phys Earth 14, 37-40

Youngs, R.R. & Coppersmith, K.J. (1985) Implications of fault slip rates and earthquake recurrence models to probabilistic seismic hazard estimates. Bull. Seism. Soc. Am., 75(4), pp.939–964.

APPENDIX A

Estimating the Magnitude Frequency Distribution (MFD) and its uncertainties.

The Maximum Likelihood as the preferred estimator of the MFD of seismic catalogues was originally proposed by Aki (1965) and Utsu (1965), as:

$$b = 1/\ln(10)(\mu - M_{min}) \qquad \text{A1}$$

where μ is the average of the magnitudes and M_{min} is the minimum magnitude for the completeness of the seismic catalogue. This assumes magnitudes are continuous. In real world catalogues magnitudes are discrete (e.g. 0.1) and when this is taken into account the Maximum Likelihood estimator becomes (e.g. Bender 1983):

$$\mathrm{b} = 1/\ln(10)[\mu - (\mathrm{M_{min}} - \Delta\mathrm{M}/2)] \qquad \text{A2}$$

where ΔM is the increment in magnitude. Note that Equation A2 should always be used in preference to Equation A1.

In regions of high activity and/or with M_{min} greater than M 5.0 the difference between Equations A1 and A2 is small but in areas where M_{min} is M 3.0 or less the difference between A1 and A2 is significant. Other estimators, that are theoretically superior to Equation A2, have been proposed, for example Tinti and Mulargia (1987):

$$b = \ln(\mathrm{p}) \left[\frac{1}{\ln(10)\Delta M}\right] \qquad \text{where } P = (1 + \Delta M/(\mu - M_{min})) \qquad \text{A3}$$

However in practice the differences between A2 and A3 are very small.

The uncertainty estimate given by Aki (1965) is

$$\sigma = b/\sqrt{\mathrm{N}} \qquad \text{A4}$$

where N is the number of earthquakes. Other, theoretically superior, formulas have been proposed (e.g. Tinti and Mulargia 1987) but in practice the results are close to Equation A4.

Equations A2 is a robust method of estimating b where there is a very clean catalogue, with a single well defined Mcomp corner and a single magnitude type. Equation A2 is sensitive to errors in Mcomp as missing earthquakes near the M_{min} will lead to an underestimate of μ and so an over estimate of b. A mixture of magnitude types has the effect of having multiple sources and this will lead to greater uncertainties in the estimation of b as well as the likelihood of a bias in the results.

Weichert (1980) proposed a method of estimating b and σ for catalogues with multiple Mcomp corners, so enabling contemporary catalogues to be combined with older instrumental and even historic catalogues – where the magnitudes can be shown to be equivalent. From my experience it also tends to be a bit less sensitive to missing earthquakes near M_{min}.. Its primary disadvantage is it requires an iterative algorithm, so cannot be implemented in a spreadsheet, but with freeware packages now becoming readily available for PCs this is now a negligible barrier.

Standard least squares analysis of the catalogue is a poor estimator of *a* and *b*. On average it underestimates *b* and requires over 1000 earthquakes to obtain a 1σ of 0.1 The modified least squares method proposed by Leonard et al. (2014) is a significant improvement on standard least squares but for ideal catalogues does not perform as well as either Equation A2 or the Weichert method. I have found that sometimes the Weichert algorithm converges to different values of *b* depending on whether the starting *b* value is smaller or larger than the actual *b* value. My preferred implementation of the Weichert method is to run twice with two starting values, take the average value of *b* and use the difference in the *b*-values as a flag that the data might need to be double checked. Based on experience with real world catalogues and extensive testing with synthetic catalogues I strongly recommend using the Weichert, with Equation A2 as a crosscheck.

Leonard et al. (2014) demonstrated that in typical Australian source zones an in increase in *b* of 0.1 results in an almost doubling of the hazard and a decrease an almost halving of the hazard. An increase in *b* of 0.05 will result in about a 1/3 increase in hazard. From Table A2 we can see that to have a confidence of 95% that the *b* is within ±0.05 we need 1400 earthquakes, 95% of within ±0.1 360 requires earthquakes and 67% within 0.1 requires 90 earthquakes. The most basic zoning of Australia into Craton, Non-Craton and Extended crust would allow *b* a 95% uncertainty in *b* of between ±0.06 and ±0.09.

Table A1 the number of earthquake required to obtain the required 1σ or 2σ uncertainty in the estimation of *b*.

1 σ	0.2	0.15	0.1	0.075	0.05	0.025
2 σ	0.4	0.3	0.2	0.15	0.1	0.05
N	23	40	90	160	360	1400

Based on 250 simulations of ideal synthetic catalogues, with a *b* of 0.95, I have found that the values of uncertainty estimated by Equation A4 and the Weichert algorithm to be correct. Table A2 summarises the results for simulations of 90, 180 and 360 earthquakes using Weichert method (W), Maximum Likelihood (ML), Equation A2, Modified Least Squares (MLS) (i.e. Leonard et al. 2014) and Standard Least Squares (SLS). The standard deviation (1σ) estimated from the 250 simulations is given for each of the methods along from 1σ from Equation A4 and from the Weichert method. The results for Maximum Likelihood (Equation A2) are consistent with other published results (e.g. Marzocchi and Sandri (2003) as are the Weichert results.

Of the 24 individual source zones used in the regional layer of the NSHM 2013, 12 had less than 60 earthquakes and only 3 had more than 160 earthquakes. Without a formal ANOVA analysis we cannot be sure but given the relatively small number of earthquakes in many of the source zones, I expect that none of the *b*-values are statistically significantly different from their respective background values. If so then much of the variation in hazard observed, for example, within SE Australia might not actually be real.

To gauge the effects of not getting Mcomp correct, the exercise was repeated with an average 33% of M3.0, 25% of M2.5, 17% or M3.2 and 9% pf M3.3 missing from the catalogue. This has only a small effect on the appearance of the MFD (compare Figures A2 and A3) and that the catalogue is incomplete could easily be overlooked. This suggests that any sign of roll-off of the cumulative MFD at lower magnitudes is a sure sign of completeness issues. The missing earthquakes have a significant effect on the Maximum Likelihood estimators with the average estimations dropping from 0.955 to 0.859. Though it lacks a sound theoretical basis, the Modified Least Squares method is relatively unaffected by the missing small earthquakes.

Table A2 a summary of the different methods of estimating *b* and their uncertainties. The true value of *b* is 0.95.

Number	Method	W	W σ	ML	A4 σ	MLS	SLS
90	*b*	0.945		0.942		0.969	0.863
	σ	0.102	0.1	0.1	0.099	0.141	0.168
180	*b*	0.958		0.955		0.977	0.899
	σ	0.072	0.072	0.072	0.071	0.111	0.143
360	*b*	0.951		0.948		0.970	0.904
	σ	0.05	0.05	0.046	0.050	0.085	0.112
180	*b*	0.862		0.859		0.96	0.888
Mc	σ	0.055	0.065	0.054	0.064	0.099	0.132

The methods used are: W Weichert (1980), ML Maximum likelihood (Aki 1965), MLS modified least squares (Leonard et al 2014) and SLS standard least squares.

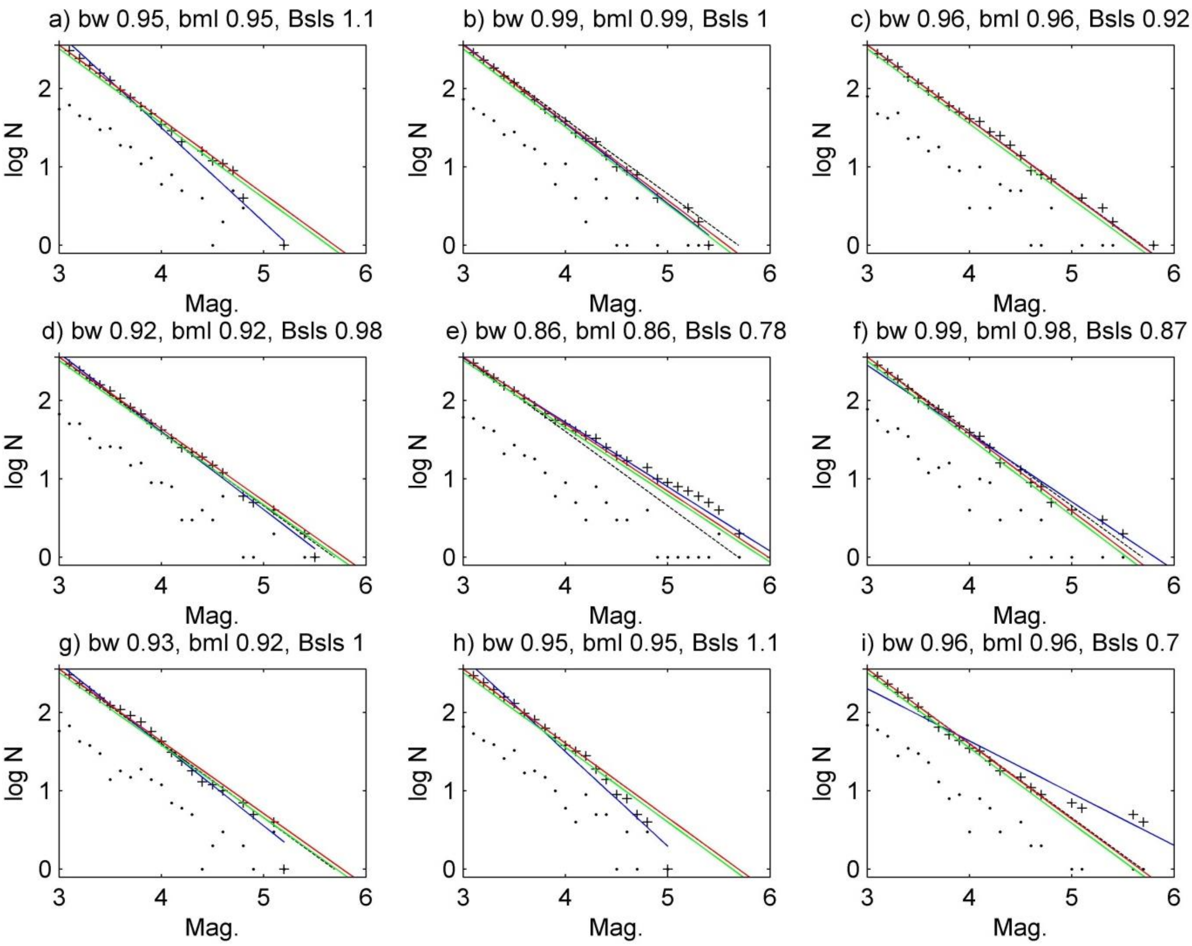

Figure A1 an ideal synthetic catalogues, with b = 0.95, was randomly sampled for 360 earthquakes and then the MFD parameters were estimated using the Weichert algorithm (red), Equation A2 (green) and standard least squares (blue). The · are the incremental data, the + are the cumulative data and the black dashed line is the actual MFD of the catalogue. Even for such relatively large numbers there is significant variation in the data and results, reflecting the 2σ range of b of ±0.1 for Weichert and Eq 2 and ±0.22 for Least Squares. Even with this large number of earthquakes the least squares methods remains unstable in both the presence (i) and absence (a & h) of large earthquakes. The purely statistical variations in seismicity could easily lead to the over-interpretation of the data to propose incorrect seismo-tectonic models. For example subfigure h) might suggest a Mmax close to M5.1 instead of the actual value of 7.5. Where subfigure i) might be incorrectly interpreted as a bilinear relation (possibly due to the seismo-tectonics or a problem with magnitudes).

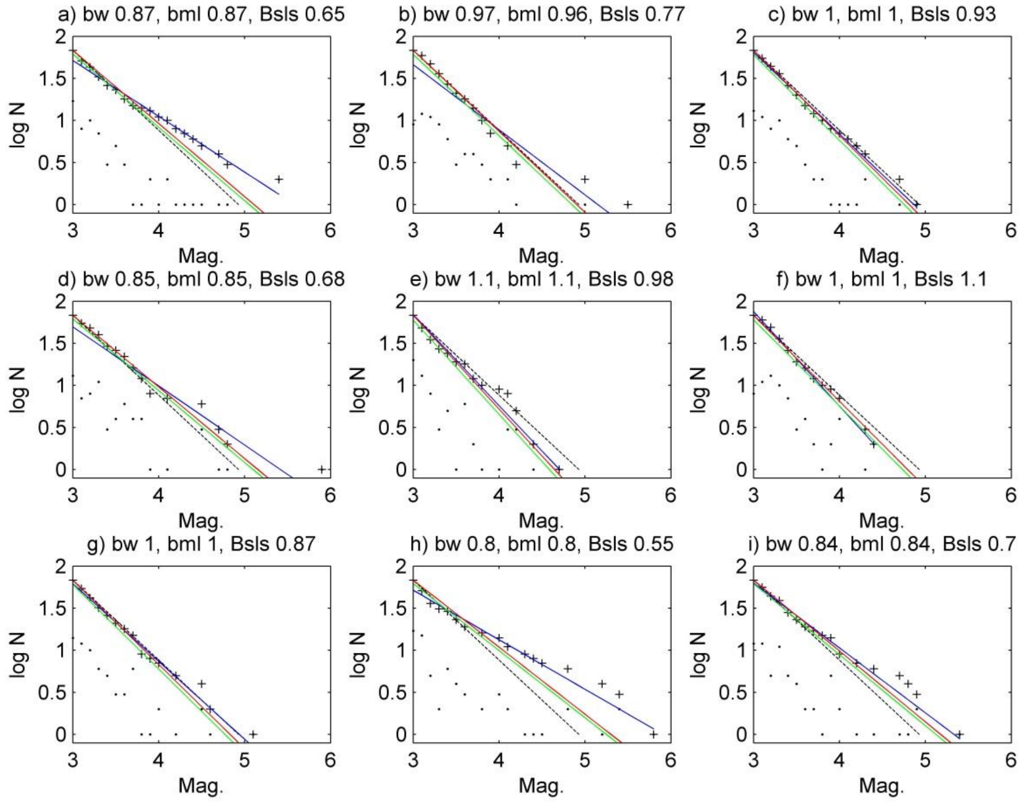

Figure A2 the synthetic catalogues of 68 earthquakes. 68 being the median number of earthquakes used to estimate the MFD parameters (a & b) for NSHM 2013. For this ideal catalogue bw and bml are very similar. While the maximum likelihood results appear to be good a decrease in *b* of 0.1 (i.e. 0.85) will result in a doubling of the hazard in most Australian zones. As 0.95 is the true value examples a, d, h & i would result in the hazard being incorrectly overestimated and examples c, e, f & g would give an underestimation of the hazard. The LS method is sensitive to the presence of larger magnitude data points and so performs poorly when these "outlier" data points are present.

Table A3 the number of earthquakes per year expected in Australia

Magnitude	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5
Num./year	65	22	8	2.7	0.9	0.32	0.11	0.04
Return Period (years)	0.015	0.045	0.13	0.38	1.1	3.2	9.2	27

Note this gives a b-value of 0.925.

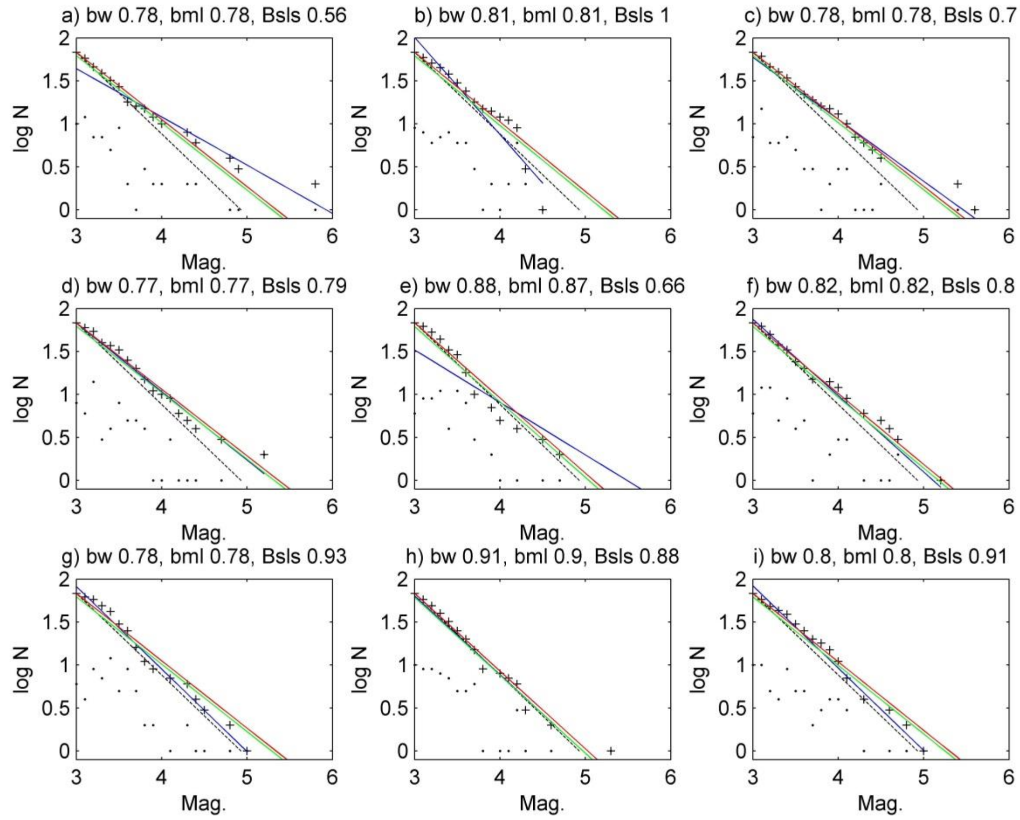

Figure A3 same as figure A2 but with master catalogue missing 33% or M3.0, 25% of M3.1, 17% of M3.2 and 9% of M3.3, to simulate the case of incorrect choice of Mcomp. The roll-off at low magnitudes can be seen in figures d and g but not in most figures. Despite the apparent complete appearance of the data, all the estimators are consistently under-estimating the b-value. This highlights the importance of correctly estimating Mcomp and the need to be cautious about pushing the Mcomp limits in order to include more data in the analysis.