Analytical Simulation of Limited Ductile RC Beam-Columns

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Abstract

A systematic analytical modelling technique for the simulation of limited-ductile beam-column elements based on the concentrated plasticity model is proposed in this paper. The modelling technique employs a backbone curve for the monotonic behaviour of the limited ductile beam-column elements based on empirical equations. The hysteretic behaviour of the beam-columns in the cyclic loading is modelled and calibrated to results from experiments on columns obtained from the literature using *OpenSEES* software. The modelling technique provides an important tool for the seismic performance assessment of existing buildings which have been designed with considerations of low to no ductile detailing.

Keywords: Lightly Reinforced Columns, Failure Mechanisms, Bond-Slip, Concentrated Plasticity Modelling Technique, OpenSEES

1 INTRODUCTION

Reinforced concrete buildings constitute a significant portion of construction in Australia. Many existing buildings in Australia and other low to moderate seismic regions have been designed with little to no considerations of ductile detailing. The majority of these buildings also possess vertical and horizontal irregularities which can exacerbate their vulnerability in an earthquake. There is a need to assess the seismic performance of buildings in these regions.

Parametric studies based on dynamic modal analyses have been conducted by the authors to investigate the effects of discontinuities in the gravity load carrying elements of multi-storey buildings (Mehdipanah et al., 2016). It has been shown that the displacement behaviour of the buildings is not significantly affected by the discontinuities in the columns. However, the behaviour of the buildings in the non-linear range is still largely unknown. Response modification factors specified in seismic design provisions (e.g., *AS 1170.4* (Australian Standard, 1993), ASCE 7 (ASCE/SEI, 2010)) are currently used for seismic design of reinforced concrete buildings to reduce the elastic base shear demand imposed on the buildings. The applicability of adopting such factors on the design of irregular buildings needs to be assessed. This paper presents interim findings of a study aimed to investigate the ductility and failure behaviour of multi-storey buildings featuring discontinuities in their vertical load resisting elements and evaluation of collapse due to the sudden shear failure in the transfer beams and formation of weak storey. In particular, this paper presents the analytical modelling technique adopted to predict the load deformation behaviour of lightly reinforced beam-column elements that are expected to fail in the flexural-shear failure or shear failure mechanisms.

Two major studies on the analytical modelling of lightly reinforced concrete columns have been conducted by Elwood et al. (2003) and LeBorgne et al. (2014). In these studies, fibre sections along with the shear springs at the ends of the elements were used to model the shear response based on the distributed plasticity modelling techniques using *OpenSEES* software (McKenna et al., 2000). The coded elements are capable for accurate detection of shear failure mechanism. However, numerical simulations using these elements are computationally expensive and are often faced with convergence issues.

This paper seeks to address the need for an accurate and robust modelling technique whilst keeping it simple for the analytical simulations of limited ductile RC beam-column elements. The proposed modelling technique was successfully implemented in nonlinear finite element program *OpenSEES*. 18 columns were numerically simulated using the program. Results demonstrate an acceptable agreement with the results of experiments of non-ductile columns obtained from the literature.

2 BEAM-COLUMN ELEMENTS FAILURE MECHANISMS

Relative amounts of shear and flexural capacities dictate the nonlinear load deformation relationship of a beam-column element. When the ultimate flexural capacity of an element is less than the shear capacity, failure can occur due to degradation of the flexural capacity (Figure 1). In the case when the shear capacity is less than the ultimate flexural capacity, the displacement behaviour is controlled by shear mechanism (Figure 2) which is characterised by a sudden decrease in the load deformation behaviour. The sudden decrease is caused by diagonal shear cracks resulting in the two sides of the

cracks sliding against each other. Hence, the element is not stable and cannot absorb extra energy. In contrasts to shear failure, reinforcements and concrete gradually reach their ultimate strain and element can sustain its stability before the onset of a flexural failure. Therefore, shear capacity degrades significantly as ductility demand increases. A beam-column element can have a load deformation behaviour that is initially governed by flexure (as the nominal shear capacity exceeds the ultimate failure capacity) but ultimately fails in shear (as shown in Figure 3).

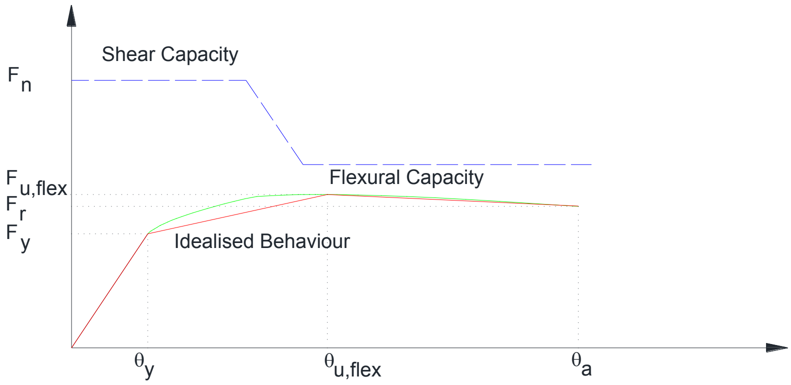


Figure 1: Flexural failure mechanism

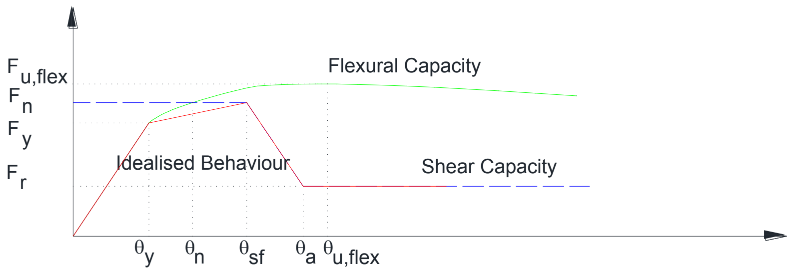


Figure 2: Shear failure mechanism

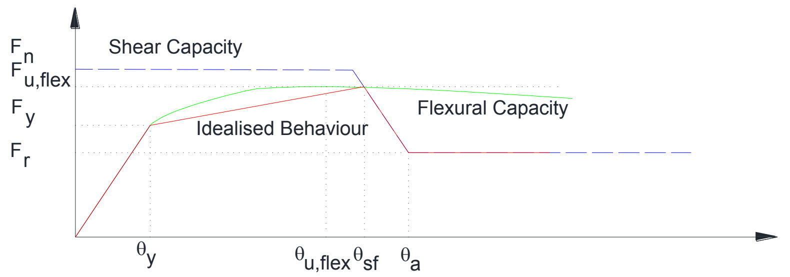


Figure 3: Flexural-shear failure mechanism

3 PROPOSED ANALYTICAL PLASTIC HINGE MODEL

The load deformation relationship can be defined by estimating the strength and deformation capacities of the beam-column elements. In this study, the monotonic load deformation curve (backbone curve) was modelled using sectional moment curvature analysis to estimate the flexural strength capacities. Empirical formulae were employed to provide estimates of the lateral drift ratios at the onset of shear and axial load failure. Furthermore, the behaviour of the beam-column components under cyclic load was calibrated to experimental results of limited ductile columns obtained from the literature.

3.1 Monotonic Behaviour-Back Bone Curve

In this study, a nonlinear finite element computer program *OpenSEES* (McKenna et al., 2000) was utilised to model the shear critical beam-column members. Two *ZeroLength Element*s were used as hinges for the concentrated plasticity zones at the ends of the beam-column element (Figure 4). The elements were used to account for the nonlinear flexural response as well as the shear response and bar-slippage at the ends of the beam-column elements.

To define the backbone curve of the load deformation relationship, *Hysteretic Uniaxial Material*, a built-in material in *OpenSEES*, was adopted. The material has the capability to model any arbitrary hysteretic response shape, including pinching behaviour and variable unloading stiffnesses. This material model is also consistent with the backbone curve recommended in the seismic performance assessment guidelines, e.g., *ASCE 41-13* (ASCE/SEI, 2013). The material model is based on three positive and three negative points to define the trilinear backbone curve in the positive and negative directions of loading (Figure 5). Having diverse parameters to account for damage and pinching behaviour has made this material an appropriate tool to model the shear controlled elements. In this study, along with the *ZeroLength Element*s, an *Elastic Beam Column Element* was also used to model the column (Figure 4).

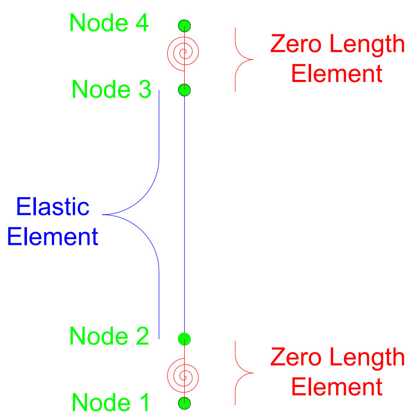


Figure 4: Elements used to model the column

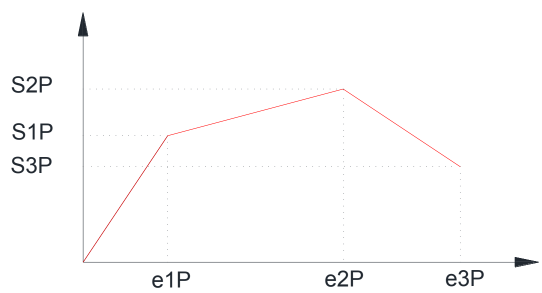


Figure 5: Monotonic behaviour of the *uniaxial Hysteretic Material*

3.1.1 Empirical Equations Predicting Shear Critical Behaviour Responses

In recent years, studies on the estimation of drift ratio at the onset of shear and axial failure have been conducted. Elwood and Moehle (2003) suggested that the drift ratio at shear ($\frac{\Delta_s}{L}$) and axial load ($\frac{\Delta_a}{L}$) failure can be found using Equations (1) and (2):

$$\frac{\Delta_s}{L} = \frac{3}{100} + 4\rho_t - \frac{1}{40}\frac{v}{\sqrt{f_c'}} - \frac{1}{40}\frac{P}{A_g f_c'} \geq \frac{1}{100} \tag{1}$$

where,

ρ_t is the transverse reinforcement ratio, v is the nominal shear stress (MPa), f_c' is concrete compressive strength (MPa), P is the axial load on the column and A_g is the column cross-sectional area.

$$\frac{\Delta_a}{L} = \frac{4}{100}\frac{1+(\tan\theta)^2}{\tan\theta + P\left(\frac{s}{A_{st}F_{yt}d_c\tan\theta}\right)} \tag{2}$$

where,

d_c is the depth of the column core from centre line to centre line of the ties, S is the spacing of the transverse reinforcement, A_{st} is the area of transverse reinforcement, F_{yt} is the yield strength of the transverse reinforcement, θ is the critical crack angle from the horizontal, assumed to be 65°.

A study on lightly reinforced columns has also been conducted by Wibowo et al. (2014). Equation (3) was proposed to compute the drift ratio at the onset of axial load failure (δ_{af}). This point corresponds to the strength equals to half of the ultimate lateral strength which can be found from sectional moment curvature analysis.

$$\delta_{af} = 5(1+\rho_v)^{-\frac{1}{1-\beta}} + 7\rho_h + \frac{1}{5n} \tag{3}$$

where,

ρ_h is the longitudinal reinforcement ratio, ρ_v is the transverse reinforcement ratio, $\beta = \frac{n}{n_b}$, where n is the axial load ratio and n_b is the axial load ratio at the balance point on the interaction diagram.

The drift at the onset of shear failure can be calculated using linear interpolation between δ_{af} and the drift ratio when the ultimate lateral strength is reached. The shear failure is defined when the lateral strength has degraded to 0.8 of the ultimate lateral strength of the column.

3.1.2 Flexural Response and Flexural Stiffness

Using a sectional moment curvature analysis, the curvature at the yield (ϕ_y) and ultimate strength (ϕ_u) can be obtained. For a cantilever column, the drift ratio at yield is defined by Equation (4):

$$\theta_y = \frac{\phi_y l}{3} \tag{4}$$

where, l is the height of the cantilever column.

The effective moment of inertia (I_{eff}) can be calculated using Equation (5), where E is the modulus of elasticity.

$$\phi_y = \frac{M_y}{EI_{eff}} \tag{5}$$

Having the values of ϕ_u and ϕ_y:

$$\theta_{pl} = (\phi_u - \phi_y)L_p \tag{6}$$

where, L_p is plastic hinge length, which can be estimated using Equation (7) (Mattock, 1965).

$$L_p = 0.05l + 0.5D \tag{7}$$

where, D is the column depth.

Equation (8) can be used to compute the contribution of the flexural response at the drift corresponding to the ultimate flexural strength ($\theta_{u,flex}$).

$$\theta_{u,flex} = \theta_y + \theta_{pl} \tag{8}$$

3.1.3 Shear Stiffness

Shear stiffness at the onset of flexural yielding can be computed using the elastic shear stiffness of the element.

$$k_{shear} = \frac{5}{6} GAl \tag{9}$$

where, G is the shear modulus of the concrete and A is the cross sectional area of the element.

3.1.4 Effect of Bar-Slippage in the Response

The effects of bar-slippage can be modelled by using springs located at the ends of the beam-column elements. Sezen & Moehle (2004a) proposed analytical equations for the computation of rigid body rotations due to the slipping of the longitudinal reinforcements. The rotational stiffness due to the bar-slippage can be computed using Equations (10) and (11).

$$k_{slip_1} = \frac{M_y 8\sqrt{f_c'}(D-C)}{\varepsilon_s f_s d_b}, \varepsilon_s \leq \varepsilon_y \tag{10}$$

$$k_{slip_2} = \frac{M_y 8\sqrt{f_c'}(D-C)}{[\varepsilon_y f_y + 2(\varepsilon_s+\varepsilon_y)(f_s - f_y)]d_b}, \varepsilon_s > \varepsilon_y \tag{11}$$

Where,

ε_s is the strain in reinforcing bar, ε_y is the yielding strain of the reinforcing bar, D is the section depth, C is the neutral axis depth, d_b is the reinforcement diameter, f_s is the stress in longitudinal reinforcements and f_y is the yield strength of the longitudinal reinforcements.

3.1.5 Total Response

The total stiffness (k_1) of the hinge before yielding occurrence in the element can be calculated using Equation (12).

$$k_1 = ({k_{shear}}^{-1} + {k_{slip}}^{-1})^{-1} \tag{12}$$

The stiffness due to the flexural response before yielding is assigned directly to the elastic element (as shown in Figure 4).

Equation (13) can be used to estimate the total stiffness (k_2) of the element after yielding.

$$k_2 = ({k_{flex}}^{-1} + {k_{shear}}^{-1} + {k_{slip}}^{-1})^{-1} \tag{13}$$

where, the post-yield flexural stiffness (k_{flex}) is:

$$k_{flex} = \frac{M_{u,flex} - M_y}{\theta_{pl}} \tag{14}$$

$M_{u,flex}$ is the ultimate flexural capacity which can be found using sectional moment curvature analysis.

3.2 IMPLEMENTATION IN *OPENSEES*

To define the trilinear backbone curve (shown in Figure 5) for the monotonic response, six inputs are needed to define the *Hysteretic Uniaxial Material* (three inputs to define the backbone in the positive loading and negative loading direction). The negative backbone curve in this study was assumed to be identical to the positive backbone curve. The pre-yield flexural stiffness was accounted for by assigning the flexural stiffness directly to the elastic element (Figure 4). The value of (*EI*) should account for the reduction in stiffness due to the cracking (EI_{eff}).

As mentioned in Section 2, the load deformation relationship depends highly on the shear strength values. The nominal shear capacity can be obtained by calculating the shear strength value (V_n) in accordance with the standard design procedure for concrete structures (*AS 3600* (Standards Australia, 2009)). The value shall then be multiplied by

the height of the element to calculate the moment corresponding to the shear capacity (M_n).

The failure mechanism can be determined by comparing $M_{u,flex}$ obtained from the sectional moment curvature analysis with M_n values (Figures 1 to 3). *S1P* in Figure 5 corresponds to the yield strength of the element (M_y). *e1P* in Figure 5 is the initial rotation (θ_i) due to bar-slippage and shear displacement before yielding and can be calculated using Equation (15).

$$\theta_i = \frac{M_y}{k_1} \qquad (15)$$

S2P is the ultimate capacity of the element, which should be taken as the lesser of M_n and $M_{u,flex}$. *S3P* is the residual strength which can be taken as $0.7 \times S2P$. Sezen and Moehle (2004b) suggested that the shear strength can be reduced linearly from the ultimate shear strength (V_n) to the minimum value which equals to the 0.7 of the ultimate strength. Equation (2) or (3) can be used to define *e3P* which corresponds to the drift at the onset of axial load failure.

3.3 VALIDATION OF PROPOSED MODELLING TECHNIQUE

The validity of the proposed modelling technique has been verified using a database of experimental tests on shear critical columns available in the literature (Ghannoum et al., 2012; SERIES, 2013). The properties of the selected column specimens are summarised in Table 1. With the exception of specimen U6 (Saatcioglu and Ozcebe, 1989) which failed in flexure, the rest of the specimens have failed in shear. Specimen U6 was selected in this study to check the capability of the technique to model the load deformation behaviour of a ductile column.

Table 1 Experimental database used to check the validity of the results (units are in mm, kN and MPa)

Test	Specimen	Section depth	Section width	ρ_l	F_{yl}	F_{yt}	ρ_t	f_c'	Axial load
Ikeda (1968)	IK 43	200	200	0.0199	434.4	558.5	0.0028	19.6	78
	IK 44	200	200	0.0199	434.4	558.5	0.0028	19.6	78
	IK 46	200	200	0.0266	434.4	558.5	0.0028	19.6	157
	IK 62	200	200	0.0197	344.8	475.8	0.0028	19.6	78
	IK 63	200	200	0.0197	344.8	475.8	0.0028	19.6	157
	IK 64	200	200	0.0197	344.8	475.8	0.0028	19.6	157
Lynn (1996)	2CLH18	457.2	457.2	0.0194	331.0	399.9	0.0007	33.1	503
	2CMH18	457.2	457.2	0.0194	331.0	399.9	0.0007	25.5	1512
	3CLH18	457.2	457.2	0.0303	331.0	399.9	0.0007	26.9	503
	3CMH18	457.2	457.2	0.0303	331.0	399.9	0.0007	27.6	1512
Nagasaka (1982)	HPRC19-32	200	200	0.0127	371.0	344.0	0.0119	21.0	294
Saatcioglu and Ozcebe (1989)	U1	350	350	0.0321	430.0	470.0	0.0030	43.6	0
	U2	350	350	0.0321	453.0	470.0	0.0030	30.2	592
	U3	350	350	0.0321	430.0	470.0	0.0060	34.8	600
	U6	350	350	0.0321	437.0	425.0	0.0085	37.3	600
Sezen (2000)	No. 1	457.2	457.2	0.0247	434.4	476.0	0.0017	21.1	667
	No. 4	457.2	457.2	0.0247	434.4	476.0	0.0017	21.8	667
Yalcin (1997)	BR-S1	550	550	0.0198	444.9	424.9	0.0012	45.0	1800

The yield and ultimate points on the backbone curve were determined using Response-2000 (Bentz et al., 2000). The hysteretic behaviour of the elements is controlled by two damage parameters, two pinching parameters and a factor namely “beta factor”. The values of these parameters were determined by calibrating the hysteretic response into the experimental results.

Results of the numerical simulations for the Saatcioglu and Ozcebe tests are plotted against the experimental results in Figure 6. It is shown that the proposed method is able to present the load deformation behaviour of the non-ductile columns. The rest of the numerical simulation results are presented in the appendix. The results of the calibration are summarised in Table 2.

Table 2 Calibrated input parameters

Test	Specimen	pinchX	pinchY	damage1	damage2	beta
Ikeda (1968)	IK 43	1	1	0	0.3	0.5
	IK 44	1	1	0	0.3	0.5
	IK 46	1	1	0.12	0.35	0.5
	IK 62	1	1	0.007	0.21	0.5
	IK 63	1	1	0	0.29	0.5
	IK 64	1	1	0	0.26	0.5
Lynn (1996)	2CLH18	1	1	0	0.01	0.6
	2CMH18	1	1	0	0.1	0.6
	3CLH18	1	1	0.12	0.08	0.6
	3CMH18	1	1	0.08	0.1	0.6
Nagasaka (1982)	HPRC19-32	1	1	0	0.5	0.2
Saatcioglu and Ozcebe (1989)	U1	1	1	0.003	0.08	0.5
	U2	1	1	0.01	0.17	0.5
	U3	1	1	0.005	0.17	0.5
	U6	1	1	0	0	0.45
Sezen (2000)	No. 1	1	1	0.057	0.19	0.55
	No. 4	1	1	0.057	0.13	0.55
Yalcin (1997)	BR-S1	1	1	0.05	0.08	0.6

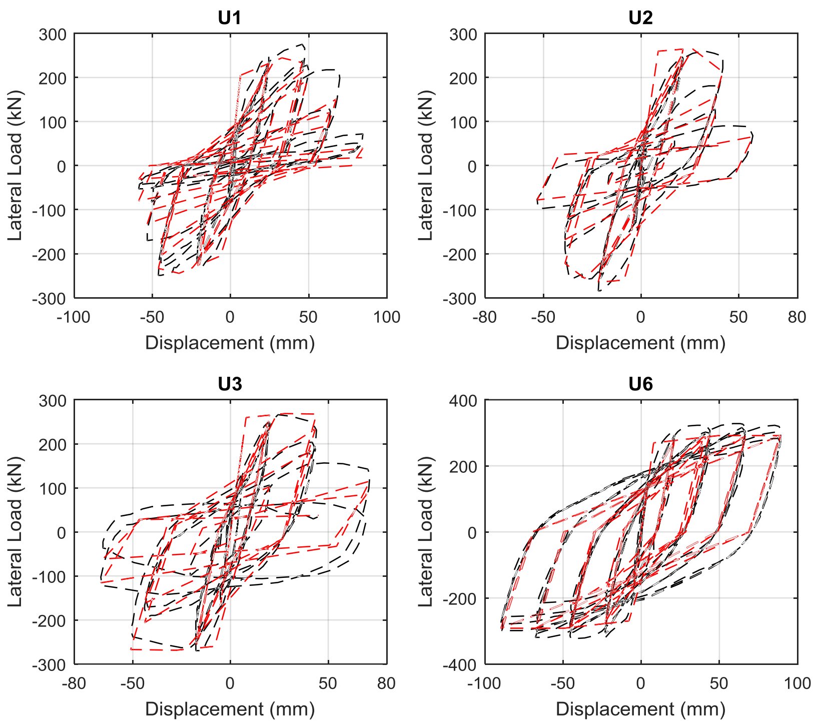
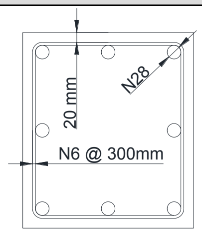


Figure 6 Results of analytical simulations of Saatcioglu and Ozcebe specimens (1989)

4 LOAD DEFORMATION RELATIONSHIP OF LIMITED DUCTILE COLUMN - EXAMPLE

In this section the load deformation relationship of a limited ductile column is presented as an example. The column has been designed in accordance with *AS3600* (Standards Australia, 2009) and hence represents a typical column that may exist in reinforced concrete buildings in Australia as a part of their gravity frames. Table 3 presents the properties of the example column. The height of the storey was assumed to be 3.2 m. Hence, for defining the backbone for a single hinge at one end of the full length column, height of cantilever column is 1.6 m.

Table 3 Properties of a random example column (units are in mm, kN and MPa)

Column	Section depth	Section width	ρ_l	F_{yl}	F_{yt}	ρ_t	f_c'	Axial load
	400	350	0.0351	500	500	0.0047	40	2300

The values of moment and curvature at yield and ultimate states were estimated by performing sectional moment-curvature analysis. By substituting the computed moments and curvatures into Equations (4)-(8), the effective moment of inertia (I_{eff}) and the plastic rotation corresponding to the ultimate flexural strength (θ_{pl}) can be calculated. The nominal shear capacity of the column was estimated using *AS 3600*.

Equations (1) and (2) were employed to estimate the drift ratio at the onset of shear and axial load failures. The calculated values are reported in Table 4.
The amount of drift ratio at the onset of flexural yielding constitutes the flexural rotation and the rotation due to shear and bar-slippage. The rotations due to the shear and bar-slippage define the initial stiffness values of the plastic hinge backbone. Equations (9), (10) and (12)-(15) were employed to estimate the total stiffness of element before and after yielding (k_1 and k_2) and these values were used as inputs to the hinge element shown in Figure 4. The elastic element (shown in Figure 4) was used to model the flexural behaviour of the element before yielding occurs. The simulated backbone curve is shown in Figure 7. The column is expected to experience flexural-shear failure mechanism.

Table 4 Calculated values of example column

$s1p = M_y$(kN.m)	$M_{u,flex}$(kN.m)	$\phi_y(\frac{rad}{km})$	$\phi_u(\frac{rad}{km})$	$\frac{EI_{eff}}{EI}$
419.4	494.4	9.493	13.899	0.748
V_n(kN)	M_n(kN. m)	$e2p = \frac{\Delta_s}{L}(rad)$	$e3p = \frac{\Delta_a}{L}(rad)$	$\theta_{pl}(rad)$
365.0	583.64	0.01313	0.01313	0.00123
$k_{shear}(\frac{\text{kN. m}}{rad})$	$k_{flex}(\frac{\text{kN. m}}{rad})$	$k_{slip}(\frac{\text{kN.m}}{rad})$before yeilding	$k_{slip}\left(\frac{\text{kN.m}}{rad}\right)$after yeilding	$e1p = \theta_i(rad)$
2361167.3	293335.9	334947.57	140848.56	0.00143

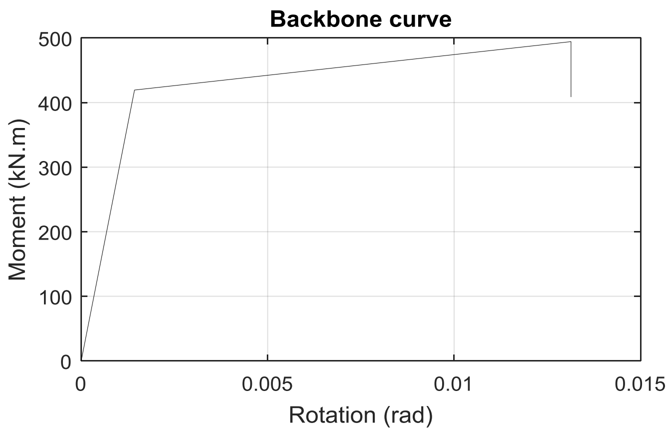


Figure 7 Suggested backbone for the example column

5 CONCLUSION

This paper presents interim findings of an ongoing study aimed to investigate the ductility and failure behaviour of reinforced concrete buildings featuring discontinuities in the gravitational load carrying elements. In particular, an analytical modelling technique for a systematic simulation of limited-ductile beam-column elements based on the concentrated plasticity modelling method has been proposed. The proposed modelling technique was successfully implemented in *OpenSEES*. Results of the numerical simulations were compared with the experimental results of shear critical columns collected from the literature. The analytical modelling technique will be adopted to assess the seismic performance of the irregular multi-storey buildings.

6 ACKNOWLEDGEMENT

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8 APPENDIX

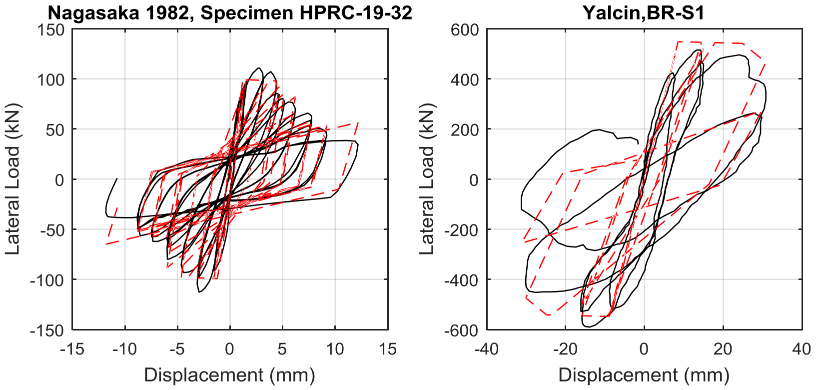

Figure 8 Results of analytical simulations of Nagasaka (1982) and Yalcin specimens (1997)

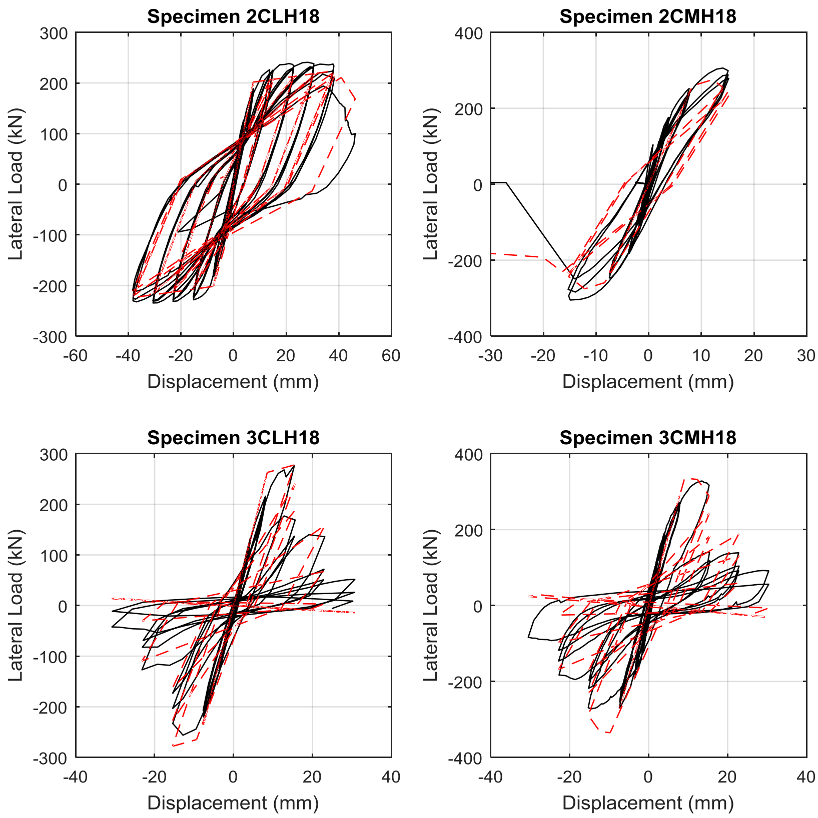

Figure 9 Results of analytical simulations of Lynn specimens (1996)

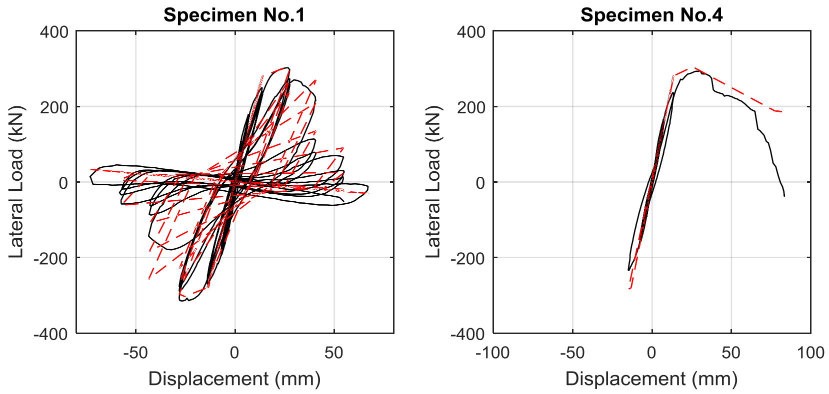

Figure 10 Results of analytical simulations of Sezen specimens (2000)

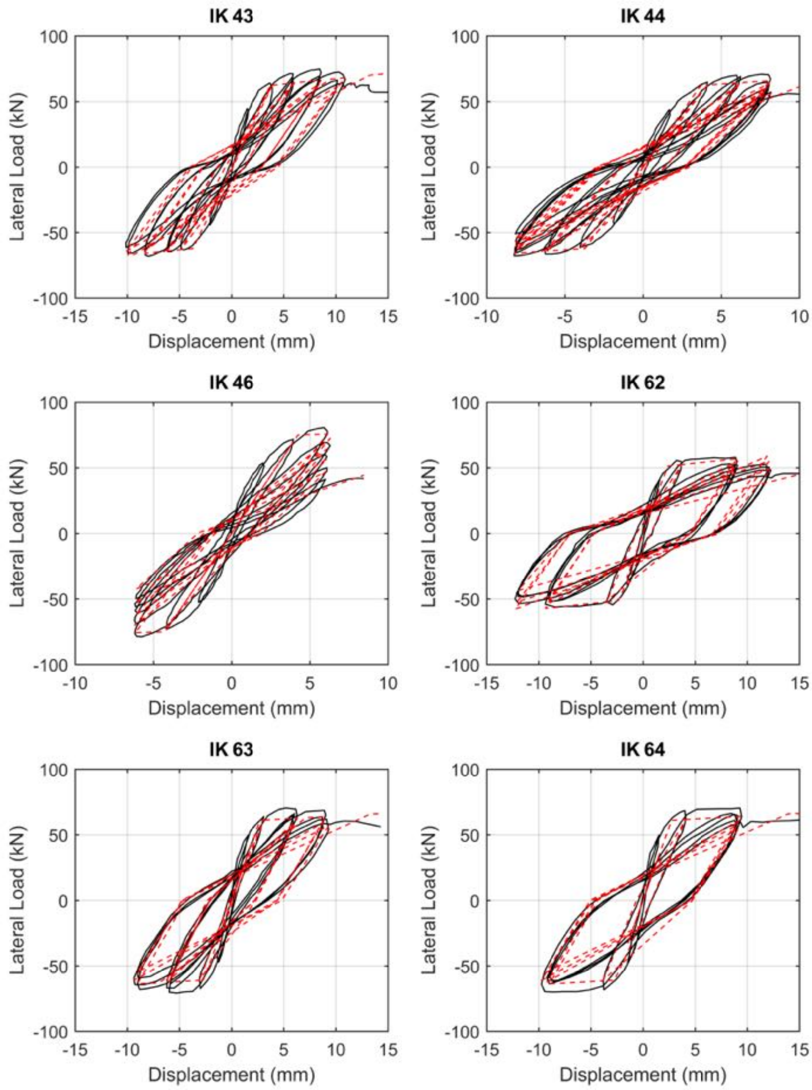

Figure 11 Results of analytical simulations of Ikeda specimens (1968)