Testing the sensitivity of seismic hazard in Australia to new empirical magnitude conversion equations

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Abstract

All modern ground motion prediction equations (GMPEs) are now calibrated to the moment magnitude scale $M_W$. Consequently, it is essential that earthquake catalogues are also expressed in terms of $M_W$ to ensure consistency between predicted ground motions and earthquake magnitude-recurrence rates for probabilistic seismic hazard analyses. However, $M_W$ is not routinely estimated for earthquakes in Australia because of the low-to-moderate level of seismicity, coupled with the relatively small number of seismic recording stations. As a result, the Australian seismic catalogue has magnitude measures mainly based on local magnitudes, $M_L$. To homogenise the earthquake catalogue based on a uniform $M_W$, a “reference catalogue” that includes earthquakes with available $M_W$ estimates from Australian earthquakes was compiled. This catalogue consists of 240 earthquakes with original $M_W$ values between 2.0 and 6.58. The reference catalogue served as the basis for the development of relationships between $M_W$ and other magnitude scales: $M_L$, body-wave magnitude $m_b$, and surface-wave magnitude $M_S$. The conversions were evaluated using general orthogonal regression (GOR), which can be used interchangeably between magnitude types. The impact of the derived magnitude conversion equations on seismic hazard is explored by generating synthetic earthquake catalogues and computing the seismic hazard at an arbitrary site. The results indicate that we may expect up to 20-40% reduction in hazard for a given ground-motion intensity measure depending on the selection and application of the magnitude conversion equations.

Keywords: Earthquake catalogue, seismic hazard analysis, magnitude conversion equation

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INTRODUCTION

In any seismic hazard assessment (SHA), an earthquake catalogue with magnitudes expressed with a uniform magnitude type is required to estimate reliable earthquake recurrence parameters and subsequently, the expected level of ground-shaking hazard through probabilistic seismic hazard analyses (PSHA). The use of ground motion prediction equations (GMPEs) is an integral component of any PSHA. Because all modern GMPEs are now calibrated to the moment magnitude scale ($M_W$), it is important that catalogue magnitudes be characterised by $M_W$. However, in practice the earthquake catalogue for a region of interest may represent a composite catalogue of earthquakes reported by different agencies that may not necessarily be characterised by a uniform magnitude scale. Furthermore, each of the seismological agencies may compute different magnitude scales due to earthquake location, distribution of recording stations, changes in instrumentation and magnitude formula, etc. Hence, for those events characterised with magnitude scales other than $M_W$, it is a common practice to compute $M_W$-equivalent values using magnitude conversion equations.

In Australia, $M_W$ is not routinely estimated for local earthquakes by seismic observatories because of the low-to-moderate level of seismicity, coupled with the relatively small number of seismic recording stations. As a result, the Australian seismic catalogue has magnitude measures mainly based on local magnitudes, $M_L$ (e.g., Michael-Leiba and Malafant, 1992). Other magnitude scales, such as body-wave magnitude $m_b$, and surface-wave magnitude $M_S$, are computed only for a relatively small number of moderate-to-large earthquakes in Australia. Such events must have recording stations within the distance range of the respective magnitude scales with acceptable level of signal-to-noise ratio (SNR).

The 2013 national seismic hazard assessment of Australia was developed assuming equivalence between $M_W$ and other magnitude types because of challenges involved in implementing first-generation $M_L$-$M_W$ conversion equations (e.g., Allen et al., 2004; 2011) for seismic hazard assessments (Leonard et al., 2014). However, subsequent studies on the relationship between $M_L$ and $M_W$ in Australia (Ghasemi et al., 2016) and in other regions of low-to-moderate seismicity (e.g., Edwards et al., 2010; Yenier 2017) agree well with early Australian studies. The aforementioned studies clearly show that there is no one-to-one relationship between $M_L$ and $M_W$ in many regions, suggesting that magnitude conversion equations should be developed and implemented to consistently express earthquake catalogues in terms of $M_W$ for seismic hazard analysis (SHA). It is therefore vital to develop region-specific and statistically robust conversion equations based on a sufficient number of data covering the magnitude range of interest. This is because the results of SHA are highly sensitive to the application of such equations (Rong et al. 2011, Leonard et al. 2014).

This paper summarizes the development of new magnitude conversion equations for Australia between $M_W$ and other magnitude scales: $M_L$, $M_S$, and $m_b$. These new conversion equations are subsequently compared with those from previous studies. We also explore different algorithms to apply such conversion equations and their impact on the calculated level of peak ground acceleration (PGA) for the 475-year return period.

REFERENCE EARTHQUAKE CATALOGUE

Compiling a reference catalogue that includes earthquakes with measured $M_W$ values forms the basis for development of magnitude conversion equations for Australia. For larger events in Australia, measured moment magnitudes can be obtained from global earthquake catalogues such as Global Centroid Moment Tensor Catalogue (www.globalcmt.org; last accessed Aug. 2017), and ISC-GEM catalogue (http://www.isc.ac.uk/iscgem/; last accessed Aug. 2017). There are also
several individual studies of seismic source parameters, including seismic moment, for moderate-to-large Australian earthquakes, i.e. Somerville et al. (2009), Sippl et al. (2015), and De Kool (personal communication). Furthermore for a small number of earthquakes in Australia, measured moment magnitudes are available from the University of St. Louis Regional Moment Tensor database of R. Herrmann (http://www.eas.slu.edu/eqc/eqc_mt/MECH.AU; last accessed Aug. 2017).

Based on the aforementioned studies, there are 38 earthquakes in Australia expressed with magnitudes in terms of $M_W$. However, robust estimation of conversion equation coefficients requires a reference catalogue adequately sampling the entire magnitude range of interest, i.e. $M_W>~3.0$. This represents a challenge in Australia, as most of the earthquakes are recorded at regional distances (>200 km) given the low density of seismic recording networks. At such distances, small-to-moderate earthquakes may not be recorded above the noise level, and thus such records cannot be used to measure $M_W$. Systematic studies on measuring $M_W$ for small-to-moderate earthquakes in Australia have been conducted by Allen et al. (2006), Allen (2012) and Ghasemi et al. (2016). Allen et al. (2006) and Allen (2012) determined $M_W$ for 164 earthquakes in Australia recorded between 1990 and 2012. For each earthquake, they determined $M_W$ using local and regional seismic data by fitting a theoretical Brune (1970; 1971) spectral shape accounting for source and local attenuation effects. In contrast, Ghasemi et al. (2016) computed $M_W$ for 60 events in Australia recorded between 2005 and 2017. For each earthquake, they retrieved earthquake source parameters, including $M_W$, by using regional seismic data and minimising the misfit between observed and synthetic displacement spectra. While there are relatively few common events in the respective catalogues, those events that are in common (mostly in southeastern Australia) have very similar estimates of $M_W$.

The reference catalogue compiled for this study consists of 240 earthquakes with original $M_W$ values between 2.0 and 6.58. In this catalogue 225 out of 240 of $M_W$ estimates are from recent studies by Allen et al. (2006), Allen (2012) and Ghasemi et al. (2016), and the rest are from other
aforementioned studies. Figure 1 shows the geographical distribution of the reference events. These events demonstrate a wide distribution in terms of spatial extent and magnitude suggesting that a nationally-applicable conversion equation can be derived.

For the earthquakes listed in the reference catalogue, we assigned corresponding $M_L$ values based on the revised $M_L$ estimates following Allen (2010) for pre-1990 earthquakes. For post-1990 events, the $M_L$ estimates of Geoscience Australia were adopted (www.ga.gov.au/earthquakes/; last accessed Aug. 2017). The method introduced by Allen (2010) corrects magnitudes using the difference between the original (inappropriate) magnitude formula (e.g., Richter, 1935; Bakun and Joyner, 1984) and the Australian-specific correction curves (e.g., Michael-Leiba and Malafant, 1992) at a distance determined by the nearest recording station likely to have recorded the earthquake. Such corrections are required to ensure that, for an earthquake of a given size, $M_L$ estimates are approximately consistent over time. For other magnitude scales, i.e. $M_S$ and $m_b$, preferred estimates provided by the International Seismological Centre (ISC) were used. Such estimates are not available for all of the events included in the reference catalogue as most of the small-to-moderate earthquakes in Australia do not have high signal-to-noise records at the distance ranges required to determine these magnitude types.

**MAGNITUDE CONVERSION EQUATIONS**

The compiled reference catalogue served as the basis for the development of magnitude conversion equations between $M_W$ and other magnitude scales: $M_L$, $m_b$, $M_S$. The conversions were evaluated using general orthogonal regression (GOR), which accounts for measurement errors in the $x$ and $y$ variables, e.g. $M_L$ and $M_W$ values, respectively. This is particularly desirable as $M_W$ estimates, similar to $M_L$ estimates, are not exact (Heimann, 2011). Unlike ordinary least-squares, GOR also provides a unique solution that can be used interchangeably between magnitude types. As a result, GOR has become the preferred approach to develop magnitude conversion equations (e.g. Storchak et al., 2013; Gasperini et al., 2013).

![Figure 2: Relationship between the calculated moment magnitudes $M_W$ for Australian earthquakes versus the preferred local magnitudes. The models of best-fit are shown relative to the conversion of Goertz-Allmann et al. (2011), and Ross et al. (2016) developed for Switzerland and California, respectively.](image-url)
Figure 2 shows the comparison of $M_L$ and $M_W$ values for earthquakes listed in the reference catalogue. The dashed line indicates a one-to-one relationship between the two magnitude scales. It is clear that $M_W$ is typically lower than $M_L$ for earthquakes larger than approximately $M_L 3.5$. In contrast, the opposite holds for earthquakes with $M_L$ estimates lower than this threshold. Interestingly, previous studies on $M_L$-$M_W$ relationship have also shown a similar pattern with the cross-over point around $M_L 3.5$ (e.g. Ben-Zion and Zhu, 2002; Edwards et al., 2010; Allen et al., 2011; Ross et al., 2016; Yenier, 2017). The fitted bi-linear model (green line), and polynomial model (black curve) fitted to our dataset are also shown in Figure 2. The bi-linear model has a fixed hinge point at $M_L 4.5$. This point has been subjectively chosen based on the visual inspection of the distribution of the data points. The polynomial model may have the advantage of not introducing a discontinuity in the conversion around the hinge point. However, the bi-linear model is our preferred model as it is a better fit to the data and has a slightly lower sum of squares error (SSE) in comparison with the polynomial model. Furthermore, the polynomial model, in comparison with the bi-linear model, applies a larger correction for earthquakes larger than approximately $M_L 5.0$ (Figure 2). Consequently, if applied to generate a corrected $M_W$ catalogue, the polynomial conversion would result in lower annual occurrence rates for larger earthquakes than the bi-linear conversion equation.

The bi-linear model suggests that for small earthquakes $M_W \propto (2/3) M_L$. This is in excellent agreement with theoretical and empirical studies demonstrating such $M_L$-$M_W$ relationships for small events (e.g. Hanks and Boore, 1984; Edwards et al., 2015; Deichmann, 2017). Figure 2 also compares the derived conversion equations with those developed by Ross et al. (2016) for California and Goertz-Allmann et al. (2011) for Switzerland. It should be noted that although the slope of the models are comparable, the actual correction factors are not the same (e.g. bi-linear model versus Goertz-Allmann et al. [2011] model). This is expected as the adopted region-specific attenuation parameters, and hence $M_L$ formula, can be significantly different. This is particularly true in near-source regions where geometric spreading functions embedded within various magnitude scales can vary considerably. In contrast, for larger events, i.e. $M_L > 4.5$, the bi-linear and

![Figure 3: Relationship between the calculated moment magnitudes $M_w$ for Australian earthquakes versus (a) $M_s$, and (b) $m_b$. The models of best-fit are shown relative to the relevant conversion equations developed by previous studies.](image)
Goertz-Allmann et al. (2011) equations begin to converge with $M_W \approx M_L - 0.3$. This may suggest that the consideration of crustal attenuation for the respective regions at larger distances is modelled appropriately, leading to more consistent magnitude estimates for larger earthquakes.

Figure 3 shows the distribution of $M_W$ versus $M_S$, and $m_b$ along with corresponding conversion equations from previous studies as well as those from this study. In both cases, there are fewer data with which to perform regression analysis relative to the $M_L$-$M_W$ case. Given the available data, a linear model was developed using GOR to convert $M_S$, or $m_b$ to $M_W$. $M_S$-$M_W$ models begin to converge for earthquakes larger than approximately $M_S$ 5.5, and there is almost a one-to-one relationship for earthquakes larger than $M_S$ 6.0 (Figure 3a). Unlike $M_S$-$M_W$ models, there is large variability between published $m_b$-$M_W$ conversion equations (Figure 3b), and this variability can be dependent on the earthquake catalogue and tectonic setting. Overall, our preferred conversion equation, i.e. the linear fit model, indicates that in general $m_b$ values are larger than those of $M_W$ for the entire considered magnitude range, i.e. 3.5<$m_b$<6.0. The $m_b$-$M_W$ difference decreases for larger earthquakes. It should be noted that, as shown in Figure 3b, there is a significant scatter in distribution of the data points suggesting that more data is required to better constrain $m_b$-$M_W$ conversion equation for Australia.

Figure 4: Schematic illustration of the area-source zone considered in this study. The rupture geometries following $M_L$-based Gutenberg-Richter model are shown as blue lines, while the red lines indicate the modified geometries following the application of the $M_L$-$M_W$ conversion equation. For illustration purposes, the catalogue is generated over the period of 500 years, with fixed strike and dip angles of 45, and 90 degrees, respectively. The seismic hazard calculation is performed over the grid points spaced at 0.1°.

**SEISMIC HAZARD SENSITIVITY ANALYSIS**

The effects of magnitude conversion equations on SHA results are explored using synthetic earthquake catalogues. In this example, we only consider the application of the $M_L$-$M_W$ conversion equations as developed for the Australian catalogue.

To perform sensitivity analysis, we calculate PGA for 10% probability of exceedance in 50 years for an arbitrary area-source zone (Figure 4) evaluating different magnitude conversion equations as well as testing different methods to implement such equations. The hazard estimates are calculated for bedrock site conditions ($V_{S30} = 760$ m/s) over a grid of sites with spacing of 0.1° in latitude and
longitude using a selected GMPE (i.e. Boore et al., 2014). We also assume that the seismic source zone is capable of generating earthquakes in the range of \( M_L \) 4.0-7.5, following a truncated Gutenberg-Richter magnitude-frequency distribution with earthquake recurrence parameters of \( a \)-value=3.5, and \( b \)-value=1.0.

To explore the application of the \( M_L-M_W \) conversion equations derived in this study, i.e. bi-linear and polynomial models, we consider three different methods:

- **Method 1:** In this approach, first a stochastic event set is generated following the earthquake recurrence model of the area-source zone as specified above. Then for each earthquake in the event set the earthquake magnitude is converted to \( M_W \) using the \( M_L-M_W \) conversion equations. This process is followed by modifying the original rupture geometry based on the assigned \( M_W \) using Wells and Coppersmith (1994) magnitude-area relationships (Figure 4). The stochastic event set generated in this study covers the period of 50,000 years. It should be noted that in SHA, for the given seismic source model, the classical approach considers a comprehensive rupture dataset that includes all possible earthquake ruptures that can be generated by the seismic source (Cornell, 1968). In contrast, a stochastic event set represents a realization of such a comprehensive dataset. In this study the choice of 50,000 years is based on the stability of the calculated hazard curves, as well as their compatibility with the hazard curve obtained by evaluating the full hazard integral using the classical approach (Figure 5).

![Figure 5](image_url): Seismic hazard curve calculated based on the classical approach (black curve) versus those from the stochastic event sets generated for a) period of 1,000 years, and b) period of 50,000 years (grey curves). For each case, the simulation is repeated 100 times using different random seeds.

Figure 6 (a-b) shows the percentage differences between PGA hazard calculated based on Method 1 and the classical approach using bi-linear and polynomial conversion equations. It can be seen that using the bi-linear model the level of hazard is reduced on average by a factor of 20-30%; whereas using the polynomial model introduces a reduction of at least 30-40%. This is expected as the polynomial model, in comparison with the bi-linear model, applies more correction for earthquakes larger than approximately \( M_L \) 5.0 (Figure 2), which should translate to lower annual rates of larger earthquakes.

- **Method 2:** This approach follows the standard procedure of applying magnitude conversion equations, i.e. applying the conversion equation to the whole catalogue, and then computing the earthquake recurrence parameters by, for example, fitting the truncated Gutenberg-Richter model. This is the method used for Geoscience Australia’s 2018 draft national seismic hazard assessment (Allen et al., 2017). Following this procedure and apply the bi-linear conversion equation, we
observe a 30% increase in the Gutenberg-Richter $b$-value, i.e. $b$-value=1.3. This is expected as our conversion equation effectively increases the number of small and moderate earthquakes relative to large events. This would in turn reduce the level of seismic hazard for the region of interest. In our example the level of reduction in PGA hazard level by applying this method is at least 30-40% (Figure 6c).

- **Method 3**: In this approach, we first estimate the earthquake recurrence parameters from the original catalogue. Based on the magnitude conversion equation and for each magnitude bin, the annual rates which are computed from the fitted earthquake recurrence model are adjusted, e.g. truncated Gutenberg-Richter model. Applying this method to our example causes 20-30% reduction in the level of PGA hazard (Figure 6d).

Overall, based on our synthetic example, Method 2 yields a greater reduction in PGA values than the other techniques. This is expected as this method reduces the number of moderate-to-large earthquakes slightly more than other two methods due to the post-magnitude conversion fitting process, i.e. fitting the truncated Gutenberg-Richter model into the catalogue with converted magnitudes. In contrast, Method 1 and 3 have comparable level of PGA values; however Method 1 has the advantage of capturing the aleatory variability of the magnitude conversion equations through a random sampling process (not addressed here). The major disadvantage of Method 1 is that is very computationally expensive and would result in very long run-times should it be implemented for a national-scale hazard assessment. The results of the synthetic example need further verification by applying the suggested techniques to the actual input models of the national seismic hazard model.

Figure 6: The percentage difference between PGA hazard values calculated based on classical approach for $M_L$-based Gutenberg-Richter model with those based on a) Method 1 and using the bi-linear conversion equation, b) Method 1
and using the polynomial conversion equation, c) Method 2 and using the bi-linear conversion equation, and d) Method 3 and using the bi-linear conversion equation.

CONCLUSION
In this study we have developed Australian specific magnitude conversion equations between \( M_W \) and other magnitude types: \( M_L \), \( m_b \), and \( M_S \). In the absence of measured \( M_W \) values for most of the earthquakes listed in the Australian earthquake catalogue, such conversion equations are needed to homogenize the catalogue in terms of moment magnitude. The compiled reference catalogue represents the \( M_L-M_W \) data distribution relatively well over the magnitude range of interest (i.e. 3.0 <\( M_L < 6.0 \)). The derived \( M_L-M_W \) conversion equation is in good agreement with the results of previous empirical and theoretical studies, and shows that there is no one-to-one relationship between the aforementioned magnitude scales. In contrast, earthquakes with preferred magnitude types of \( m_b \) and \( M_S \) are less-well represented in the reference catalogue. Hence care should be taken in applying published magnitude conversion equations in the Australian context. The coefficients of the magnitude conversion equations (and the reference catalogue) may be further refined and will be published upon completion of the 2018 National Seismic Hazard Assessment of Australia.

Using synthetic earthquake catalogues, the sensitivity of the SHA results to the selection and application of magnitude conversion equations were explored. For our arbitrary scenario, the results indicate that we may expect up to 20-40\% reduction in PGA hazard, depending on the method selected for applying the magnitude conversion equations. To further explore the results of the sensitivity analysis, we suggest applying similar techniques to the actual input seismic source models of the national seismic hazard assessment. Furthermore it should be noted that such conversion equations are associated with uncertainties that can be quantified (i.e. aleatory variability). Taking into account such variability may have significant impacts on SHA results and has not been fully explored in previous studies. Hence it is crucial to develop a framework to capture the aleatory variability of such magnitude conversion equations and examine its effects on SHA results.

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REFERENCES


