

# Influence of spandrel modelling on the seismic assessment of existing masonry buildings

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**ABSTRACT:** The paper discusses different modelling assumptions for spandrels in equivalent frame models and their effect on the global response of the wall. The topic is motivated by the following issues: i) reliable and practice oriented numerical tools constitute a key instrument to support mitigation policies and plan effective (in terms of cost and impact) retrofit interventions; ii) spandrels have lately been increasingly recognized to play a significant role on the load bearing capacity of masonry walls; iii) despite the experimental evidence of last years that highlighted significant differences between spandrels and piers, numerical tools available in literature and recommendations of codes do not yet account properly for the progress achieved with regard to spandrel models. To deepen such issues, various strength criteria specifically formulated for spandrels and available in literature have been implemented in the Tremuri program, including those proposed by Beyer (2012) that have been recently incorporated in the updated release of the NZSEE recommendations. With the so enhanced program, parametric nonlinear static analyses on 2D masonry walls have been performed to quantitatively assess the influence of spandrel modelling.

## 1 INTRODUCTION

The seismic vulnerability of existing masonry buildings to earthquakes has been demonstrated by past and recent earthquakes. To assess the seismic performance and plan retrofit interventions, good predictions of the expected seismic behaviour are necessary: in particular, over-conservative seismic strength estimates of URM buildings may potentially lead to unnecessary or inadequate interventions. This is undesirable from a socio-economic point of view as the retrofit costs are unnecessarily high and, in case of cultural heritage structure, also the value might be negatively impacted by such interventions.

Traditionally, one distinguishes between two structural elements in a masonry wall: *piers*, the vertical panels, which are the most important elements, resisting both to static and seismic loads; *spandrels*, the horizontal elements between two vertically-aligned openings that couple two vertical piers. Although traditionally classified in the hierarchy of structural walls as secondary elements, spandrels have lately been increasingly recognized to play a significant role on the load bearing capacity of masonry walls while an increasing number of experimental campaigns proved that they are characterized by specific features which are different from the seismic behavior of piers.

Despite this, numerical tools available in literature and recommendations of codes do not yet account properly for the progress achieved with regard to spandrel models.

Within this context, the paper discusses different modelling assumptions for spandrels in Equivalent Frame (EF) models and their effect on the global response of the wall. The choice to focus on EF models mainly derives from its effective capability to perform nonlinear static analyses with a limited computational effort that promoted its widespread use also in engineering practice, being moreover explicitly recommended by various codes at the national and international level.

To this aim, various strength criteria specifically formulated for spandrels, that constitute the most up-

to-date state of the research available in literature, have been implemented in the Tremuri program (Lagomarsino et al. 2013), including those proposed by Beyer (2012) that have been recently incorporated in the updated release of the NZSEE recommendations (in the following namely NZSEE 2015). With the so enhanced program, parametric nonlinear static analyses on 2D masonry walls have been performed in order to quantitatively assess how spandrel modelling may affect the seismic behaviour of URM buildings and the outcome of safety verifications.

## 2 KEY FEATURES OF SPANDREL SEISMIC RESPONSE AND OVERVIEW OF STRENGTH CRITERIA PROPOSED IN LITERATURE AND CODES

As aforementioned, in the last decade an increasing number of experimental campaigns investigated the response of masonry spandrels (e.g. Beyer and Dazio 2012, Graziotti et al. 2012, Knox 2012, Parisi et al. 2014). These campaigns showed that there are some significant differences between piers and spandrels. In fact, the behavior of piers is essentially controlled by their geometry, masonry characteristics and the boundary conditions. For spandrels, the following further parameters have an important influence on the force-displacement response: i) the interlocking of bricks at end-sections with the contiguous masonry portions; ii) the type of lintels if present (masonry arches or lintels in stone, timber, steel or reinforced concrete); iii) the axial restraint provided by other structural elements (e.g. reinforced concrete beams, steel tie-rods or adjacent piers).

Experimental results for spandrels have only been available relatively recently. As a result, only very few specific criteria are proposed in codes, which usually assume the spandrel behaviour as that of a pier rotated by  $90^\circ$  (provided that an effective lintel/architrave is supporting the spandrel, as explicitly indicated in EC8-3 2005, NTC 2008, ASCE 41-13 2014). More precisely, NTC 2008 makes a distinction in the strength criterion to be adopted for spandrels as a function of the acting axial load (if known or unknown from the analysis) and ASCE 41-13 (2014) points out the relevance of the issue indicating some literature references without explicitly adopting any of them. The NZSEE 2015, for which the updating process has been recently completed, constitutes the most up-to-date recommendation with respect the research progress in this field, having incorporated the criteria proposed by Beyer (2012) that, founded on aforementioned experimental evidences, provide a comprehensive set of criteria mechanically based for the assessment of both peak and residual strength, including also the lintel's contribution. A comprehensive review of various criteria proposed in literature for spandrels is illustrated in Beyer and Mangalathu (2013).

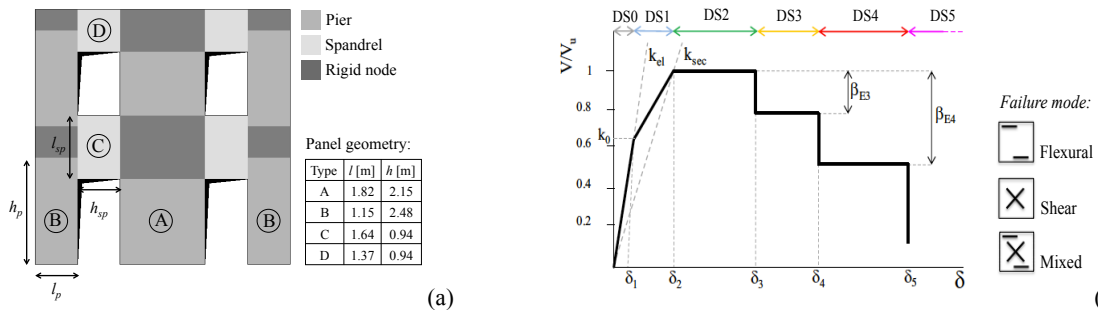
In particular, it is recognized by the authors that a reliable interpretation of the flexural spandrel response plays one of most critical issues since it affects the interaction with piers and is mainly responsible of a reliable prediction of the overall base shear of masonry walls/buildings. In Cattari et al. (2015a) an elementary system composed by one pier and two adjacent spandrels has been presented to highlight the repercussions of such interaction. Indeed, due to very low axial loads that usually characterize spandrels (in particular when tensile resistant elements coupled to them are absent), the adoption of a *Rocking/Crushing* failure criterion analogous to that of pier may lead to a significant underestimation of the actual strength associated to this mechanism. The consequence is that, very often, in EF models that simulate the response of panels as nonlinear beams, spandrels are ineffective just from the beginning of the analysis, thus inducing the cantilever conditions of piers. Indeed, thanks to the interlocking phenomena aforementioned, spandrels may rely on the contribution of an "equivalent tensile strength" ( $f_{t,sp}$ ) that alters the flexural strength domain usually adopted for piers (based on the beam theory, neglecting the tensile strength of the material and assuming an appropriate normal stress distribution at the compressed toe), at least with reference to the peak strength. This is recognized by different proposals in literature (Cattari and Lagomarsino 2008, Beyer 2012, FEMA 306 1998) and also confirmed by some experimental evidences (Beyer and Mangalathu 2013). Main differences and analogies among such proposals may be summarized as follows. On the computation of  $f_{t,sp}$ , if only the frictional contribution (Cattari and Lagomarsino 2008) or also the cohesive one (FEMA 306, Beyer 2012) are considered and if a limitation associated to the tensile failure of blocks is included or not in addition to the checks on joints. On the computation of the peak strength, if a linear distribution of stresses along the length of the spandrel (FEMA 306, Beyer 2012) or an elasto-perfectly plastic constitutive law with limited ductility (Cattari and Lagomarsino 2008) are assumed and if the

the effect of the axial load acting on the spandrel is neglected (FEMA 306) or not. Finally, on the computation of the residual strength, differences again arise in the stress distribution assumed.

Parametric analyses illustrated in Section 4 aim to highlight the potential effects of such issues at the scale of the URM wall response.

### 3 KEY FEATURES OF THE NUMERICAL TOOL ADOPTED

Parametric study illustrated in the paper have been carried out using the Tremuri program which is a software specifically oriented towards the seismic assessment of masonry buildings. Tremuri builds on the equivalent frame approach and allows to perform nonlinear static and dynamic analyses on 2D and 3D structures. A comprehensive description of general features of the software is illustrated in Lagomarsino et al. (2013), while in the following the attention is focused only on criteria adopted to describe the nonlinear response of masonry panels and the novelties recently implemented for spandrels. Among various options, masonry panels are herein modelled as nonlinear beam elements with lumped plasticity and a piecewise-linear behaviour (Cattari and Lagomarsino 2013). In particular, the latter allows to describe the non linear response until very severe damage levels (from 1 to 5) through progressive strength decay ( $\beta_{Ei}$ ) in correspondence of assigned values of drift ( $\delta_{Ei}$ ), that may be different for piers and spandrels and that are a function of various failure modes that may occur (*flexural*, *shear* and also *mixed* failure modes). The stiffness degradation is computed in an approximate way by assigning (see figure 1b): a proper ratio between the initial ( $k_{el}$ ) and secant stiffness ( $k_{sec}$ ) (the latter corresponding to the point in which the maximum strength is reached); a ratio between the shear at the end of the elastic phase and the shear strength ( $k_0$ ). The yielding strength is computed according to common criteria also adopted in codes to interpret different failure modes (*Rocking/Crushing*, *Diagonal Shear Cracking*, *Bed Joint Sliding*). Moreover, also a hysteretic response is formulated, through a phenomenological approach, useful for performing nonlinear dynamic analyses. Figure 1 illustrates the equivalent frame idealisation of the 2D URM wall analyzed in Section 4, that shows the nodes that are assumed as rigid, the structural elements piers and spandrels where the deformations are concentrated and the piecewise-linear behavior that is assumed for them.



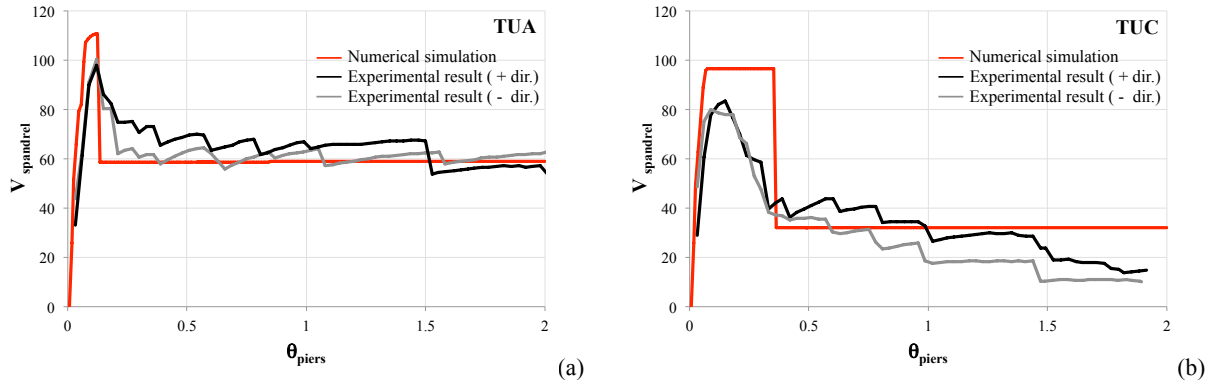
**Figure 1.** Equivalent frame idealisation of the “Door wall” (a) and idealisation of the multi-piece constitutive law adopted for masonry panels (b)

In addition to the options already available in Tremuri, a comprehensive set of new tools specifically oriented to spandrels elements have been recently implemented by the authors:

- the possibility of selecting different criteria for the flexural response: in particular, that proposed in Beyer (2012) has been added to those already available of Cattari and Lagomarsino (2008) and that proposed in NTC (2008) and EC 8-3 (2005). Moreover, different options are available that allow to include or not: the cohesive and/or frictional contribution of mortar joints (distinguishing by that of head and horizontal joints); the tensile contribution provided by the architrave element coupled to the spandrel;
- the computation of the residual strength according to the criteria proposed in Beyer (2012) as a function of two different architrave types, i.e., a lintel beam or an arch system (instead of assigned values of strength decay through  $\beta_{Ei}$ ).
- the definition of the extension of the plastic branch (that is defined by the yielding condition and the failure of the spandrel) on the basis of an assigned ductility value instead of a given drift value as in the original formulation of the multi-piece constitutive laws available in

Tremuri before. That complies with the evidences of some numerical studies illustrated in Beyer and Mangalathu (2014).

The correct implementation in Tremuri has been tested by simulating the experimental campaign at the scale of single spandrels illustrated in Beyer and Dazio (2012). For now, the attention has been focused on the response of masonry spandrels characterized by brick blocks with mortar joints. As illustrated in Figure 2, the comparison between numerical (in which the strength criteria for the flexural response proposed in Beyer 2012 have been adopted) and experimental results are very satisfactory: indeed, despite some slight overestimation of the peak strength, the use of simplified and mechanically based strength criteria presents the advantage to be versatile and can be directly correlated to mechanical parameters usually available from in-situ tests.



**Figure 2.** Numerical simulation of spandrel test units tested by Beyer and Dazio (2012) by the enhanced Tremuri program: (a) TUA test unit with timber lintel and constant axial force applied equal to 80 kN; (b) TUC test unit with masonry arch and constant axial force applied equal to 80 kN

#### 4 PARAMETRIC NONLINEAR STATIC ANALYSES ON 2D WALLS

The impact of the enhanced numerical tool has been tested to analyze the response of a 2D two storey masonry wall by nonlinear static analyses. In particular, the full-scale two storey masonry building prototype with openings experimented by Calvi and Magenes at the University of Pavia in 1994 (Calvi and Magenes 1994) has been used as a reference structure. The building, having a plan of 6 m x 4.4 m and a height of 6.4 m, has flexible floor diaphragms and contains an almost independent in-plane loaded shear wall. The wall here considered (named as “Door wall”) is 25 cm thick and has two door openings on the first storey and two window openings on the second storey (Fig. 1a); the experimental test has been performed by applying a constant vertical load to the two floors ( $P_1=14.1$  kN,  $P_2=13.8$  kN) followed by a cyclic history in which the imposed displacement at the two floor levels was such that the two corresponding forces were equal (“uniform” load pattern). Differently from the experimental tests, since the focus was the evaluation of the spandrel role than a rigorous simulation of the test, the non-linear static analysis has been performed monotonically.

Together with the results provided by the experimental tests (in terms of overall pushover curve and damage pattern), also those carried out through a numerical simulation with a Finite Element (FE) model have been used as target reference to check the reliability of the EF model. A detailed description of such numerical modelling is presented in Calderini et al. (2009). Table 1 summarizes the main parameters necessary for defining the multi-piece constitutive laws adopted for masonry panels (Fig.1b); it is worth noting that masonry mechanical properties have been adopted coherently with those available from the experimental campaigns, including that illustrated in Anthoine et al. (1995) on cyclic static tests performed on masonry piers characterized by a masonry type and geometry analogous to that of “Door wall”. Mechanical parameters adopted in the FE model are consistent with those adopted for the EF model.

The EF model was first validated against these experimental/FE numerical results and then the spandrel configurations were varied in order to investigate their effect on the overall response as a function of different assumptions. Table 2 lists all the examined cases.

**Table 1. Mechanical parameters adopted for describing the multi-piece constitutive law of piers and spandrels**

Masonry mechanical properties <sup>(+)</sup>						Drift limits and strength decay piers/spandrels [%]					
Element	$f_m$	$c$	$\mu$	$\phi^{(c)}$	$f_{bt}$	Failure	$\delta_{E3}$	$\delta_{E4}$	$\delta_{E5}$	$\beta_{E3}$	$\beta_{E4}$
type	[MPa]	[MPa]			[MPa]	mode					
pier	6.2	0.23	0.58	0.5	1.22	flex.	0.6/ <sup>(*)</sup>	1/2	1.5/3	-/ <sup>§</sup>	15/ <sup>§</sup>
spandrel <sup>(++)</sup>	6.2	0.23	0.58	0.86	1.22	shear	0.3/ <sup>(*)</sup>	0.5/2	0.7/3	30/ <sup>§</sup>	60/ <sup>§</sup>

<sup>(+)</sup> Besides to define the mechanical parameters ( $f_m$  compressive strength,  $c$  cohesion,  $\mu$  friction coefficient,  $f_{bt}$  tensile strength of block), the results of Anthoine et al. (1995) were useful to choose the value of  $\beta_{Ei}$  for piers.

<sup>(c)</sup> The interlocking parameter  $\phi$  has been defined on the basis of the brick dimensions, in case of spandrels (as  $2\Delta_y/\Delta_x$  being  $\Delta_y$  and  $\Delta_x$  the height and length of the block, respectively), and taking into account also the masonry pattern (“*english bond*”) in case of piers.

<sup>(++)</sup> Value of  $c$  and  $\mu$  refer to the horizontal mortar joints; for the head joints the same values have been assumed apart from Case III-a and Case III-b where the head joints contribution have been neglected or halved.

<sup>(\*)</sup> Value of  $\delta_{E3}$  in case of spandrels have been computed by assigning a ductility equal to 4 (computed from the yielding point). Such value has been defined according to evidences of numerical results illustrated in Beyer and Mangalathu (2014).

<sup>§</sup> Value of  $\beta_{E3}$  in case of spandrels have been computed as specified in Table 2

**Table 2. Set of examined configurations for the “Door wall” analysed by the EF model**

Case	Flexural strength criterion <sup>(*)</sup>	Computation of $f_{t,sp}$		Residual strength <sup>(+)</sup>	Steel beams
	Flexural	Horiz. joints	Head joints		
I-a	NTC 2008	No	No	Conv.	No
I-b	NTC 2008	No	No	Conv.	Yes
II-a	Beyer 2012	Yes	Yes	Mech.	No
II-b	Beyer 2012	Yes	Yes	Mech.	Yes
II-c	Cattari & Lagomarsino 2008	Yes	No	Mech.	No
III-a	Beyer 2012	Yes	No	Mech.	Yes
IV-a	Beyer 2012	Yes (0.7)	Yes (0.7)	Mech.	Yes
IV-b	Beyer 2012	Yes (0.5)	Yes (0.5)	Mech.	Yes

<sup>(\*)</sup> For the shear response it has been assumed a criterion consistent with that of Mann and Müller (1980) for interpreting the *Diagonal Shear Cracking*. In Cases IV a reduced value of the cohesion and the friction coefficient has been adopted coherently with the assumption of a poor quality of spandrel masonry (by adopting a reduction factor equal to 0.7 or 0.5).

<sup>(+)</sup> “Conv” means that the residual strength of spandrels have been conventionally assumed by adopting  $\beta_{E3}=\beta_{E4}= 50$ , while “Mech.” means that the residual strength has been defined on the basis of the mechanical criteria proposed in Beyer (2012).

In particular eight different configuration have been analyzed as a function of: a) the criteria adopted for the flexural strength criterion if the contribution of the equivalent tensile strength of spandrels  $f_{t,sp}$  was included or not; b) the contributions eventually considered in the computation of  $f_{t,sp}$  as listed in Section 2; c) the presence or not of a steel beam coupled to the spandrels. Indeed, in the experimental campaign two steel beams were used to introduce the horizontal forces that seemed to work as “tie-rods”, inhibiting the flexural failure of spandrels (as testified by the actual damage occurred).

#### 4.1 Influence of spandrels on the seismic behaviour and pushover curves

Figure 3 summarizes the main results of the parametric analyses in terms of pushover curves, while Figure 4 shows the results in terms of the obtained damage pattern. Figure 3 highlights that the global force-displacement response of the wall is strongly dependent on the assumed spandrel behaviour. In particular it is worth noting:

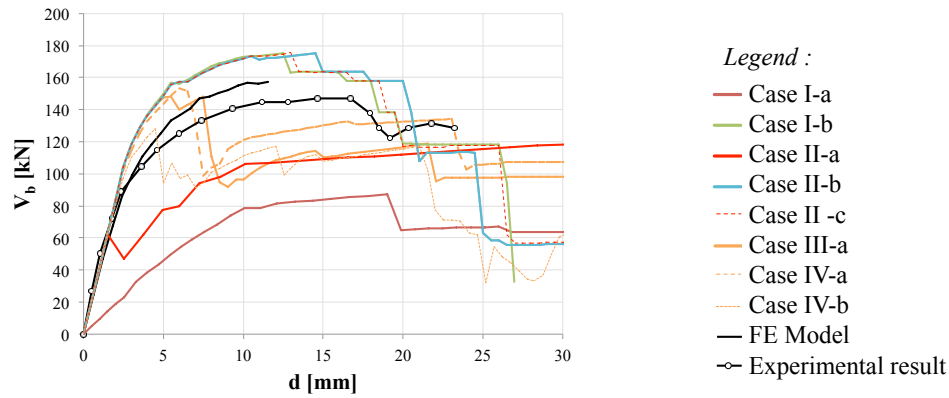


Figure 3. Effect of spandrel modelling on the pushover curve of the wall

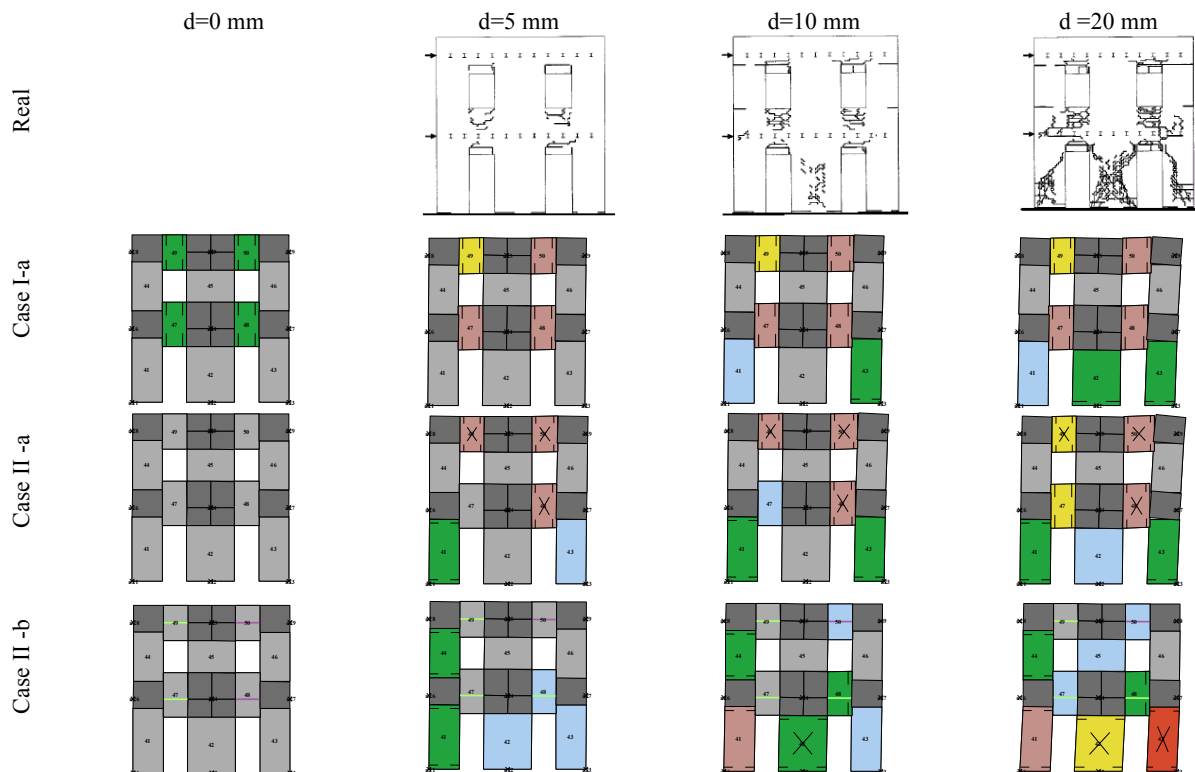


Figure 4. Effect of spandrel modelling in terms of damage pattern (for the damage legend – colours and symbols adopted for representing failure modes - see also Figure 1b)

- the high variability of seismic response predicted by adopting the criteria actually proposed in Italian code NTC 2008 (which can be taken as a reference for the most common assumptions that are used in today's codes). In Case I-a, where no tensile resistant element is coupled to the spandrels, the flexural strength of spandrels is almost null. As a result, spandrels reach flexural yielding very early on and the wall response corresponds almost to that of uncoupled piers (Fig.4). Such response seems unrealistic if compared to the real damage testified after earthquakes that does not confirm such dominance of the flexural response of spandrels in existing buildings. Indeed, such assumption leads potentially to a significant underestimation of the actual base shear capacity of the masonry buildings—and also potentially to an overestimation of the displacement capacity of the wall. The predicted response changes completely if a tie-rod or a steel beam is coupled to the spandrel (Case I-b);
- as a result of the equivalent tensile strength  $f_{t,sp}$  (Case II-a), spandrels are at the beginning elastic and able to couple the piers in an effective manner (Fig.5). Such effect, even in the absence of tensile resistant elements, is confirmed by various experimental studies. The strong influence of this assumption on the initial response of the wall highlights the usefulness of the im-

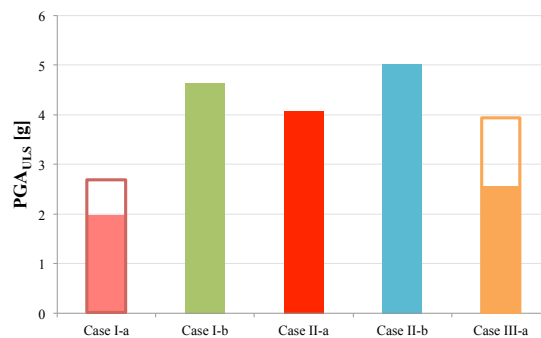
plementation of such updated criteria for the flexural response. Case II-b reflects the effect of the introduction of steel beams (comparable to Case I-b).

- the role of the head mortar joints to the computation of  $f_{t,sp}$ , even if a steel beam is coupled to spandrels (highlighted by Case III). Indeed, to neglect (Case III-a) such contribution significantly affects the response and is potentially equivalent to assume reduced mechanical strength properties of spandrels (Case IV).

Cases II-b and III are the most consistent with the experimental results in terms of strength and damage pattern (with a spread damage both in spandrels and piers). This is due to the aforementioned role played in the experimental campaign by two steel beams used to apply the horizontal forces.

#### 4.2 Consequences on the safety verification

Such differences in terms of pushover curves impact of course the safety verification, illustrated in Figure 5 in terms of maximum peak ground acceleration compatible with the ultimate limit state ( $PGA_{ULS}$  computed according to the procedure adopted in NTC (2008) and EC8-3 (2005) based on the N2 Method originally proposed in Fajfar (1999).



**Figure 5.** Impact of spandrel modelling on the assessment of the maximum peak ground acceleration compatible with the fulfilment of the ultimate limit state

In particular, the ultimate limit state has been conventionally defined as the displacement associated with a 20% loss of strength (filled rectangles in Fig.5). Figure 3 shows that in some cases (particularly evident for Cases III and IV) after a first sudden decay (associated to the flexural cracking of the spandrels and loss of  $f_{t,sp}$  contribution) the base shear remains stable: this is mainly due to the change of the boundary conditions of the piers from a fixed-fixed condition to a cantilever system (with a change also in the prevailing failure mode from shear to flexural). If this residual base shear capacity is considered when computing the bilinear pushover curve, another—higher—value of  $PGA_{SLU}$  can be computed, which is indicated in Figure 5 by the unfilled rectangles for Case I-a and III-a. The figure shows that the criteria adopted to define the limit states may lead to significant variation of the value of  $PGA_{SLU}$ . Although the case study is rather simple, such aspect may appear secondary, in case of more complex URM buildings, especially in presence of irregularities and/or flexible diaphragms, it becomes really crucial as recently discussed in Lagomarsino and Cattari (2015) and Cattari et al. (2015b). Apart from the reflections with regard to the definition of the limit state, Figure 5 stresses how strength criteria proposed to date in codes risk to lead to an over-conservative assessment if the spandrels are not coupled to other tensile resistant elements.

## 5 FINAL REMARKS

In the paper an enhanced version of the Tremuri program, which includes the new state-of-art strength models for spandrel elements, has been presented. In particular, it allowed to quantitatively assess the effect of different choices on the strength criteria adopted for spandrels. The hypotheses assumed for computing the flexural strength were found to be the most crucial points that affect the interaction between piers and spandrels and, thus, the global response of URM buildings. Indeed, the assumption of most codes to neglect the contribution of the equivalent tensile strength—as in the case of piers—appeared too conservative. Such conservatism leads potentially to unnecessary retrofit interventions, which are often irreversible.

## ACKNOWLEDGMENTS:

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