

# Simulation of seafloor seismic motions in southwest of Western Australia

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**ABSTRACT:** A simulation method of seafloor seismic motions is presented in this paper. The ground motion transfer functions of an offshore site are theoretically derived using the fundamental hydrodynamics equations and one-dimensional wave propagation theory. The effect of seawater layer and water saturation of subsea soil layers on seabed ground motions are taken into account in the derived model. In the simulation, the three-dimensional seismic motions are assumed to consist of out-of-plane SH-wave or in-plane combined P- and SV-waves. The base rock motion is simulated by the seismological model for the area of southwest of Western Australia (SWWA). The ground motions on typical onshore and seafloor sites are stochastically simulated based on the SWWA model and the calculated transfer functions of respective sites. The simulation results show that the vertical-to-horizontal (V/H) PGA ratios of seafloor motions are much lower than those of onshore motions, due to the significant suppression effect of seafloor vertical motions near the P-wave resonant frequencies of the seawater layer. The characteristics of generated seafloor motions are consistent with those of available seafloor earthquake recordings. The simulated seafloor motions can be used as more realistic inputs in the seismic analyses of offshore engineering structures.

## 1 INTRODUCTION

A large number of offshore platforms and pipelines have been constructed for energy exploitation use during the past few decades in southwest of Western Australia (SWWA). In the design of these offshore engineering structures to resist earthquake loadings, the definition of input ground motions is very crucial. Onshore seismic motions are commonly employed as inputs in the seismic response analyses of offshore structures due to the lack of seafloor earthquake ground motion recordings. This may result in unreliable seismic response predictions because the seismic motions at seafloor can be very different from those on an onshore site. Previous studies (Crouse and Quilter 1991, Boore and Smith 1999) on the statistical analyses of seafloor earthquake recordings have revealed that the vertical component of seafloor seismic motion can be significantly suppressed near the resonant frequencies of seismic P-waves in the overlying seawater layer. Moreover, seawater can indirectly increase the water saturation of offshore sites and affect the P-wave propagation in porous soil layers, which also influences the ground motion amplification effect of the offshore sites (Yang and Sato 2000, Wang and Hao 2002). Therefore, it is very important to properly define seafloor motions and develop a simulation method of artificial seafloor seismic motions in SWWA for more realistic seismic response analysis and design of offshore structures.

The spectral representation method (SRM) is one of the most commonly used methods for the simulation of seismic ground motions. The representation approach of a random process was firstly put forward by Rice (1954). Shinozuka (1971) extended this methodology in the simulations of random processes. Hao et al. (1989) utilized the SRM and developed a method to simulate spatially varying ground motions on a flat site in the frequency domain. To consider the local site effect on the ground motions, Deodatis (1996) synthesized seismic motions on different sites by using the evolutionary power spectra with different power spectral densities at different locations. The drawback of this method is that it can only approximately model the local site effect because only the fundamental vibration frequency of the site can be considered. In fact, the energy of seismic motions

on the site surface concentrates at multiple frequency bands corresponding to the various vibration modes of the local site. To realistically represent the ground motion amplification effect of a site with multiple soil layers, the one-dimensional (1D) wave propagation theory (Wolf 1985) is employed by Bi and Hao (2012) in the simulation of seismic ground motions. However, the methodology regarding the simulation of seafloor seismic motions, in which the effects of the overlying seawater layer and water saturation of porous soil layers should be taken into account, cannot be found in the literature.

In this paper, a simulation method of seafloor seismic motions in SWWA is proposed. The ground motions on the free surface of base rock are simulated by the SWWA seismological model presented by Hao and Gaull (2009). The base rock motions are assumed to consist of out-of-plane SH-wave and in-plane combined P- and SV-waves propagating into the layered soil site with an assumed incident angle. The ground motion transfer functions of an offshore site, which is assumed to compose of the base rock, the porous soil layers and a upper seawater layer, are theoretically derived based on the fundamental hydrodynamics equations (Batchelor 2000) and 1D wave propagation theory (Wolf 1985). The effects of seawater layer and water saturation of subsea soil layers are both considered. Seismic ground motions on typical SWWA onshore and offshore sites are stochastically simulated using spectral representation method. The characteristic differences of generated onshore and seafloor motions are discussed in detail.

## 2 THEORETICAL DERIVATION OF GROUND MOTION TRANSFER FUNCTIONS OF A BASE ROCK SITE OVERLAID WITH SEAWATER

Because the viscosity coefficient is very low for common fluids such as seawater, the influence of viscosity is normally neglected in most of the engineering hydrodynamics problems and the fluid can be regarded as an ideal fluid, in which only the P-wave can propagate. The linear wave equation of P-wave propagating in seawater can be expressed as (Jensen et al. 2011)

$$\nabla^2\psi - \frac{1}{c_p^2} \frac{\partial^2\psi}{\partial t^2} = 0 \quad (1)$$

in which  $\nabla^2$  represents the Laplace operator;  $\psi$  is the displacement potential, which can be defined by  $\mathbf{u}=\nabla\psi$  with  $\mathbf{u}$  denoting the fluid particle displacement;  $c_p=K/\rho$  is the velocity of seismic P-wave propagating in seawater with  $K$  and  $\rho$  denote the fluid bulk modulus and density, respectively.

For a sinusoidal wave, the linear wave equation can be rewritten as the Helmholtz Equation (Lurton 2002):

$$\nabla^2\psi + \frac{\omega^2}{c_p^2} \psi = 0 \quad (2)$$

where  $\omega$  is the circular frequency, Equation (2) can be solved with the P-wave trail function:

$$\psi = A_p \exp\left[\frac{i\omega}{c_p}(-l_x x - l_y y - l_z z)\right] \quad (3)$$

in which  $A_p$  is the amplitude of P-wave,  $l_x$ ,  $l_y$  and  $l_z$  are the direction cosines of seismic P-wave in the Cartesian coordinate system. The wave equation is satisfied by providing

$$l_x^2 + l_y^2 + l_z^2 = 1 \quad (4)$$

With Equation (3), the displacement vector of fluid particle can be expressed as

$$u_m = -l_m A_p \frac{i\omega}{c_p} \exp\left[\frac{i\omega}{c_p}(-l_x x - l_y y - l_z z)\right] \quad (5)$$

where  $m$  represents  $x$ ,  $y$  or  $z$  direction in the Cartesian coordinate system.

The fluid wave pressure can be formulated with the bulk modulus  $K$  and the displacement potential  $\psi$  (Jensen et al. 2011):

$$p = -K\nabla^2\psi \quad (6)$$

As stated in the classical fluid dynamics (Batchelor 2000), all the stress tensor components of the Newtonian fluid in the Cartesian coordinates can be represented as

$$\sigma_{ij} = -p_f\delta_{ij} + \mu \left[ \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) - \frac{2}{3}\delta_{ij}\nabla \cdot \mathbf{v} \right] \quad (7)$$

where  $\mu$  is the coefficient of viscosity;  $\delta_{ij}$  equals 1 when  $i=j$  and 0 when  $i \neq j$ ;  $p_f$  denotes a measure of compressive stress in a flowing fluid and it is a scalar whose value equals the average compressive stress acting on the surface of an infinitesimal fluid particle.

Under the ideal fluid assumption, the viscosity coefficient  $\mu$  is zero and only the first part, namely the normal stress remains in Equation (7). Moreover,  $p_f$  can be replaced by the scalar of P-wave induced fluid pressure  $p$  as expressed in Equation (6). Thus, the fluid normal stress caused by the seismic P-wave is written as

$$\sigma_{nn} = -KA_p \frac{\omega^2}{c_p^2} \exp \left[ \frac{i\omega}{c_p} (-l_x x - l_y y - l_z z) \right] \quad (8)$$

in which  $\sigma_{nn}$  denotes the normal stresses in the Cartesian coordinates along any  $x$ ,  $y$  or  $z$  axis. Thus, the fluid particle displacements and stresses under seismic excitations are both obtained as expressed in Equations (5) and (8).

The 1D wave propagation theory presented by Wolf (1985) can be employed to calculate the ground motion transfer functions of an onshore site with multiple soil layers. However, the influence of seawater is not considered in Wolf's theory. In this study, the approach presented in the 1D wave propagation theory is combined with the above hydrodynamics formulas to theoretically derive the ground motion transfer functions of a base rock site overlaid with a seawater layer. For seismic P-wave propagating in the vertical  $x$ - $z$  plane ( $l_y=0$ ,  $l_x^2 + l_z^2=1$ ), the vertical fluid displacement and normal stress can be formulated as

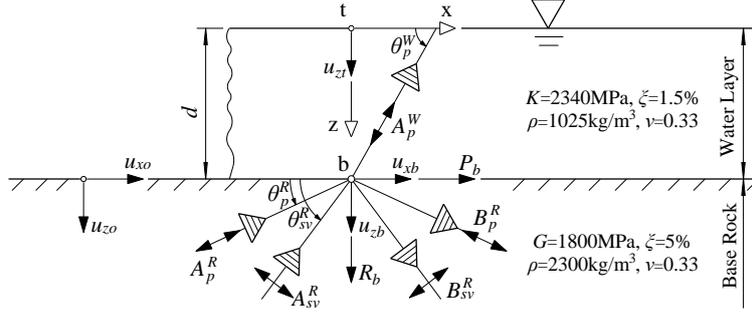
$$u_z = -l_z A_p \frac{i\omega}{c_p} \exp \left[ \frac{i\omega}{c_p} (-l_x x - l_z z) \right] \quad (9)$$

$$\sigma_{zz} = -K^* A_p \frac{\omega^2}{c_p^{*2}} \exp \left[ \frac{i\omega}{c_p^*} (-l_x x - l_z z) \right] \quad (10)$$

where  $K^*$  and  $c_p^*$  are the bulk modulus and P-wave velocity in which the fluid hysteretic damping is introduced. Using the approach presented in 1D wave propagation theory, the dynamic stiffness matrix of the seawater can be derived by analysing the relationship between the amplitudes of fluid displacements and normal stresses on the top and bottom of the seawater layer:

$$\begin{Bmatrix} R_1 \\ R_2 \end{Bmatrix} = [S_p^w] \begin{Bmatrix} u_{z1} \\ u_{z2} \end{Bmatrix} = \frac{K^* \omega}{l_x s c_p^* \sin ksd} \begin{bmatrix} \cos ksd & -1 \\ -1 & \cos ksd \end{bmatrix} \begin{Bmatrix} u_{z1} \\ u_{z2} \end{Bmatrix} \quad (11)$$

where the  $R_1$ ,  $u_{z1}$  and  $R_2$ ,  $u_{z2}$  denote the vertical load amplitudes and displacements on the top and bottom of the seawater layer, respectively;  $k$  is the wave number and  $s$  is the tangent value of the incident angle.  $[S_p^w]$  is the dynamic stiffness matrix of the seawater layer, which is determined by the circular frequency  $\omega$ , the seismic incident angle  $\theta_p$  and the property of seawater, i.e., the bulk modulus  $K$ , the density  $\rho$ , the water depth  $d$  and the hysteretic damping ratio  $\zeta$ .



**Figure 1. Nomenclature for in-plane motions in a base rock site overlaid with seawater**

Assuming that the base rock motions are composed of out-of-plane SH-wave and combined in-plane P- and SV-waves, the dynamic equilibrium equation of the site can be expressed in the frequency domain as (Wolf 1985)

$$[S_{SH}]\{u_{SH}\} = \{P_{SH}\} \quad \text{or} \quad [S_{P-SV}]\{u_{P-SV}\} = \{P_{P-SV}\} \quad (12)$$

in which  $\{u_{SH}\}$ ,  $\{u_{P-SV}\}$  and  $\{P_{SH}\}$ ,  $\{P_{P-SV}\}$  are the displacements and load vectors of the in-plane SH-wave and out-of-plane combined P- and SV-waves, respectively. For a base rock site overlaid with seawater, the in-plane total dynamic stiffness matrix of the site  $[S_{P-SV}]$  can be obtained by assembling the derived dynamic stiffness matrix of seawater layer with that of the base rock (Wolf 1985). The out-of-plane dynamic stiffness matrix  $[S_{SH}]$  is not affected by the seawater since the S-wave cannot be propagated in ideal fluid.

Figure 1 shows an underwater base rock site with in-plane combined P- and SV-waves. Solving Equation (12) at every discrete frequency, the in-plane vertical transfer functions of the underwater site, namely the ratio of underwater rock vertical motion  $u_{zb}$  to outcropping vertical motion  $u_{zo}$  can be theoretically derived as

$$H_z(\omega) = \frac{u_{zb}}{u_{zo}} = \frac{1}{1 + \frac{abs' \tan ksd}{(2kG^* - b)^2 + b^2 s' t'} \cdot i} \quad (13)$$

and its modulus is

$$|H_z(\omega)| = \left| \frac{u_{zb}}{u_{zo}} \right| = \frac{1}{\sqrt{1 + \left[ \frac{abs' \tan ksd}{(2kG^* - b)^2 + b^2 s' t'} \right]^2}} \quad (14)$$

in which  $a = K^* \omega / l_x^W s c_p^{*W}$  and  $b = kG^*(1+t^2)/(1+s't)$ ; the superscript  $W$  denotes the water layer;  $s'$  and  $t'$  are the tangent values of P- and SV-wave incident angles in the base rock.

It can be seen that the modulus of the vertical transfer function is equal to or less than 1, and it tends to zero when  $\tan ksd \rightarrow \infty$ , namely

$$\tan ksd = \tan \left[ \frac{2\pi f l_x^W}{c_p^{*W}} (-i) \sqrt{1 - \frac{1}{l_x^{W2}} \cdot d} \right] = \tan \left[ \frac{2\pi f d}{c_p^{*W}} \sin \theta_p^W \right] \rightarrow \infty \quad (15)$$

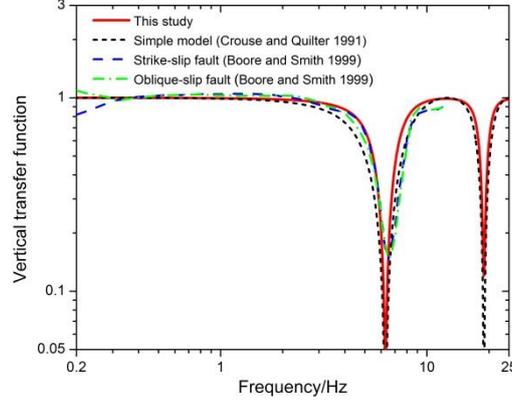
Equation (15) is satisfied when

$$f = n \frac{c_p^{*W}}{4d \sin \theta_p^W}, \quad (n=1, 3, 5) \quad (16)$$

where  $n$  is an odd number,  $f$  can be defined as the P-wave resonant frequencies of the seawater layer. At these frequencies, a destructive interference is induced by a phase change of P-wave motions at the

water and base rock interface (Boore and Smith 1999). This effect results in a significant reduction in the underwater vertical motion.

Analogously, the transfer function for in-plane horizontal motion can also be derived. The derived vertical transfer function of the underwater base rock site is compared with those suggested in previous studies (Crouse and Quilter 1991, Boore and Smith 1999). The parameters of base rock and seawater are given in Figure 1. The incident angle of seismic P-wave and seawater depth are assumed to be  $90^\circ$  and 60m, respectively. The comparison results are shown in Figure 2.



**Figure 2. Comparison of the derived vertical transfer function model with those presented in previous studies**

As shown, the vertical transfer function match well with those suggested in previous studies. In the model by Boore and Smith (1999), only the fundamental resonant frequency of the water layer is considered. However, the models suggested by Crouse and Quilter (1991) and this study are capable of including multiple resonant frequencies (6.25 Hz, 18.75 Hz, ...). Since the material damping is considered in the derived model, the minimum values of the transfer function decrease with frequency. Moreover, the fundamental frequency will be larger than 6.25 Hz if the incident angle of P-wave in seawater is less than  $90^\circ$ , as can be inferred from Equation (16). It should be noted that the model by Crouse and Quilter (1991) can only consider the vertically incident waves. Therefore, it can be concluded that the previous models can be considered as special cases of the model derived in this study. The present model can take into consideration the effect of SV-wave, material damping and wave incident angle. Therefore, it can be utilized to more realistically represent the influence of seawater on the underwater seismic motions.

### 3 GROUND MOTION TRANSFER FUNCTIONS OF A LAYERED SUBSEA SITE

Due to the effect of seawater, there exists a widespread of fully or nearly saturated sediments in an actual subsea site. Previous studies have revealed that the water saturation of soil layers can significantly influence the ground motion site amplification effect (Wang and Hao 2002). In this study, the Poisson's ratio and P-wave velocity model of porous soils suggested by Yang and Sato (2000) is employed to consider the effect of water saturation on the offshore site transfer functions:

$$v = \frac{1}{2} \frac{\alpha^2 M / G + 2v' / (1 - 2v')}{\alpha^2 M / G + 1 / (1 - 2v')} \quad (17)$$

$$V_p = \sqrt{\frac{\lambda + 2G + \alpha^2 M}{\rho}} \quad (18)$$

where  $G$ ,  $\lambda$  and  $v'$  are the shear modulus, Lamé constant and Poisson's ratio of the soil skeleton, respectively;  $\rho = (1-n)\rho_s + n\rho_f$ , with  $\rho_s$  and  $\rho_f$  denoting the densities of the soil grains and pore fluid;  $\alpha$  and  $M$  are represented as

$$\alpha = 1 - K_b / K_s, \quad M = K_s^2 / (K_d - K_b), \quad K_d = K_s [1 + n(K_s / K_f - 1)] \quad (19)$$

where  $K_s$  and  $K_b$  are the bulk modulus of soil grains and skeleton, respectively;  $n$  represents the porosity;  $K_f$  denotes the bulk modulus of the pore fluid, which is defined as (Verruijt 1969)

$$K_f = 1 / \left[ 1 / K_w + (1 - S_r) / p_a \right] \quad (20)$$

where  $K_w$  is the bulk modulus of pore water;  $S_r$  is the degree of saturation and  $p_a$  is the absolute fluid pressure. It is worth noting that the pore fluid does not affect the propagation of S-wave.

The total dynamic stiffness matrix of an offshore site in Equation (12) can be formulated by assembling the stiffness matrices of the base rock, the porous soil layers and the seawater layer. The water saturation effect of porous soil layers are considered by substituting Equations (17) and (18) into Equation (12). Thus, the offshore site transfer functions can be calculated by solving Equation (12) at every discrete frequency.

#### 4 GROUND MOTION SIMULATION

Based on the recorded strong ground motions in the area of southwest Western Australia, Hao and Gull (2009) proposed a seismological model for SWWA. In this study, the SWWA model is utilized to simulate the base rock motions. Two typical onshore and offshore sites in SWWA are employed to simulate the onshore and seafloor seismic motions. The profiles of the two sites are shown in Figure 3, in which  $d$  denotes the layer depth,  $K$  bulk modulus of seawater,  $G$  shear modulus of soil,  $\rho$  density,  $\zeta$  damping ratio,  $\nu'$  Poisson's ratio of soil skeleton,  $n$  porosity and  $S_r$  degree of saturation. The depth of seawater layer is 60m and the saturation degree of soil layers is higher than that of the onshore site.

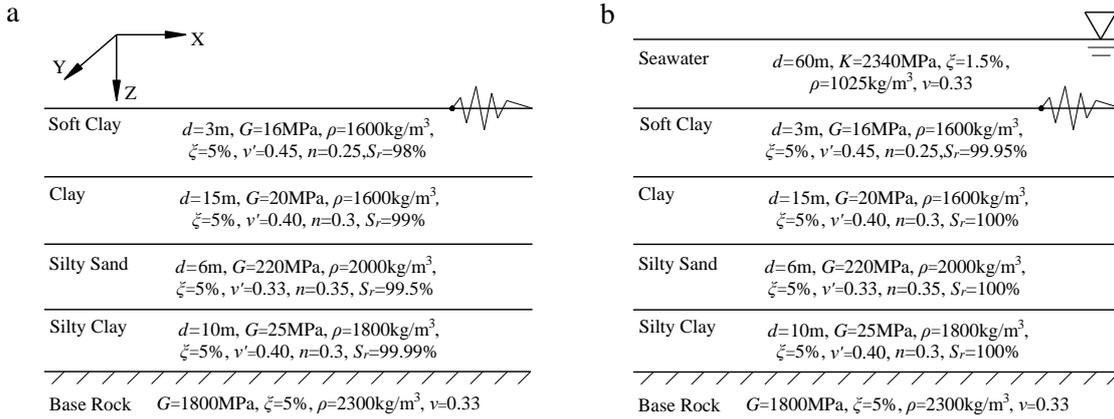


Figure 3. Typical sites in SWWA, (a) onshore site; (b) offshore site

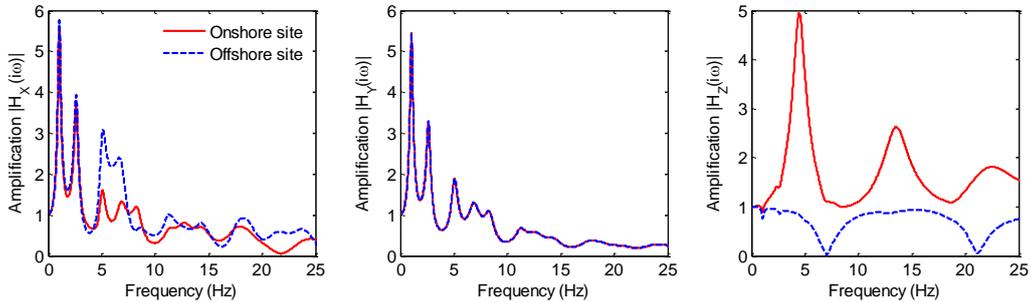
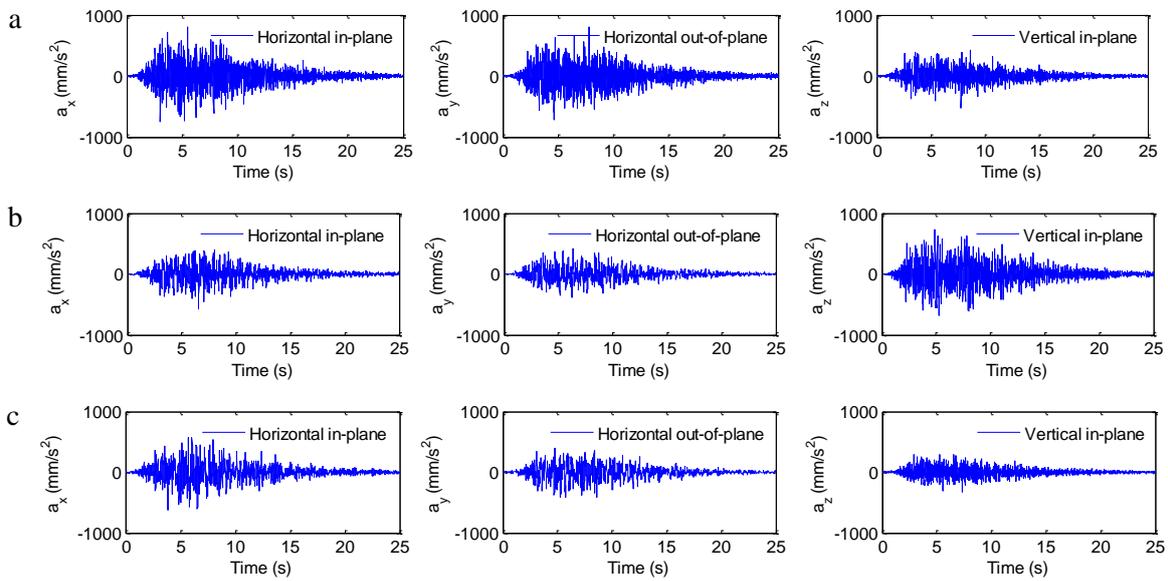


Figure 4. Onshore and offshore site transfer functions

The ground motion transfer functions of the two sites are firstly calculated by considering the effect of seawater layer and water saturation of porous soil layers, as shown in Figure 4. The base rock motions are assumed to consist of out-of-plane horizontal (X) SH-wave or in-plane horizontal (Y) and vertical (Z) combined P- and SV-waves. The incident angles of out-of-plane SH wave and in-plane P-wave in the base rock are assumed to be  $60^\circ$ . It can be observed that the horizontal transfer functions of the onshore and offshore sites are similar, indicating that the effect of seawater and water saturation on the horizontal motions is very slight. However, the transfer function of vertical in-plane motion is

significantly affected by the seawater; the vertical transfer function of the offshore site achieves minimum values at the resonant frequencies of the 60m seawater layer, namely 7.06 Hz and 21.18 Hz.

Multiplying the SWWA model of base rock with the calculated onshore and offshore site transfer functions, the Fourier amplitude spectra of onshore and seafloor motions can be obtained. The onshore and seafloor seismic motions are simulated based on the spectral representation method (Bi and Hao 2012). In the simulation, the Richter magnitude ML7.2 and epicentral distance 75 km is assumed in the SWWA model to synthesize the base rock ground motion time histories. This assumption corresponds to the upper bound event in SWWA with a design PGA of 0.09g. The sampling frequency and upper cut-off frequency are assumed to be 100 Hz and 25 Hz, respectively. The time duration is estimated to be 25.2s according the suggested model (Atkinson and Boore 1998). The phase angles of generated motions are assumed to be randomly distributed in 0 to  $2\pi$ . For each site, four independent groups of three-dimensional seismic motions are generated. A typical group of simulated acceleration time histories is shown in Figure 5. It should be noted that the Fourier amplitude spectra of simulated motions match well with the respective target model spectra.



**Figure 5. Generated three-dimensional ground motions on base rock free surface, (a) base rock motion; (b) onshore motion; (c) seafloor motion**

The average horizontal PGAs of the simulated base rock motions are  $802\text{mm/s}^2$  and  $827\text{mm/s}^2$ , which are very close to the target PGA of  $0.09g$  ( $882\text{mm/s}^2$ ). The average PGAs of horizontal onshore and seafloor motions are lower than those of the base rock motions. This is because the abundant horizontal base rock motions are deamplified at frequencies higher than 5 Hz, as can be seen in the transfer functions in Figure 4. The onshore site considerably amplifies the base rock vertical motions, but the seafloor vertical motion is significantly deamplified due to the suppression effect near the resonant frequencies of the overlying seawater layer. The average vertical-to-horizontal (V/H) PGA ratio of the seafloor motions is 0.63, which is much lower as compared to onshore V/H PGA ratio of 1.39. The characteristics of simulated seafloor motion are consistent with those of available seafloor earthquake recordings (Boore and Smith 1999), namely the intensity of vertical seafloor motion is much lower than that of the horizontal components, due to the destructive interference effect of P-wave motions near the resonant frequencies of the overlying seawater layer.

## 5 CONCLUSIONS

This paper presents a simulation method of seafloor seismic motions in SWWA. The offshore site transfer functions are derived by considering the effect of seawater layer and water saturation of subsea soil layers. Three-dimensional seismic motions on two typical SWWA onshore and offshore sites are stochastically simulated. The results show that the simulated seafloor motions are in line with

recorded seafloor motions, namely the vertical component of seafloor motion is significantly suppressed near the resonant frequencies of the overlying seawater layer. Therefore, it is believed that the seafloor seismic motions generated in this study can be utilized as inputs in the seismic response analyses of offshore engineering structures in SWWA.

#### ACKNOWLEDGMENTS:

The authors acknowledge the support from the National Basic Research Program of China (Grant No. 2011CB013605) and Australian Research Council Discovery Early Career Award DE150100195. The first author also acknowledges the scholarship from China Scholarship Council.

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