Proceedings of the Tenth Pacific Conference on Earthquake Engineering
Building an Earthquake-Resilient Pacific
6-8 November 2015, Sydney, Australia

Fuzzy Probability Analysis of the Performance of Reinforced Concrete Frame Buildings in Bhutan

K. Thinley & H. Hao

Centre for Infrastructure Monitoring and Protection, School of Civil and Mechanical Engineering, Curtin University, Bentley, Perth WA 6102.

ABSTRACT:

Seismic performance of reinforced concrete (RC) buildings is mostly assessed based on the interstorey drift limits. Many existing guidelines have specified distinct interstorey drift limits corresponding to the performance level of buildings. In reality, a structure cannot have a fixed damage boundary since damage is dependent on many factors and is a continuous process under the action of load. It is not logical to designate performance level based on the distinct interstorey drift limit which is bivalent in nature. In this paper, probability and fuzzy set theory is used to estimate failure probabilities of the buildings to address the ambiguity of damage boundaries. Three typical RC frame buildings are considered for structural response analysis. Statistical variation of material and geometrical parameters are considered and the ground motions obtained from PSHA in Thimphu are used for the structural response prediction. Soil structure interaction (SSI) is incorporated using the uncoupled spring model for shallow foundation. Typical buildings in general experience irreparable and severe damages at rock, shallow stiff soil and soft rock sites and complete damage at soft soil site for the ground motions considered.

1 INTRODUCTION

Inter storey drift is the most important response quantity that is commonly used to indicate the performance of buildings under seismic action. It is defined as the relative lateral displacement of two consecutive floors normalised by the storey height. Many existing guidelines such as ASCE 41-06, ATC-40 and Vision 2000 document have specified interstorey drift limits corresponding to the performance level of buildings and accordingly many researchers have assessed the performance of buildings based on these guidelines. Moreover, several seismic design codes have also imposed drift limitation during the design stage to ensure the intended performance of buildings. For example, Indian Seismic Code, IS 1893:2002 restricts the inter storey drift of less than 0.004 times the storey height at any floor. Similarly, Australian Seismic Code, AS1170.4 (2007) limits the inter storey drift at any floor to 1.5%.

However, performance assessment of buildings based on the distinct interstorey drift limits is not realistic and logical. Under the action of earthquake ground motion, structures deform continuously. Moreover, interstorey drift of building is dependent on the number of uncertain events. The most common uncertain events are modelling, ground motion, material, geometrical and failure criterion uncertainties. These uncertainties are basically classified into randomness, fuzziness and incomplete information (Zhao et al. 1996). Owing to these uncertainties, interstorey drift of buildings could vary to a large extent. Hence, assessing the performance of buildings based on the distinct interstorey drift limit and on the single value of interstorey drift is not logical in reality.

This study is geared towards assessing the seismic performance of reinforced concrete buildings in Bhutan in more holistic and realistic manner by taking into account both random and fuzzy events. Three typical RC frame buildings termed as ‘6 storey’, ‘3 storey new’ and ‘3 storey old’ buildings are considered for the seismic performance assessment in this paper. They represent the general RC building stocks in Bhutan that were constructed before and after the adoption of Indian Seismic Code in Bhutan. Ground motions predicted for the site conditions in Bhutan for 475 and 2475 year return periods are used for the analyses. Rosenbluth Point Estimate Method is used for the statistical variation of random variables and dynamic nonlinear analysis and performance assessment software, Perform 3D is used for the response prediction of the buildings. The structural response predicted from the statistical variation of random variables by Rosenbluth Point Estimate Method is verified using Monte Carlo Simulation technique.

The fuzzy probability analysis is used to estimate damage probabilities of buildings by introducing commonly used triangular membership function. The drift limits specified by Structural Engineers Association of California (SEAOC) in Vision 2000 document is used for the definition of damage boundaries and construction of membership function. Based on the damage probabilities of typical buildings, it is observed that '3 storey new' and '6 storey' buildings in general suffer irreparable to severe damages at different soil sites under the given ground motions while '3 storey old' building undergoes severe damage and also collapses for the same.

2 GROUND MOTION

Located right on the junction of tectonic plates where Indo-Australian plate is continuously being subducted into the Eurasian plate, Bhutan has experienced number of earthquakes of various sizes in the past. However, owing to its isolation for the larger part of its history combined with the lack of in-house technical capabilities, none of the earthquakes occurred in Bhutan were recorded or studied in detail. In absence of recorded ground motions, the earthquake ground motions predicted by Hao and Tashi (2010) from probabilistic seismic hazard analysis (PSHA) are used in this study. They considered 18 seismic source zones within a distance of 400Km from Thimphu, Bhutan for PSHA. The response spectrum of ground motions predicted in Thimphu at various soil sites for the return period of 475 and 2475 years are shown in Figures 1 below.

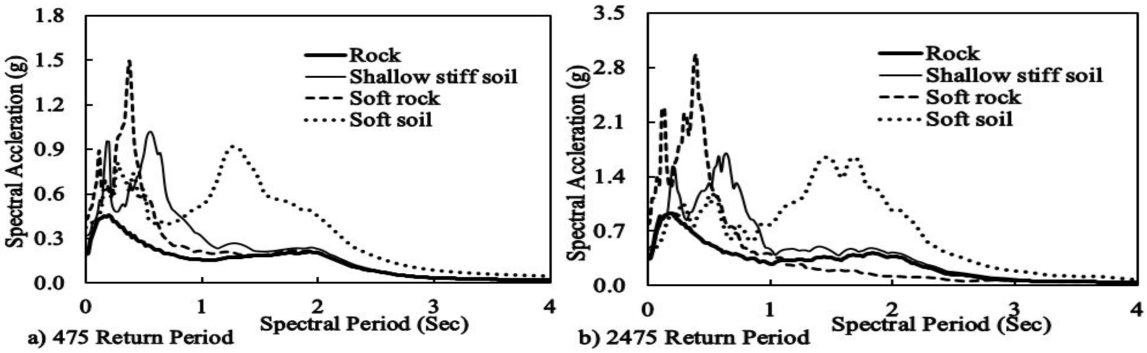


Figure 1. Ground motion response spectra at different sites for 475 and 2475 year return periods.

3 STRUCTURAL MODEL FOR PERFORMANCE ASSESSMENT

RC frame buildings are very common in the urban areas of Bhutan and their construction have been started as early as 1970s. However, it is to be noted that only in 1997, the country has adopted Indian Seismic Code for the seismic design of RC buildings. Until 1997, RC buildings in Bhutan were only designed for gravity load or were just built based on some primitive thumb rules. As such, in addition to the RC buildings that were designed and built according to Indian Seismic Code, there are number of RC buildings which were built without any kind of seismic provision. In order to represent these building stocks in Bhutan realistically, three typical RC frame buildings in Bhutan are considered for the performance assessment. They are denoted as '6 storey' and '3 storey new' buildings which were designed according to Indian Seismic Code and '3 storey old' building designed only for gravity load. The '6 storey' and '3 storey new' buildings are currently standing in Thimphu and their details are obtained from Thimphu City Corporation in the form of architectural and structural drawings. Since, there are no records available for buildings built prior to 1997, structural details of '3 storey old' building are adopted from the result of non-destructive tests conducted on 15 such old buildings in

Thimphu under the Thimphu Valley Earthquake Risk Management Project in 2005 (UNDP Report 2006). For the purpose of comparison, plan and elevation of '3 storey old' building are adopted identical to that of '3 storey new' building. Plan of these buildings are shown in Figures 2. The loading and reinforcement details can be referred to Thinley et *al*. (2014). The member dimensions of typical buildings are given in Table 1.

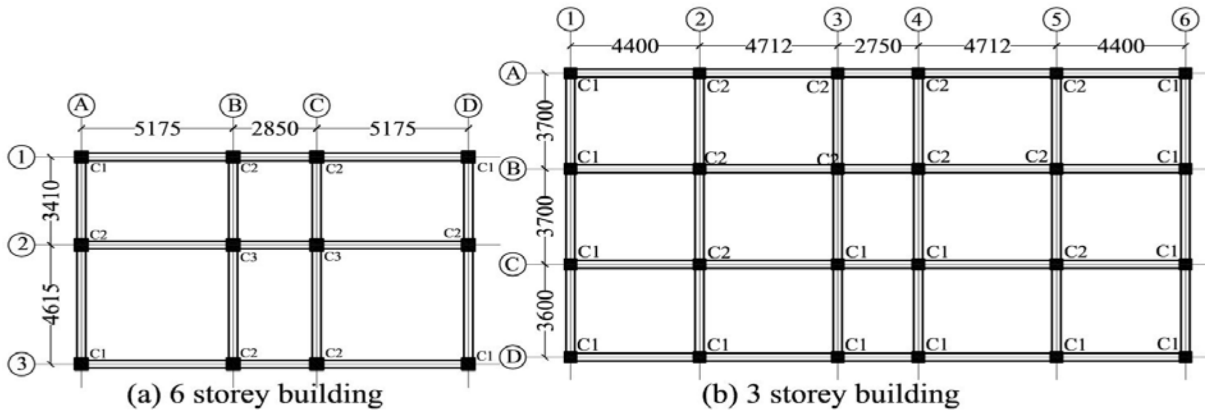


Figure 2. (a) Plan of '6 storey' buildings; (b) Plan of '3 storey new' and '3 story old' buildings.

Table 1. Member dimensions of typical buildings.

	Dimension (bxD) in mm		
Members	**6 storey**	**3 storey new**	**3 storey old**
Column, C1	450x450	400x400	250x250
Column, C2	450x450	400x400	250x250
Column, C3	500x500	-	-
Beam along longer direction	300x450	300x400	250x350
Beam along shorter direction	300x400	300x350	250x300
Slab depth, mm	150	150	100
Storey height, mm	3060	3060	3060

With all the structural details available, nonlinear analysis and performance assessment software, Perform 3D is used to carry out nonlinear analyses and estimation of structural responses of the buildings. In order that the structural response is predicted correctly, the numerical model used for the nonlinear analysis is first calibrated with the experimental results. The details of model calibration can be found in Thinley et *al*. (2014). The most commonly used lumped plasticity model with trilinear force deformation (F-D) relationship is used for numerical simulation. The 5% modal damping is used in combination with a small amount of stiffness proportional Rayleigh damping (0.1%) which is used to damp out high frequency vibrations.

To account for the contribution of slab to the stiffness of structure, interior and exterior monolithic beams and slab are approximated as T and L beams respectively. The effective width of T and L beams are calculated as per ACI 318-02 depending on the span of beam and depth of slab. The columns of typical buildings have square strip footings and are founded at a shallow depth of 1.5 to 2m from the ground level. They are initially considered as fixed support at all sites for the prediction of structural responses. To study the effect of soil structure interaction (SSI), uncoupled spring support is considered at the soft soil site. The stiffness of spring is calculated as per ASCE/SEI-41 and is taken as the product of stiffness at the surface and embedment correction factor.

4 PROBABILISTIC STRUCTURAL RESPONSE ESTIMATION

4.1 Modelling of uncertainities

In probabilistic seismic assessment of buildings, there are number of uncertainties that are required to be taken into consideration. These are loading, ground motion, modelling, material and geometrical uncertainties. In this study, only material and geometrical uncertainties are considered for the response estimation. Loading and modelling uncertainties are expected to have limited effect, since the numerical model used in this study was previously calibrated with the experimental results. Ground motions used in this study are considered as deterministic as they were specifically predicted for the site conditions in Bhutan from PSHA for both 475 and 2475 year return periods.

Among the material uncertainties, modulus of elasticity of concrete, compressive strength of concrete and yield strength of steel reinforcement are considered since they are reported to have higher coefficient of variation (CoV) and significantly influence the response of the structures. Hence for the probabilistic estimation of structural response, statistical variation of modulus of elasticity of concrete and compressive strength of concrete, yield strength of steel and dimension of beams and columns are considered. The designed values of these parameters are considered as mean, while standard deviation and coefficient of variation are adopted based on the number of past studies such as Basu et al. (2004), Mirza et al. (1976), Mirza et al. (1979), Barlett and MacGregor (1996), Nielson and DesRoches (2007) and Indian Standards for Plain and Reinforced Concrete, IS 456:2000. All these parameters are considered as random and statistically independent from one another. The mean, CoV and probability distribution of these parameters are given in Table 2.

Table 2. Mean and coefficient of variation of material and geometrical parameters

	6 storey building columns		3 storey new and beams of 6 storey buildings*		3 storey old building		
Variables	**Mean (Mpa)**	**CoV**	**Mean (Mpa)**	**CoV**	**Mean (Mpa)**	**CoV**	**Probability Distribution**
Ec	25000.00	0.20	22361.00	0.20	19365.00	0.23	Normal
fc	25.00	0.20	20.00	0.20	15.00	0.23	Normal
fy	415.00	0.09	415.00	0.09	415.00	0.09	Normal
Dimension	As in Table 1	0.05	As in Table 1	0.05	As in Table 1	0.05	Normal

**Concrete used for all members of '3 storey new' building and beams of '6 storey' building are same.*

4.2 Structural response estimation

Estimation of probabilistic dynamic response of structures is not straight forward owing to the variation of number of input parameters. In this study, Rosenbluth Point Estimate Method (Rosenbluth 1981) is used to statistically vary the material and geometrical parameters. For each combination of these parameters, dynamic nonlinear analysis is carried out using Perform 3D to estimate the structural responses of buildings. The number of combination or simulation depends on the number of variables according to the formula 2^n, where 'n' is the number of variables. Since, four variables are considered in this study, 16 combinations of these variable parameters are formed and 16 dynamic nonlinear analyses are run resulting in the 16 response quantities for each building. Hence mean and standard deviation of interstorey drift is calculated based on the intersotrey drifts estimated from 16 combinations. This method is used for the estimation of interstorey drift of all buildings since its application is simple and computationally very efficient. Its reliability is however validated using Monte Carlo Simulation.

Monte Carlo Simulation method is straight forward and more reliable than Rosenbluth Point Estimate Method. However, it is computationally very expensive and requires large number of simulations in the order of 200 and above to arrive at the converged estimations. Hence, this method is only applied to ‘3 storey new’ building just to validate the reliability of Rosenbluth Point Estimate Method. Since it is not practical to carry out large number of dynamic nonlinear analyses especially for 3 dimensional building frame with large number of structural members, a variance reduction technique called stratified sampling is used to reduce the number of simulations. It is initially assumed that 500 simulations are required to arrive at the converged response quantity and accordingly 500 random samples are generated for each variable. Using the stratified sampling technique, 50 random samples are selected from the initial 500 random samples generated by the Monte Carlo Simulation for each variable. Dynamic nonlinear analyses are carried out using Perform 3D for each of these 50 random combination to estimate the structural responses of the building. The popularly used Kolmogorov-Smirnov test is conducted and confirmed the normal distribution of interstorey drifts with significance levels of 0.05 for rock, soft rock and soft soil sites and 0.01 for shallow stiff soil site.

Figure 3 shows the interstorey drift computed from Rosenbluth Point Estimate and Monte Carlo Simulation Methods at rock and shallow stiff soil sites for 475 year return period ground motions. It can be seen from the figure that interstorey drift estimated from two methods are very close indicating the correct validation of Rosenbluth method by the Monte Carlo Simulation method.

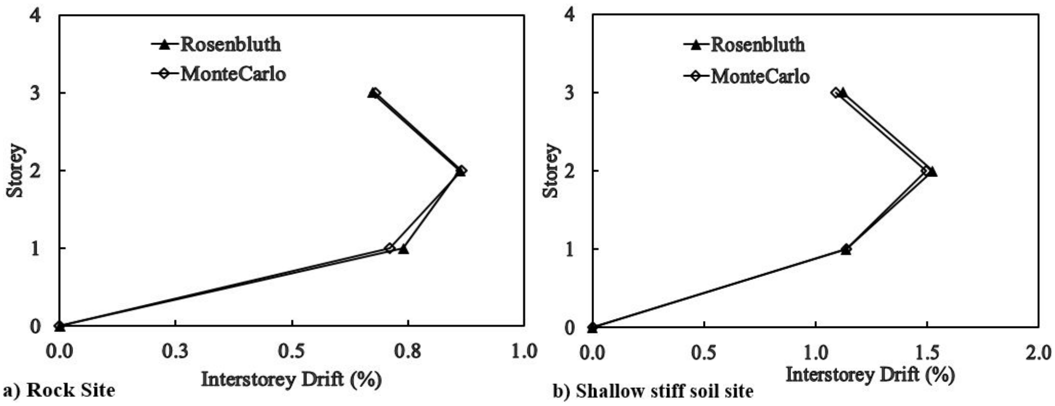


Figure 3. Comparison of interstorey drift estimated from Rosenbluth Point Estimate Method and Monte Carlo Simulation Method.

After validation, Rosenbluth Point Estimate Method is used in combination with Perform 3D to estimate responses of the typical buildings at different soil sites for both 475 and 2475 year return periods. Effect of soil structure interaction (SSI) is also studied by introducing flexible support at the soft soil site. The mean and standard deviation of interstorey drift obtained at different soil site for 475 and 2475 year return period are given in Tables 3- 5 for the three typical buildings. The analysis failed to converge at soft soil site for ‘3 storey old’ building due to large excessive responses. Hence, structural response at soft soil site for ‘3 storey old’ building is shown in the table.

Table 3. Mean maximum interstorey drift of 6 storey building at different soil sites

		475 year return period			**2475 year return period**		
Site class	**Support Type**	**Mean (%)**	**Std. Dev. (%)**	**Floor Level**	**Mean (%)**	**Std. Dev. (%)**	**Floor Level**
Rock	Fixed	1.518	0.210	2	2.268	0.171	2
Shallow stiff	Fixed	1.854	0.344	2	2.537	0.335	3
Soft rock	Fixed	1.607	0.237	2	1.552	0.156	3
Soft soil	Fixed	2.985	0.485	2	4.642	0.424	3
Soft soil	Spring	2.976	0.475	3	4.727	0.685	3

Table 4. Mean maximum interstorey drift of '3 storey new' building at different soil sites

		475 year return period			2475 year return period		
Site class	**Support Type**	**Mean (%)**	**Std. Dev. (%)**	**Floor Level**	**Mean (%)**	**Std. Dev. (%)**	**Floor Level**
Rock	Fixed	0.862	0.289	2	1.704	0.516	2
Shallow stiff	Fixed	1.525	0.344	2	2.399	0.588	1
Soft rock	Fixed	1.162	0.285	2	1.925	0.520	3
Soft soil	Fixed	2.887	1.221	1	6.320	1.447	1
Soft soil	Spring	3.277	1.081	1	7.190	0.874	1

Table 5. Mean maximum interstorey drift of '3 storey old' building at different soil sites.

		475 year return period			2475 year return period		
Site class	**Support Type**	**Mean (%)**	**Std. Dev. (%)**	**Floor Level**	**Mean (%)**	**Std. Dev. (%)**	**Floor Level**
Rock		1.854	0.401	1	2.285	0.640	1
Shallow stiff soil		2.110	0.939	1	2.428	0.603	1
Soft rock		1.930	0.453	1	2.152	0.461	2

5 FUZZY FAILURE PROBABILITY AND PERFORMANCE ASSESSMENT

The performance of buildings is mostly evaluated based on the inter-storey drift demand. Many guidelines such as ASCE 41-06, ATC-40 and Vision 2000 document have provided performance levels of buildings with corresponding damage states and inter-storey drifts. Since performances of buildings described by these guidelines are similar in concept, only Vision 2000 is used for defining the damage boundary in this paper. Given the probabilistic information of maximum interstorey drift obtained from the random variation of material and geometrical parameters, damage probabilities of typical buildings can be estimated from

$$P_f = \int_{D_c}^{\infty} f_D(D) dD \tag{1}$$

where D = maximum inter-storey drift obtained for typical buildings given in Tables 3-5, D_c = inter-storey drift limits defined in Table 6 and $f_D(D)$ is the probability density function.

Table 6. Performance levels, damage states and interstorey drift limits from Vision 2000

Performance level	**Damage state**	**Interstorey drift (%)**
Fully operational	Negligible (No Damage)	<0.20
Operational	Light (Repairable Damage)	<0.50
Life safe	Moderate (Irreparable)	<1.50
Near collapse	Severe	<2.50
Collapse	Complete	>2.50

Expression (1) provide damage probabilities based on the distinct damage boundaries wherein structure is considered to be damaged if $D \geq D_c$ and not damaged if $D \leq D_c$. In reality, a structure cannot have a fixed damage boundary since damage is dependent on many factors. For instance, it is not logical to describe a structure as moderately damaged when the maximum inter-storey drift is 1.499% and severely damaged when the maximum inter-storey drift is 1.501%. Hence, it is logical to define fuzzy region in between the damage boundaries given in Table 6 to logically estimate the damage probabilities of the structures. Infact, Zadeh (1965) first introduced fuzzy sets to tackle with the real world situations which are virtually imprecise in nature. According to fuzzy set theory, a fuzzy region is defined by a lower and upper fuzzy region in combination with membership function (Wu et *al*. 1999). Introducing fuzzy region to the distinct damage boundary limits in Table 6, the fuzzy failure probabilities of typical buildings can be estimated from Zhao et al. (1995) as

$$P_{ff} = P(D \geq D_c) = \int_{D_L}^{D_U} \mu(D) f_D(D) dD \quad (2)$$

where, D_U and D_L are respectively upper and lower fuzzy limit and μ *(D)* is the membership function.

In fuzzy region, structure may fail even when $D<D_c$ or may not fail even when $D>D_c$. It is to be noted that definition of membership function is quite complex and subjective. It is often based on some experts' knowledge (Wu et *al*. 1999). In this study, triangular membership function is adopted due to its simplicity and high level of accuracy. As shown in Figure 4, triangular membership function is constructed by linearly extending each damage state to the midpoint of the next damage state (Kirke & Hao 2004). The midpoint of damage limit is assigned the membership function of 1 indicating the most likelihood of occurring damage at the midpoint of the respective damage state.

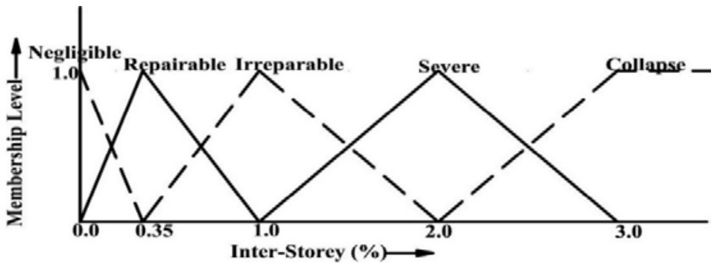
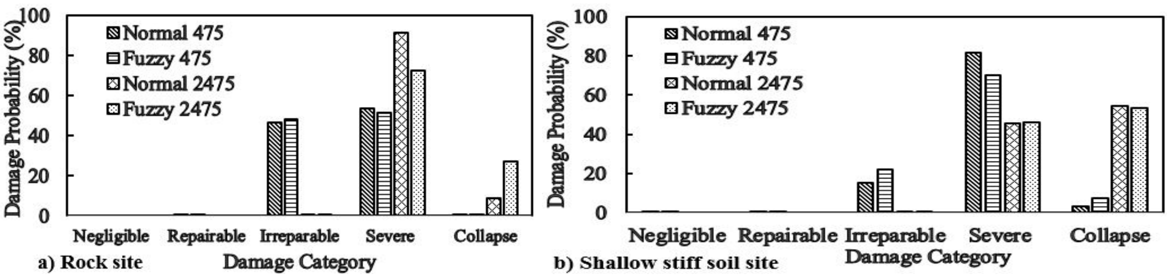


Figure 4. Triangular membership function adopted in this study.

Based on the probabilistic information in Tables 3-5 and using normal probability density function and triangular membership function in equations (1) and (2), damage probabilities of typical buildings are estimated. Figures 5-7 depict the damage probabilities of '6 storey', '3 storey new' and '3 storey old' buildings at different soil sites with fixed support.



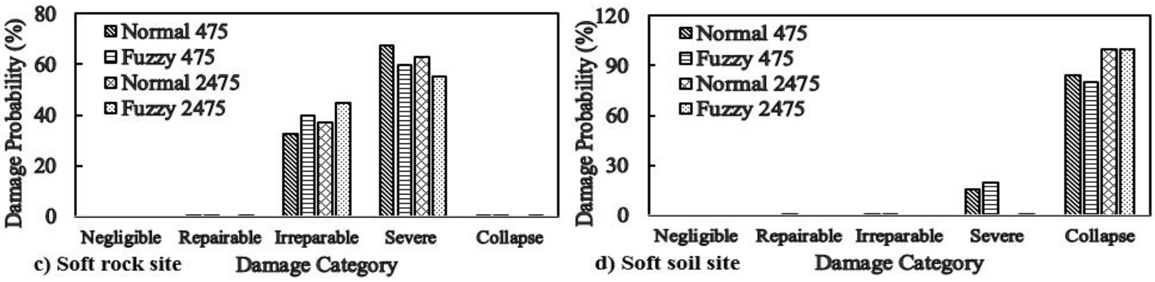


Figure 5. Damage probabilities of '6 storey' building for 475 and 2475 return periods.

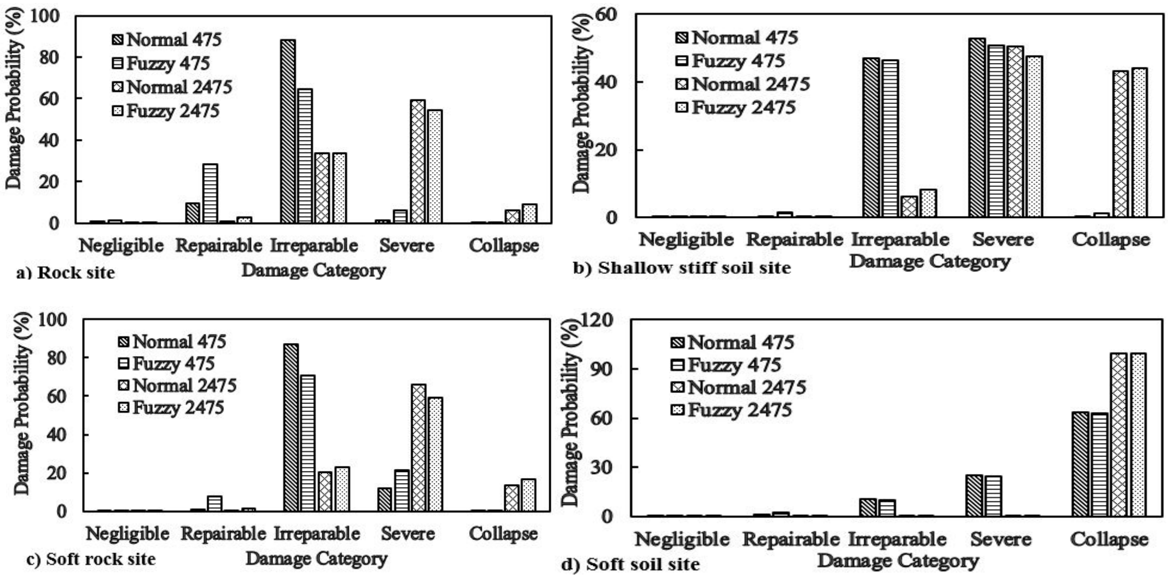


Figure 6. Damage probabilities of '3 storey new' building for 475 and 2475 return periods

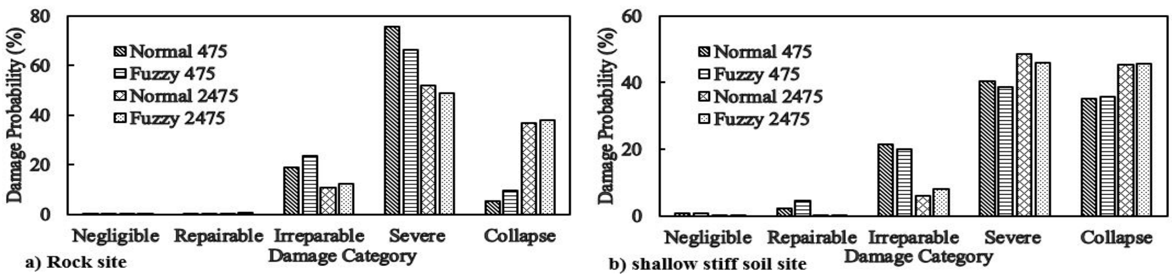


Figure 7. Damage probabilities of '3 storey old' building for 475 and 2475 return periods

6 DISCUSSION

Observing the mean maximum interstorey drift of typical buildings in Tables 3-5 and comparing with the drift limits of Vision 2000, it can be seen that '3 storey old' building is the most vulnerable structure followed by '6 storey' building. As noted earlier, analysis failed to converge for '3 storey old' building at soft soil site due to large excessive response and at other sites the building undergoes severe damage. Similarly, '6 storey' building also undergoes severe damage at rock, shallow stiff soil and soft rock sites. For '3 storey new' building, irreparable damage is predicted at rock and soft rock sites, while severe damage is predicted at shallow stiff soil site. Both '6 storey' and '3 storey new' building experience complete damage at soft soil site. The extremely large interstorey drift predicted at soft soil site for '3 storey new' building is found to be due to the soil resonance. As observed from Tables 3-4, soil structure interaction is slightly beneficial for '6 storey' building and detrimental for '3 storey new' building. It is actually found to be highly dependent on the site natural period of soil and the period of the structure.

However, the damages predicted above are solely based on the distinct drift limits and a single interstorey drift value. Referring Table 3 for '6 storey' building, it can be observed that severe damage is predicted at rock site for both 475 and 2475 year return periods although there is significant difference in their interstorey drift values. The predicted interstorey drift value for 475 year return period is 1.518% while it is 2.268% for 2475 year return period. In practice, building would undergo different damages since 2475 year return period ground motion is more severe than that of 475 year return period ground motion. Similarly, severe damage is predicted for both '6 storey' and '3 storey old' buildings, although '6 storey' building has lower interstorey drift value of the two. This is the main limitation of assessing the performance of structures based on the distinct damage boundary and using the deterministic or single interstorey drift value.

Figures 5-7 represent the damage probabilities predicted by considering random variation of material and geometrical parameters. They are more practical than the deterministic approach since probability of structure undergoing different damages are predicted instead of a single damage state. Further, fuzzy region is introduced to address the impracticality of distinct damage boundary. Figures 5-7 depict the comparison of damages predicted from normal and fuzzy probabilities. From the figures, it can be observed that there is a difference in the probability of damages predicted by normal and fuzzy probabilities. Although, difference is not very significant, yet in cases where damages are converted into monetary figure, the difference could be very significant. Hence, it is more practical and rational to assess structures based on fuzzy probability and the same is described for typical buildings here.

As shown in the Figures 5-7, '6 storey' building has the higher probability of undergoing severe damage at rock, shallow stiff soil and soft rock site, while it has higher probability of complete damage at the soft soil site. On the other hand, '3 storey new' building has the higher probability of experiencing irreparable damage at rock and soft rock site, while it has almost an equal probability of experiencing severe and irreparable damages at the shallow stiff soil. At the soft soil site, '3 storey new' building has the higher probability of complete damage. '3 storey old' building has the higher probability of experiencing severe damage at rock and soft rock site, while it has nearly equal probability of severe and complete damages at shallow stiff soil site.

While, higher probability of damage of '3 storey old' building is expected, it is not expected for '6 storey' and '3 storey new' buildings to experience irreparable and severe damages. They were designed according to Indian Seismic Code and are expected to perform better than that predicted in this study. The higher probability of damage could mainly be due to the use of low strength concrete which results in higher interstorey drift. The performance of buildings could also have been better had ductile detailing been included in the modelling. Since it is not possible to include all bits and pieces of reinforcement at the specified position of RC members in the modelling, ductile detailing is ignored in this study. The higher damage probability also questions that adequacy of directly using Indian Seismic Code in Bhutan.

7 CONCLUSION

Three typical reinforced concrete buildings in Thimphu that represent the RC building stocks in Thimphu are considered for performance assessment. Unlike in many cases where performance of buildings are assessed based on the distinct damage boundary limits, a more rational approach called fuzzy probability analysis is used in this study. Ground motions predicted from PSHA in Thimphu at rock, shallow stiff soil, soft rock and soft soil sites for 475 and 2475 year return periods are used for the dynamic nonlinear analyses. Rosenbluth Point Estimate Method is used for the statistical variation of material and geometrical parameters and Perform 3D is used to estimate structural response of the typical buildings. The structural response estimated from Rosenbluth Point Estimate Method is verified using Monte Carlo Simulation.

As expected, gravity designed '3 storey old' building is found to be more susceptible to damage than that of '6 storey' and '3 storey new' buildings which were designed according to Indian Seismic Code. The ‘3 storey new’ building performs better than the other buildings considered, but it also experiences irreparable and severe damages. In general, typical buildings have the high probability of irreparable and severe damages at rock, shallow stiff soil and soft rock sites, while they experience complete damage at the soft soil site.

Although, damage probabilities predicted here are quite rational for being estimated from the fuzzy probability, yet it is not very rational to directly assign the predicted damage tag to the respective buildings. In other words, the buildings may or may not experience the predicted damage under the ground motions considered. Hence, it is more rational to conclude that the damage probabilities predicted in this study are just indicative since they are based on Vision 2000 documents whose performance levels are also indicative.

8 ACKNOWLEDGEMENT

The Endeavour Postgraduate Award is gratefully acknowledged for the full scholarship provided to the first author in undertaking PhD study.

9 REFERENCES

ACI. Building code requirements for Structural Concrete (ACI 318-02) and Commentary (ACI 318R-02). *American Concrete Institute*, Farmington Hills, MI, 2002.

AS1170.4-2007. Australian Standard for structural design actions. Part 4: Earthquake actions in Australia. *Standards Australia.*

ASCE. Seismic Rehabilitation of Existing Buildings 2007a. ASCE Standard ASCE/SEI 41-06. *American Society of Civil Engineers*, Reston, Virginia.

ATC-40. Seismic Evaluation and Retrofit of concrete Buildings volume 1. *Applied Technology Council*, Redwood City, California 1996.

Barlett, F.M. & MacGregor, J.G. 1996. Statistical analysis of the compressive strength of concrete in structures. *ACI Material*, 93(2), 158-68.

Basu, P.C., Shylamoni, P. & Roshan, A.D. 2004. Characterisation of steel reinforced for RC structures: An overview and related issues. *The Indian Concrete Journal*, January 2004.

Hao, H. & Tashi, C.2012. Earthquake ground motion prediction and its influence on building structures in Bhutan. *Proceedings of the 12th International Symposium on Structural Engineering*, ISSE 2012, Nov 17, China.

IS 456:2000. Indian Standards for Plain and Reinforced Concrete. *Bureau of Indian Standards*, New Delhi, India.

IS-1893. Criteria for Earthquake Resistant Design of Structures. *Bureau of Indian Standards*, New Delhi 2002.

Kirke, A. & Hao, H. 2004. Estimation of failure probabilities of RC frame structures in Singapore to the simulated largest credible ground motion. *Engineering Structures*, 26, 139-150.

Mirza, S.A. & MacGregor J.G. 1979. Variability of mechanical properties of reinforcing bars. *Journal of the Structural Division*, 105: 751-66.

Mirza, S.A. & MacGregor, JG. 1979. Variations in dimensions of reinforced concrete members. *Journal of the Structural Division*, 105: 921-37.

Mirza, S.A., Hatzinikolas, M & MacGregor, JG. 1979. Statistical descriptions of strength of concrete. *Journal of the Structural Division*, ASCE 1979; 80: 167-76.

Nielson, B.G. & DesRoches, R. 2007. Seismic fragility methodology for highway bridges using a component level approach. *Earthquake Engineering & Structural Dynamics*, 36(6), 823-839.

Rosenbluth, E. 1981. Two-point estimates in probabilities. *Applied Mathematical Modelling*, 5(5), 329-335.

Thinley, K., Hao, H & Tashi, C. 2014. Seismic performance of reinforced concrete buildings in Bhutan. *Proceedings of Australian Earthquake Engineering Society 2014 Conference*, 21-23November, Lorne, Victoria.

UNDP. 2006. Report on Thimphu Valley Earthquake Risk Management Program. *Standards and Quality Control Authority, Ministry of Works and Human Settlement*, Thimphu, Bhutan.

VISION-2000. Performance Based Seismic Engineering of Buildings. *Structural Engineers Association of California* 2000.

Wu, C., Hao, H. & Zhou, Y. 1999. Fuzzy-random probabilistic analysis of rock mass responses to explosive loads, *Computers and Geotechnics*, 25, 205-225.

ZADEH, L. A. 1965. Fuzzy sets. *Information and control,* 8**,** 338-353.

Zhao, G., Li, Y. & Wang, H. 1996. Application of fuzzy-random probability theory to structural reliability. *Application of statistics and probability*. Rotterdam; Balkema; 1996 ISBN 9054105631.