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Comparison of alternative assessment procedures to predict seismic performance of RC columns

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ABSTRACT: As part of a more general effort to refine current assessment guidelines and various approaches to estimate ultimate/plastic rotation angle and/or drift-based limit states for members and overall structural systems in Reinforced Concrete (RC) existing buildings, a dedicated research effort is given to the improvement of the understanding of seismic response and failure mechanisms and on the refinement of assessment procedures for columns, super columns and shear walls.

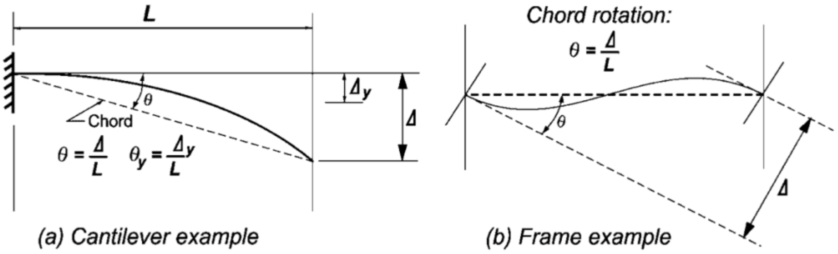
This paper presents preliminary results of analyses focusing on 2D response of square columns. For this purpose, force - displacement curves and yield and ultimate rotation angle/drift ratios of square RC columns experimentally tested by various researchers are investigated. Results are then compared to those reported experimentally and predictions by national and international standards (namely ASCE41-13, EN 1998-3: 2005 and NZSEE2006). Cumbia is used as the analytical tool to predict the response of the member. In the analytical model developed in Cumbia, both section and member analysis approaches are based on plane sections-remain-plain assumption and estimation of the plastic hinge length. Moreover RC square columns designed with NZS3101: 1970 – 2006 are investigated with the proposed method and results (in terms of ultimate drift ratio) are compared to ones predicted by other available procedures. This method will later be used to propose a simple procedure in order to predict the drift capacity of RC columns without determining the response of the member.

1 INTRODUCTION

Recent earthquakes in Christchurch placed further emphasis on the need of understanding the behaviour and seismic performance of RC structures and the requirement of improving the New Zealand current assessment guideline, NZSEE06. ASCE41-13 and EN 1998-3: 2005 are two of the current international seismic assessment guidelines for RC structures. ASCE41-13 introduces a table of plastic rotations angle for different amount of axial load ratio, transverse reinforcement ratio and shear stress ratio. On the other hand, EN 1998-3: 2005 and NZSEE06 are based on the flexural response of the member along with controlling shear capacity. In this paper a simple procedure to predict the drift capacity of a column (yield and ultimate) based on the flexural response of the member while controlling bar buckling and shear capacity is proposed. The procedure is then validated against experimental results. Moreover, the reliability of NZSEE06 and other available assessment guidelines (ASCE41-13 and EN 1998-3: 2005) is investigated.

2 METHODOLOGY

In this section the procedure that is used to predict the rotation angle/drift capacity of RC square columns under uni-directional loading is explained briefly. Fig. 1 shows the chord rotation otherwise known as the rotation angle which is equal to the story drift.



(a) Cantilever example (b) Frame example

Figure 1. Definition of chord rotation (rotation angle) (ASCE41-13)

2.1 Material modelling

Mander et al. (1988) and King et al. (1986) models (both developed at the University of Canterbury) are used for confined/unconfined concrete and steel reinforcement, respectively. Ultimate compression strain in concrete (ε_{cu}) is calculated using Eq. (1) which is proposed by Priestley et al. (1996) and is a conservative estimate of Mander et al. (1988) equation.

$$\varepsilon_{cu} = 0.004 + 1.4\rho_s f_{yh} \varepsilon_{su} / f'_{cc} \qquad (1)$$

Where ρ_s = volumetric ratio of confining steel $(\rho_s=\rho_x+\rho_y)$; $\rho_x=A_{vx}/(d_c\times s)$; $\rho_y=A_{vy}/(b_c\times s)$;$d_c$ and b_c = core dimensions to centrelines of perimeter hoop in x and y directions; f_{yh} = yield stress of transverse reinforcement; ε_{su} = steel strain at maximum tensile stress; and f_{cc}' = maximum confined concrete stress (Mander et al. 1988).

According to Priestley et al. (1996), when the member is subjected to bending or combined bending and axial compression, ultimate compression strain resulting from Eq. (1) tends to be conservative by at least 50%. Therefore in this study, the ultimate compression strain of concrete is considered 1.5 times of the ultimate strain resulting from Eq. (1).

2.2 Section analysis

The section analysis is performed using Cumbia (2007) by calculating moment and curvature of the column section while concrete strain is increasing. It is assumed that plane sections remain plane and tension is ignored in concrete.

2.3 Member response

The member response is obtained using section analysis results along with an equivalent plastic hinge length (l_p) as proposed by Priestley et al. (2007). Shear deformation is also calculated and added to the flexural deformation using Priestley et al. (2007) method. It should be noted that the response is strongly dependant on the plastic hinge length. In a comprehensive study on the methods of calculating the plastic hinge length conducted by Bae and Bayrak (2008), large variations reported among different methods. Paulay and Priestley (1992), Bae and Bayrak (2008) and EN 1998-3: 2005 methods are chosen to evaluate different methods of predicting plastic hinge length (Eq. 2-4 respectively).

$$l_p = 0.08L + 0.022 d_b f_y \geq 0.044 d_b f_y \qquad (2)$$

$$l_p = \left(0.3P/P_0 + 3A_s/A_g - 0.1\right)L + 0.25h \geq 0.25h \qquad (3)$$

$$l_p = L/30 + 0.2h + 0.11 d_b f_y / \sqrt{f'_c} \qquad (4)$$

Where L = distance from the column base to the point of contraflexure; d_b = bar diameter of the longitudinal reinforcement; f_y = yield stress of the longitudinal reinforcement; h = section height; P = axial load; $P_0 = 0.85f'_c(A_g-A_s)+A_sf_y$; f_c' = compressive strength of the concrete; A_g = gross area of the cross section; and A_s = longitudinal reinforcement area.

For this purpose, eight columns tested by different researchers are selected and the plastic hinge length predicted by each method is compared to the experimental results (Table 1). The plastic hinge length predicted by each method is given in Table 2 (also see Fig. 2 for graphical presentation). It should be noted that all specimens were tested as a cantilever column.

As seen in Table 2, Fig. 2 and also concluded by Bae and Bayrak (2008), none of these methods are able to predict the plastic hinge length with proper accuracy. Eq. (2) and (4) seems to have similar accuracy. However, in this study Eq. (2) proposed by Paulay and Priestley (1992) is used to calculate the plastic hinge length. More investigation is required for predicting the plastic hinge length of columns with a better accuracy.

Table 1. Characteristics of each specimen

Specimen number	Specimen name	Reference	Geometry L×h×b (mm)	P/P_0*	ρ**	ρ_s***	vertical spacing of ties, s (mm)
1	Unit 5	Tanaka (1990)	1650×550×550	0.093	0.0125	0.0075	110
2	Unit 7	Tanaka (1990)	1650×550×550	0.236	0.0125	0.0091	90
3	Unit 1	Li (1994)	1650×450×450	0.286	0.0157	0.0112	70
4	Unit 4	Li (1994)	1650×450×450	0.481	0.0157	0.0143	55
5	S24-2UT	Bae & Bayrak (2008)	3048×610×610	0.500	0.0125	0.0204	95
6	S17-3UT	Bae & Bayrak (2008)	3067×438×438	0.500	0.0125	0.0176	86
7	S24-4UT	Bae & Bayrak (2008)	3048×610×610	0.200	0.0125	0.0072	152
8	S24-5UT	Bae & Bayrak (2008)	3048×610×610	0.200	0.0125	0.0130	152

* Axial load ratio, ($P_0 = 0.85f'_c(A_g-A_s)+A_sf_y$)

*** Longitudinal reinforcement ratio,* $\rho = A_s/A_g$

**** Transverse reinforcement ratio,* $\rho_s = A_v/(b\times s)$

Table 2. Predicted l_p/h by each method

Specimen number	Experiment	Paulay and Priestley (1992)	Bae and Bayrak (2008)	EN 1998-3: 2005
1	0.465	0.818	0.250	0.655
2	0.751	0.818	0.274	0.619
3	1.050	0.990	0.386	0.767
4	1.225	1.012	0.627	0.761
5	0.660	0.807	0.688	0.673
6	0.910	0.955	0.863	0.733
7	0.490	0.721	0.250	0.632
8	0.470	0.721	0.250	0.616

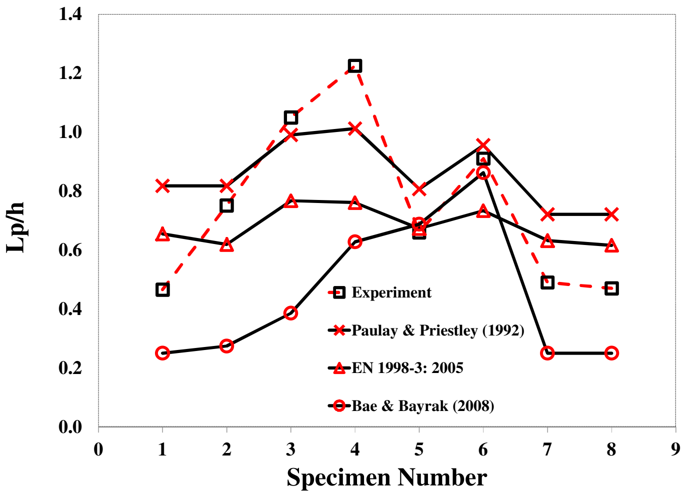


Figure 2. l_p/h predicted by each method

2.3.1 *Yield displacement*

Yield displacement (Δ_y) is calculated corresponding to the yield curvature (ϕ_y); defined based on the method proposed by Priestley et al. (2007). It should be noted that as it was explained in section 2.3, shear deformation is also considered in yield displacement calculation. ASCE41-13, EN 1998-3: 2005 and NZSEE2006 have different method of calculating the yield displacement. Each method is presented in Eq. (5)-(7) respectively.

$$\Delta_y = \delta_y \times L = \left(\delta_{flex} + \delta_{slip} + \delta_{shear}\right)L = \left(\phi_y L/3 + \phi_y d_b f_s/(8u) + 12V_p/(5A_g G)\right)L \tag{5}$$

$$\Delta_y = \delta_y \times L = \left(\phi_y (L + a_V z)/3 + 0.0013(1 + 1.5h/L) + 0.13\phi_y d_b f_y/\sqrt{f_c^{'}}\right) \times L \tag{6}$$

$$\Delta_y = \phi_y L^2/3 = 2.12\varepsilon_y L^2/(3h) \tag{7}$$

Where δ_y = yield drift ratio; f_s & u= refer to Elwood & Eberhard (2006); V_p = Plastic shear demand on the column, defined as the shear demand at flexural yielding of plastic hinges; G = shear modulus; a_V = 1 if shear cracking is expected to precede flexural yielding at the end section, otherwise, 0; $z = d-d'$; d = section effective depth; d' = depth to the compression reinforcement; and ε_y = longitudinal yield strain.

2.3.2 *Ultimate displacement*

In order to determine the ultimate displacement, three different criteria are controlled. The first criterion is based on the displacement at the onset of buckling in longitudinal reinforcement (Δ_{bb}). For this purpose, two methods proposed by Moyer and Kowalsky (2003) and Berry and Eberhard (2005) are investigated. In order to scrutinize these two methods, the first four columns introduced in Table 1 are chosen. Table 3 shows the predicted displacement at the onset of buckling by each method compared to experimental results. It is worth noting that these two methods are already implemented in Cumbia (2007).

Table 3. Predicted Δ_{bb} by each method

Specimen number	Experiment	Moyer and Kowalsky (2003)	Berry and Eberhard (2005)
1	73.8	64.2	93.1
2	82.4	133.9	79.7
3	88.8	277.9	94.2
4	76.0	279.7	81.2

As it is seen in Table 3 and also concluded in another study by Deyanova et al. (2015), method proposed by Berry and Eberhard (2005) leads to results closer to the experimental observations compared to the procedure by Moyer and Kowalsky (2003). In the following, the method proposed by Berry and Eberhard (2005) is presented briefly in Eq. (8) and (9). It should be noted that Eq. (9) is an empirical expression developed using a dataset of square and circular columns.

$$\Delta_{bb} = \Delta_y + \theta_{P\_bb} \times L \tag{8}$$

$$\theta_{P\_bb} = C_0\left(1 + C_1\rho_{eff}\right)\left(1 + C_2\frac{P}{A_g f_c^{'}}\right)^{-1}\left(1 + C_3\frac{L}{h} + C_4\frac{f_y d_b}{h}\right) \tag{9}$$

Where θ_{P_bb} = plastic rotation at the onset of bar buckling; ρ_{eff} = effective confinement ratio ($\rho_{eff}=\rho_s f_{yh}/f_c^{'}$).

According to regression analysis results by Berry and Eberhard (2005), C_0, C_1, C_2, C_3 and C_4 are chosen as 0.019, 1.65, 1.797, 0.012 and 0.072, respectively.

The second criterion is the point in which shear failure happens. As it was proven by Elwood and Moehle (2005), using shear strength models to estimate the drift ratio at shear failure can lead to unreliable predictions. Accordingly, Elwood and Moehle (2005) method which is developed based on a displacement-capacity point of view is used (Eq. 10). This equation is developed using an experimental dataset of square and circular columns.

$$\delta_s = 0.03 + 4\rho_a - v_s/\left(40\sqrt{f_c^{'}}\right) - P/\left(40A_g f_c^{'}\right) \geq 0.01 \text{ (MPa units)} \tag{10}$$

Where δ_s = drift ratio at shear failure; ρ_a = transverse steel ratio ($\rho_a=A_v/(b\times s)$); v_s = nominal shear stress ($v_s=V_p/(b\times d)$); and b = section width.

The third criterion is when a 20% reduction in the lateral resistance is observed compare to the measured peak shear capacity (ASCE41-13).

It should be noted that the limitations of each method (Eq. 9 and 10) including shear span to depth ratio, longitudinal and transverse reinforcement ratio, axial load ratio and etc. can be found in Berry and Eberhard (2005) and Elwood and Moehle (2005).

3 VERIFYING THE ANALYTICAL METHOD

In order to verify the analytical method, the first four columns introduced in Table 1 plus specimen 5 tested by Sezen (2002) are chosen and results in terms of drift ratios are compared with the experiments. Tables 4 and 5 show the characteristics of each specimens and the yield and ultimate drift ratio ($\delta=\Delta/L$), respectively. Fig. 3 and 4 also show the ultimate and yield drift ratio predicted by each method (current study, ASCE41-13, EN 1998-3: 2005 and NZSEE2006) and the experiment.

Table 4. Characteristics of each specimen

Specimen number	**Specimen name**	**Reference**	**Geometry L×h×b (mm)**	v*	ρ**	ρ_s***	**vertical spacing of ties, s (mm)**
1	Unit 5	Tanaka (1990)	1650×550×550	0.1	0.0125	0.00748	110
2	Unit 7	Tanaka (1990)	1650×550×550	0.3	0.0125	0.00914	90
3	Unit 1	Li (1994)	1650×450×450	0.3	0.0157	0.0112	70
4	Unit 4	Li (1994)	1650×450×450	0.5	0.0157	0.0143	55
5	Specimen-1	Sezen (2002)	2946×457×457	0.15	0.0245	0.00205	304.8

* *Axial load ratio,* $v = P/(A_g f'_c)$
** *Longitudinal reinforcement ratio,* $\rho = A_s/A_g$
*** *Transverse reinforcement ratio,* $\rho_s = A_v/(b \times s)$

Table 5. Evaluating analytical methods vs experiment

Specimen Number	**1**		**2**		**3**		**4**		**5**	
	Yield drift, δ_y	**Ultimate drift, δ_u**	**Yield drift, δ_y**	**Ultimate drift, δ_u**	**Yield drift, δ_y**	**Ultimate drift, δ_u**	**Yield drift, δ_y**	**Ultimate drift, δ_u**	**Yield drift, δ_y**	**Ultimate drift, δ_u**
Experiment	0.0075	0.0447	0.0062	0.0499	0.0067	0.0538	0.0046	0.0461	0.0089	0.0256
Current Study	0.0069	0.0508	0.0067	0.0483	0.0081	0.0571	0.0065	0.0492	0.0090	0.0248
ASCE41-13	0.0098	0.0448	0.0084	0.0334	0.0098	0.0348	0.0055	0.0205	0.0150	0.0343
NZSEE06	0.0062	0.1715	0.0062	0.1168	0.0067	0.2057	0.0071	0.1638	0.0062	0.0479
EN 1998-3: 2005	0.0095	0.0857	0.0095	0.0594	0.0108	0.1002	0.0090	0.0776	0.0140	0.0423

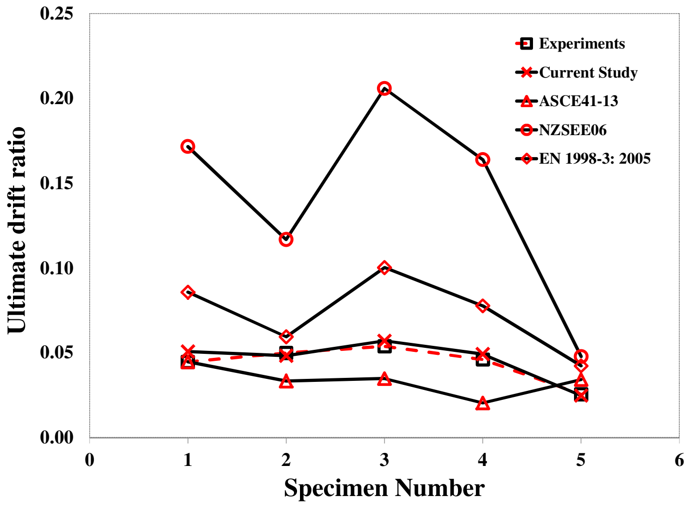


Figure 3. Evaluating ultimate drift ratio predicted by each method vs experiment

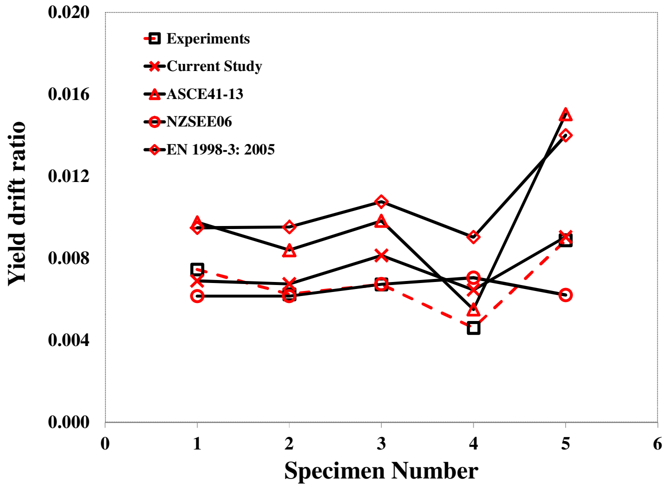
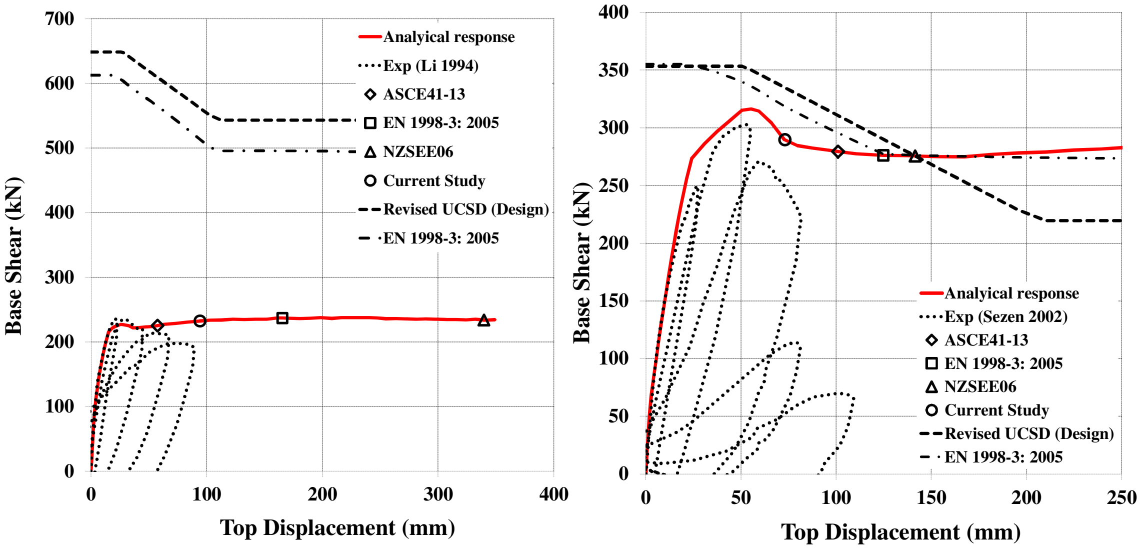


Figure 4. Evaluating yield drift ratio predicted by each method vs experiment

Fig. 5 shows the force-displacement curves of two of these specimens (Specimens 3 and 5). It should be noted that in this figure, revised UCSD (Kowalsky and Priestley 2000) and EN 1998-3: 2005 refers to shear capacity estimation in NZSEE2006 and EN 1998-3: 2005.



(a) (b)

Figure 5. Force – Displacement curve of (a) Specimen 3 and (b) Specimen 5

The failure modes predicted by ASCE41-13, current study and those reported in the experimental tests are presented in Table 6. It should be noted that all specimens were tested as a cantilever column except specimen 5 (Sezen 2002) which was tested as a double-curvature. In Table 5, for some cases (1 and 3) the predicted ultimate displacements were overestimated by the proposed method in comparison to the experimental results. However, these tests were stopped before observing 20% reduction in strength; the third criterion of this study.

Table 6. Failure mode of each specimen

Specimen number	Failure mode (Experiment)	Failure criterion (current study)	Failure mode (ASCE41-13)
1	Flexure	Elwood and Moehle (2005)	Flexure
2	Flexure	Berry and Eberhard (2005)	Flexure
3	Flexure	Berry and Eberhard (2005)	Flexure
4	Flexure	Berry and Eberhard (2005)	Flexure
5	Shear	Elwood and Moehle (2005)	Shear

When flexural response of a column along with controlling shear capacity is used to find the ultimate drift ratio (NZSEE06 and EN 1998-3: 2005), an additional procedure to control buckling is also required; especially when the failure mode is not shear (Fig. 5a). Also a more accurate method to predict the plastic hinge length is necessary since it affects the member response significantly.

ASCE41-13 seems to predict the ultimate rotation angle (drift ratio) conservatively which is acceptable for practical engineering purposes.

As explained before and also shown in Table 4 and Fig. 4, if calculating ultimate drift ratio using plastic rotation angle requires calculating yield rotation angle, it can result in different ultimate rotation angles when using different methods.

4 COLUMNS DESIGNED BY NZ GUIDELINES

In this section, columns which were designed in accordance with NZS3101: 1970 – 2006 and reported in another study by Niroomandi et al. (2015) are investigated (see Table 7, Fig. 6 and 7) and the ultimate drift ratio of each column is compared to the ones predicted by national and international standards (namely ASCE41-13, EN 1998-3: 2005 and NZSEE2006). Boys and Bull (2012) also proposed plastic rotation angle for columns based on the yield curvature and plastic hinge length; this one is also compared to the other methods. It should be noted that Boys and Bull (2012) method can only predicts the plastic rotation angle of columns with ductile and limited ductile design (NZS3101: 2006). Therefore this method cannot predict the drift capacity of columns with other ductility. Table 8 shows the ultimate drift ratio of the columns predicted by each method (see Fig. 8 for graphical presentation).

Table 7. Characteristics of each column

Column number	Column name	Geometry L×h×b (mm)	v	ρ	ρ_s	vertical spacing of ties, s (mm)
1	NZS3101: 1970	1500×400×400	0.3	0.0157	0.00049	320
2	NZS3101: 1982 (Gravity column with ϕ=0.7)	1500×400×400	0.3	0.0157	0.00123	320
3	NZS3101: 1982 (Gravity column with ϕ=0.9 and seismic design)	1500×400×400	0.3	0.0157	0.0174	65
4	NZS3101: 1995 & 2006 (Nominally ductile column)	1500×400×400	0.3	0.0157	0.00679	125
5	NZS3101: 1995 & 2006 (Ductile design column)	1500×400×400	0.3	0.0157	0.01508	75
6	NZS3101: 2006 (Limited Ductile design column)	1500×400×400	0.3	0.0157	0.00998	85

Table 8. Ultimate drift ratio predicted by each method

Column number	1	2	3	4	5	6
			Ultimate drift ratio, δ_u			
Current Study	0.0171	0.0200	0.0689	0.0421	0.0656	0.0544
ASCE41-13	0.0181	0.0201	0.0357	0.0355	0.0357	0.0356
NZSEE06	0.0119	0.0129	0.1095	0.0501	0.0960	0.0692
EN 1998-3: 2005	0.0174	0.0196	0.2359	0.1030	0.2056	0.1455
Boys and Bull	-	-	0.1650	-	0.1650	0.0950

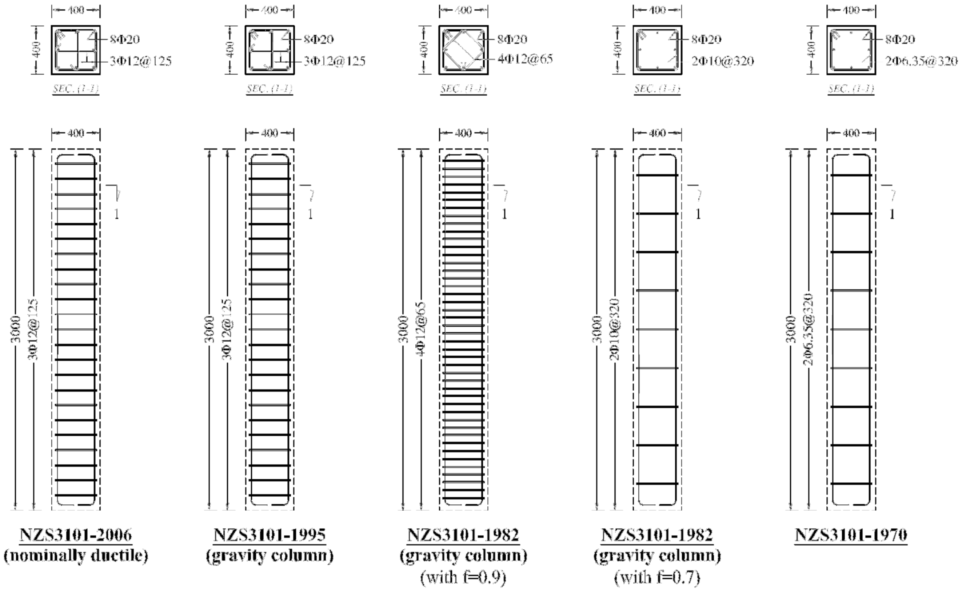


Figure 6. Typical gravity columns according to NZS 3101 (Niroomandi et al. 2015)

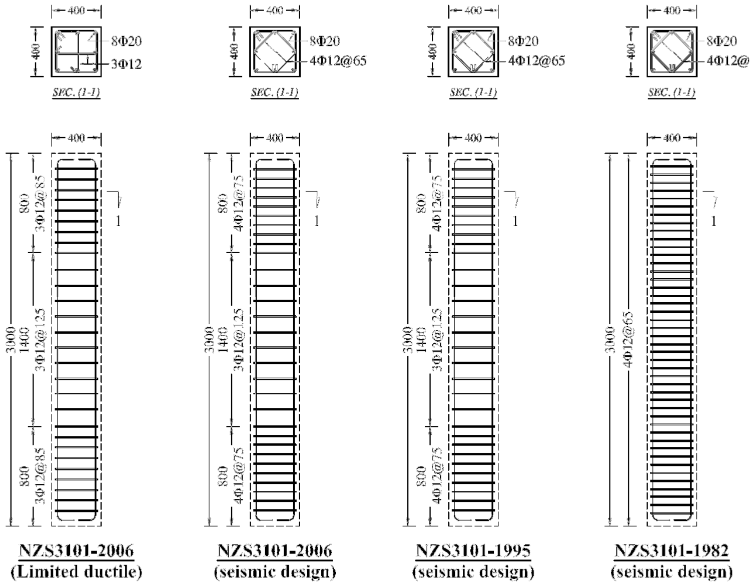
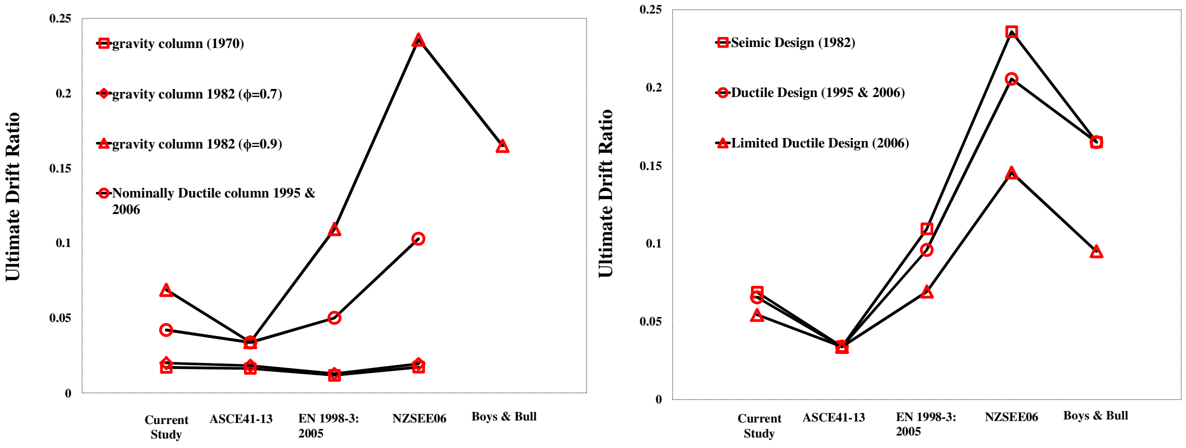


Figure 7. Typical columns with seismic design according to NZS 3101 (Niroomandi et al. 2015)



(a) (b)

Figure 8. Ultimate drift ratio predicted by each method (a) gravity and (b) seismic design columns

Figure 9 also shows the evolution of NZ history design codes (NZS3101: 70-06) in terms of force – drift ratio predicted by the method proposed in this paper. As it is seen, except columns designed according to NZS3101: 1982 with ϕ=0.9, the performance of the columns designed from 1970 to 2006 enhanced significantly in terms of strength and displacement capacity.

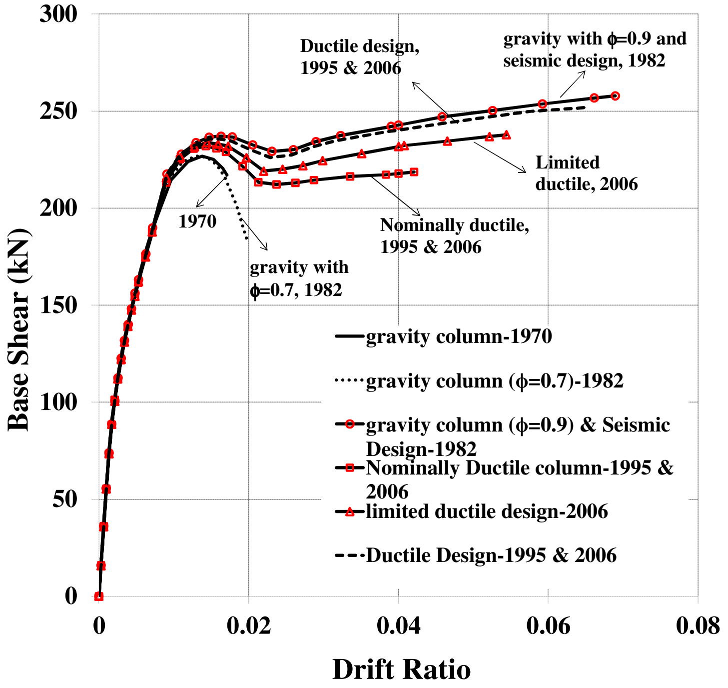


Figure 9. Evolution of Force – Drift ratio of columns designed by NZ codes (NZS3101: 70-06)

5 CONCLUSION

In this study, the reliability of the available national and international assessment guidelines (namely ASCE41-13, EN 1998-3: 2005 and NZSEE2006) is investigated against the experimental results available in the literature. The results of these investigations are summarised as follows:

1. Large variations exist among different methods of calculating the plastic hinge length. Therefore, there is a need for an accurate plastic hinge length expression.

2. If calculating the ultimate displacement is dependent on yield displacement then using different methods of calculating yield displacement will affect the ultimate displacement.

3. ASCE41-13 predicts the drift capacity of columns conservatively which is acceptable for practical engineering purposes.

4. It was found that NZSEE06 and EN 1998-3: 2005 may overestimate the ultimate drift ratio of columns, especially when the failure mode is not shear.

5. A reliable and simple assessment procedure capable of predicting the seismic performance of RC columns taking into account bar buckling and shear failure is proposed. This will later be utilized to develop a simplified procedure for engineering practitioners.

REFERENCES:

ASCE 41-13. 2013. Seismic Evaluation and Retrofit of Existing Buildings. American Society of Civil Engineers, Reston, Virginia.

Bae, S. & Bayrak, O. 2008. Plastic hinge length of reinforced concrete columns. *ACI Structural Journal*, 105(3), pp. 290–300.

Berry, M.P., & Eberhard, M.O. 2005. Practical performance model for bar buckling. *Journal of Structural Engineering*, 131(7).

Boys, A. & Bull, D. 2012. Verification of the proposed acceptance criteria for the assessment of reinforced concrete columns. SESOC NZ Conference.

Cumbia. 2007. Set of codes for the analysis of reinforced concrete members. Alaska department of transportation and public facilities, Juneau, Alaska.

Deyanova, M., Pampanin, S. & Nascimbene, R. 2015. Assessment of seismic response of slender reinforced concrete columns in precast industrial buildings. Proceeding of the NZSEE15, Rotorua, New Zealand.

Elwood, K.J. & Eberhard, M.O. 2006. Effective stiffness of reinforced concrete columns. PEER Research Digest 2006/01.

Elwood, K.J. & Moehle, J.P. 2005. Drift capacity of reinforced concrete columns with light transverse reinforcement. *Earthquake Spectra*, 21(1): 71–89.

King, D.J, Priestley, M.J.N & Park, R. 1986. Computer programs for concrete column design. Research report 86/12, Department of Civil Engineering, University of Canterbury, New Zealand.

Kowalsky, M.J. & Priestley, M.J.N. 2000. Improved analytical model for shear strength of circular reinforced concrete columns in seismic regions. *ACI Structural Journal, 97(3).*

Li, X. 1994. Reinforced concrete columns under seismic lateral force and varying axial load. PhD thesis, University of Canterbury, Christchurch, New Zealand.

Mander, J.B., Priestley, M.J. & Park, R. 1988. Theoretical stress-strain model for confined concrete. *Journal of structural engineering*, 114(8), 1804-1826.

Moyer, M.J. & Kowalsky, M.J. 2003. Influence of tension strain on buckling of reinforcement in concrete columns. *ACI Structural Journal 100(1): 75-85.*

Niroomandi, A., Pampanin, S. & Dhakal, R. 2015. The History of Design Guidelines and Details of Reinforced Concrete Column in New Zealand. Proceeding of the NZSEE15, Rotorua, New Zealand.

NZS 3101, 1970. Code of Practice for the Design of Concrete Structures. Standards Association of NZ.

NZS 3101, 1982. Code of Practice for the Design of Concrete Structures. Standards Association of NZ.

NZS 3101, 1995. Code of Practice for the Design of Concrete Structures. Standards Association of NZ.

NZS 3101, 2006. Code of Practice for the Design of Concrete Structures. Standards Association of NZ.

NZSEE. 2006. Assessment and Improvement of the Structural Performance of Buildings in Earthquake. Recommendations of a NZSEE Study Group on Earthquake Risk Buildings.

Paulay, T. & Priestley, M.J.N. 1992. Seismic Design of Reinforced Concrete and Masonry Buildings, *John Wiley and Sons*, New York.

Priestley, M.J.N., Calvi, M.C., & Kowalsky, M.J. 2007. Displacement-Based Seismic Design of Structures. *IUSS Press*, Pavia.

Priestley, M.J.N, Seible, F. & Calvi, G.M. 1996. Seismic design and retrofit of bridge structures. *John Wiley and Sons*, New York.

Sezen, H. 2002. Seismic behavior and modelling of reinforced concrete building columns. PhD thesis, Dept. of Civil and Environmental Engineering, Univ. of California at Berkeley, Berkeley, California.

Standard, British. 2005. Eurocode 8: Design of structures for earthquake resistance - Part 3: Assessment and retrofitting of buildings.

Tanaka, J. 1990. Effect of lateral confining reinforcement on the ductile behaviour of reinforced concrete columns. PhD thesis, University of Canterbury, Christchurch, New Zealand.