A Design Spectrum Model Featuring Resonant-Like Soil-Amplification

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ABSTRACT

Design spectrum model in AS1170.4 and other major codes of practice for the structural design of buildings stipulate empirical site factors for each of the five, or six, site classes. The criteria for site classification are based on qualitative descriptions of the sedimentary layers along with average shear wave velocity values of the soil/rock materials. This simple format of modelling the effects of site amplification is widely accepted albeit complexity of the wave modification processes and multiple reflections of shear waves within the soil medium. The phenomenon of resonant like amplification behaviour of the structure caused by multiple wave reflections is well known. However, potentials for such periodic amplification behaviour are typically not factored by code models in the determination of the site factors. This is partly because of expert opinion that such effects are very “localised” in the frequency domain and can be suppressed readily by damping. Thus, influential factors such as energy absorption behaviour of the soil sediments, shear wave velocity properties of the bedrock and properties of the bedrock excitations are not parameterised in most code models. Importantly, investigations into the risk of collapse (overturning) of non-ductile, and irregular, structural systems typifying constructions in regions of low and moderate seismicity revealed extensive influence of periodic base excitations on some soft soil sites. An alternative design spectrum model which addresses the important phenomenon of soil resonance without the need of numerical site response analysis of the subsurface model of the site is introduced in this paper.

Keywords: Design spectrum, code, site factor, soil amplification, resonance, displacement

INTRODUCTION

Design response spectrum models for soil sites are typically of the flat-hyperbolic form as for rock sites. Site factors are applied uniformly over the flat and the hyperbolic sections of the response spectrum respectively. Design response spectra for rock and soil sites are similar in shape except that the corner period of the response spectrum is shifted as a result of applying different site factors to the acceleration and velocity (and displacement) controlled regions of the response spectrum. Seismic action models in AS1170.4 (2007) and other major codes of practice (e.g. Eurocode 8; NZS1170.5, 2004; International Building Code, 2012) for the structural design of buildings stipulate empirical site factors for each of the five, or six, site classes. Value of the empirically derived site factor is expressed simply as function of the site class each of which is identified with a range of shear wave velocity values. The International Building Code of the United States (International Code Council, 2012) also stipulates different sets of site factors for different intensities of ground shaking in order that conditions of low and high seismicity are covered under one model.

This simple format of modelling site effects is widely accepted albeit that in reality the modification of seismic waves through soil sediments is well known to be highly frequency selective and under the influence of many factors. The linear filter (frame) analogy can be used to explain the observed phenomena that only wave components in a certain range of frequencies are amplified (Lam *et al.*, 2001). It is also shown by this analogy that the extent of the amplification can be very dependent on the energy absorption behaviour of both the soil sediments and the superstructure itself. Thus, the amount of shear strains (i.e. nonlinearity) imposed on the soil material and (for cohesive soils) the plasticity index are amongst the controlling parameters.

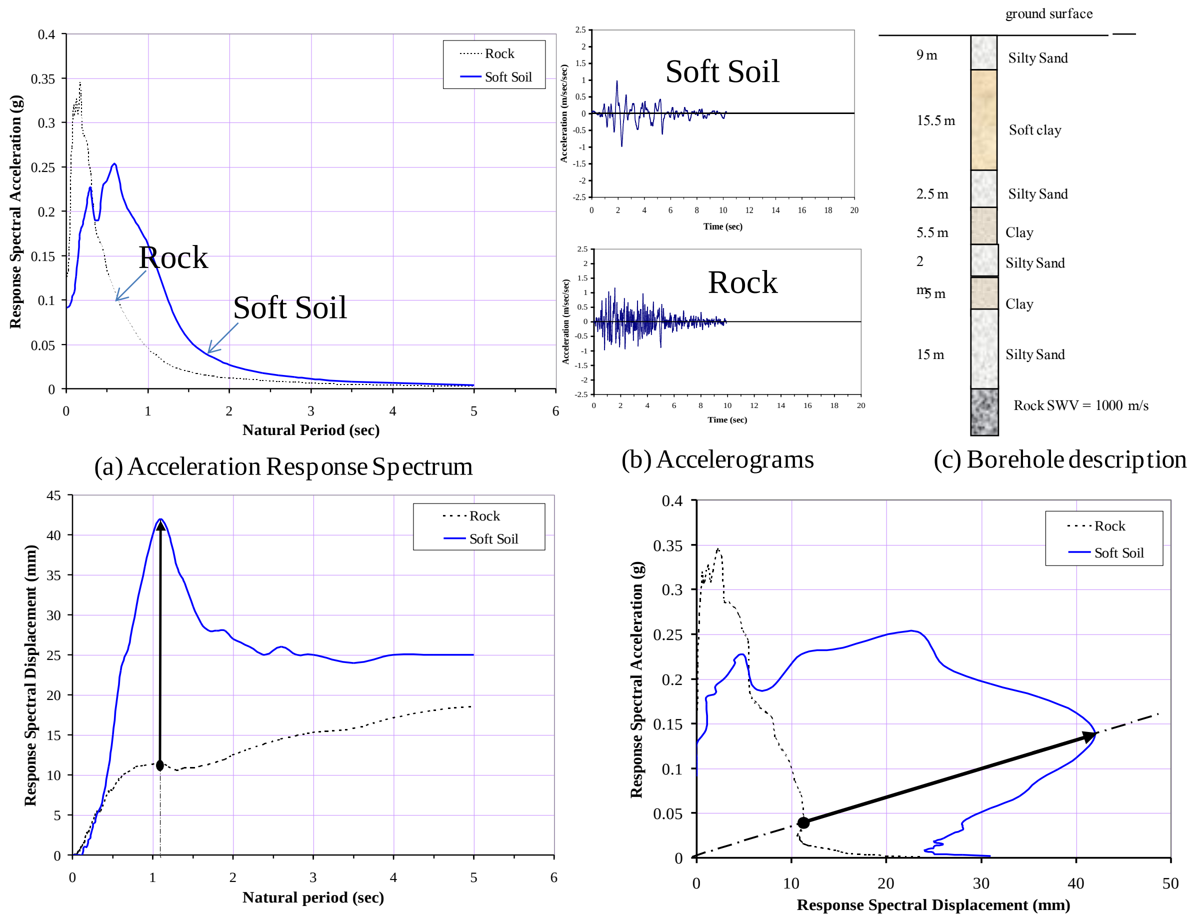
Resonant like amplification behaviour of the structure found on the soil surface can be resulted from the superposition of reflected waves. Thus, factors such as seismic impedance ratio of the soil-bedrock interface and thickness of the soil layers can also have important influence on the behaviour of ground motions on the soil surface given that these factors control the reflections of shear waves within the soil medium.

The wave modification mechanisms as described are well known and can be simulated by simple one-dimensional equivalent-linear dynamic analysis of the soil sediments (Schnabel *et al.*, 1972) but potentials for such periodic amplification behaviour are typically not factored into code provisions for the modelling of site effects. This is partly because of preference to simplicity and partly because of expert opinion that such effects are only “localised” (in the frequency domain) and can be suppressed readily by damping in the soil along with ductile behaviour of the structure.

However, non-ductile, and irregular, structural systems typify constructions in regions of low and moderate seismicity. The priority in design, and retrofitting, in such built environments is to safeguard these structural systems from total or partial collapse in a rare event for avoiding loss of lives and minimizing casualties. Until recently, the post-elastic behaviour of these systems in ultimate conditions featuring significant strength degradation and P-delta effects were uncertain and could only be modelled by rigorous non-linear time-history analyses. Reliable predictions can now be made using simple hand calculation techniques which employ displacement-based principles (Lumantarna *et al.*, 2010). Importantly, investigations into the risk of collapse (overturning) of this type of structure employing a kinematic-based calculation technique revealed significant influence by the peak displacement demand of the ground motion (Kafle *et al.*, 2011). This peak displacement demand can be amplified to a very high value by the periodic excitations of a soft soil site. Central to the calculation technique is the use of response spectrum in the displacement format (or ADRS format) which is much more effective in representing the displacement demand behaviour of the ground motion than response spectrum in the conventional acceleration format.

Figures 1a – 1e present the results of a case study of a soft soil site experiencing resonant-like amplification behaviour. Clearly, the frequency contents of ground motions have been dramatically modified by the soft soil sediments as revealed by the displacement response spectrum (Figure 1d) and ADRS diagram (Figure 1e). In contrast, the phenomenon is not clearly shown on response spectrum presented in the conventional acceleration format (Figure 1a). Furthermore, it should be noted that the resonant phenomenon as described can be easily masked by taking ensemble averages of accelerogram records (unless all the recording sites feature distinct effects of resonance at the same site natural period which is usually not the case). For this reason, the phenomenon of resonant-like soil amplification behaviour is not featured in empirically developed codified models for site effects.

A response spectrum model of the displacement format which takes into account the described amplification phenomenon as depicted in Figure 1d is introduced in this paper. Relationships for estimating the site factor and the site natural period are presented in the form of algebraic expressions and design charts in order that a representative response spectrum can be constructed readily without the need of numerical dynamic analyses. The important effects of soil depth have been parameterised whereas the effects of damping in the soil and impedance ratio at the soil-bedrock interface have also been taken into account.



(a) Acceleration Response Spectrum (b) Accelerograms (c) Borehole description

(d) Displacement Response Spectrum (e) Acceleration-Displacement Response Spectrum

Figure 1. Response spectra showing the effects of resonant-like soil-amplification behaviour

RESPONSE SPECTRUM MODEL FOR SOIL SITES IN THE DISPLACEMENT FORMAT

The recommended site-specific displacement response spectrum model (of *RSD* format) as depicted in Figure 2 has a theoretical basis. The construction of the soil response spectrum involves estimation of the site natural period (T_S) and site factor (S) as annotated in the figure. The site factor (S) encapsulates the effects of multiple wave reflections in the soil medium leading to resonant like soil amplification behaviour. The extent of amplification which is represented by the value of S is function of the seismic impedance (product of shear wave velocity and density) of the soil and bedrock materials, intensity of shaking, soil thickness and energy dissipation behaviour which for clayey soil is dependent on the plasticity index. Algebraic expressions have been developed for estimating the value of S (Tsang *et al.*, 2006a). In addition, expressions have also been developed for modelling the degradation of the shear modulus of the soil in order that the amount of site period shift (expressed in terms of the period-shift ratio) can be estimated along with the amount of damping in the soil (Tsang *et al.*, 2006b). The analysis of a soil column often involves modelling bedrock as a homogenous “half space” which is often characterised by a distinct shear wave velocity (V_R) value in order that seismic impedance at the soil-bedrock interface can be calculated. In reality, bedrock is not a homogenous material since its shear wave velocity value increases with depth. A method for calculating the value of V_R as function of the shear wave velocity profile has been presented in Tsang *et al.* (2012). The methodology to be presented in the rest of this paper is based on these developed expressions which are collectively known as the *Single Period Approximation* (*SPA*) Model.

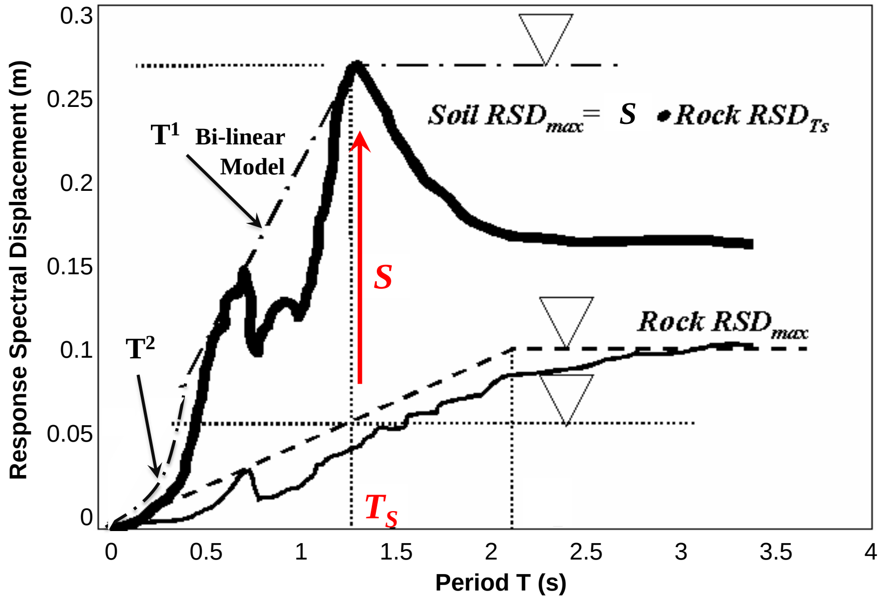


Figure 2. Parameters for representing the effects of resonant-like soil-amplification behaviour

SPA Model in its original form as presented in Tsang *et al.* (2006a) has been validated by comparison with results obtained from dynamic analyses of soil column models based on the use of the well known computer program SHAKE (Schnabel *et al.*, 1972). The great majority (around 95%) of results estimated by the model were within some +/- 20% of the computed values. This accuracy is considered very satisfactory given the high level of uncertainties surrounding ground motion and response spectrum modelling. Significantly, the model has been validated by comparison with some 200 strong motion accelerograms that have been recorded during the 1994 Northridge earthquake. This single event has provided abundant data for a wide range of shaking intensity (with peak ground velocity values on bedrock ranging between 20 and 450 mm/s). The model estimates have been found to be in good agreement with recorded results, being within +/- 50%. Achieving this level of accuracy is considered very satisfactory given the complex nature of the soil modification processes and the dynamic properties of the soil sediments.

DETERMINATION OF SITE NATURAL PERIOD

The site natural period of the soil sediment (T_i) can be estimated using Eqn. (1) when reliable information over the thickness (d_i) and estimated shear wave velocity (V_i) value of each individual soil layer (as denoted by subscript '*i*', and '*n*' is the total number of layers) are available.

$$T_i = \sum_{i=1}^{n} \frac{d_i}{V_i} \times 4 = \frac{4H_S}{V_{S,i}} \qquad (1)$$

H_S is the total thickness of the soil sediment, and $V_{S,i}$ is the initial weighted average shear wave velocity of soil layers. Values of V_i can be inferred from SPT-*N* values which are often recorded on borehole logs using a conversion relationship such as the well-known expression of Eqn. (2) as per recommendations by Imai and Tonouchi (1982):

$$V_i = 97 \times N^{0.314} \qquad (2)$$

The value of T_i provides a general indication of the nature of the sediments. It is recommended that thin/stiff soil layers where $T_i \le 0.15$ s can be treated as rock outcrops.

All the parameters introduced in Eqns. (1) and (2) are based on "initial" conditions meaning that the effects of degradation in the value of the shear modulus (and shear wave velocity) with increasing intensity of shaking (or straining of the soil materials) have not been taken into account. Thus, V_i can be described as the initial shear wave velocity and T_i the initial site natural period.

The amount of shear strains induced in the soil materials is best estimated by considering the soil sedimentary layer as an elastic single-degree-of-freedom (SDOF) system which is found on bedrock and of initial natural period value equal to T_i (Lam *et al.*, 2001; Tsang *et al.*, 2006b). The estimated amount of drift of the imaginary SDOF system (i.e. response spectral displacement at T_i, which can be read off from the rock spectrum as RSD_{Ti}) may be taken to be indicative of the shear deformation of the soil materials. It can be shown that the amount of shear strains experienced by the soil in an earthquake is proportional to the ratio $\frac{RSD_{Ti}}{H_S}$ or $\frac{RSV_{Ti}}{V_{S,i}}$ (whilst RSV_{Ti} is the response spectral velocity at T_i). The latter ratio has been defined in Tsang *et al.* (2006b) as the *dimensionless spectral parameter* which has been given the notation: ψ.

Expressions for determining the value of the shifted site natural period (T_S) and the degraded soil shear wave velocity (V_S) as function of ψ along with other dimensionless parameters have been developed in Tsang *et al.* (2006b). The original expressions have been simplified herein into Eqns. (3 or 4) and (5) for conditions where soil layers of low plasticity index overlie bedrock of much higher shear wave value values (V_R = 1800 m/s). It is noted that recent data and studies suggest less dependency of dynamic soil properties on plasticity index than previously thought (Darendeli, 2001; Zhang *et al.*, 2005; Vardanega and Bolton, 2011), such simplification can be considered reasonable.

$$T_S = T_i + \pi \frac{RSD_{Ti}}{V_{S,i}} \quad (3) \qquad \text{or} \qquad \frac{T_S}{T_i} = 1 + \frac{\pi}{4}\frac{RSD_{Ti}}{H_S} \quad (4)$$

$$\frac{V_S}{V_{S,i}} = \frac{T_i}{T_S} \quad (5)$$

It is noted that the value of RSD_{Ti} is expressed in mm, $V_{S,i}$ in m/s and H_S in m.

The ratio of period-shift T_S / T_i can be conveniently estimated using Figure 3 in which $V_{S,i}$ is shown as the only controlling parameter as RSV_{Ti} is held constant. Taking the current design spectrum model in AS1170.4 for rock sites as a reference, RSV_{Ti} is constant between natural period of 0.3 s and 1.5 s, which cover the majority of soil sites. The presented relationship is based on Eqn. (4) and an assumed value of RSV_{Ti} of 200 mm/s at period T_i or a design peak ground velocity on rock of the order of 100 – 120 mm/s. This peak ground velocity value is consistent with the hazard level of a number of major cities in Australia including Canberra, Melbourne and Sydney for a return period of 2500 years (AS1170.4). Similar curves can be constructed for other ground motion intensity values.

DETERMINATION OF SITE FACTOR

The non-linear site amplification factor (S) is defined herein as the ratio of the maximum response spectral displacement value on the soil spectrum (*Soil* RSD_{max}) and that on the rock spectrum at the shifted site natural period (T_S) (i.e. *Rock* RSD_{Ts}) as illustrated in Figure 2. Expressions that can be used for estimating the value of S have been developed in Tsang *et al.* (2006a) forming part of the *SPA* Model. Summary of the model is presented in the followings:

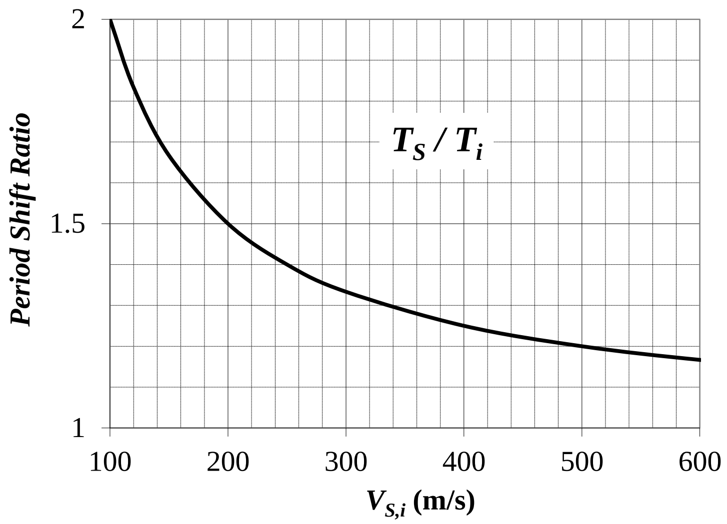


Figure 3. Chart for determination of the value of T_S / T_i

$$S = f(\alpha) \cdot \frac{2\alpha}{1+\alpha} \sqrt{\frac{\beta}{1-R^4\beta^4}} \qquad (6)$$

where $f(\alpha) = \alpha^{0.3} \leq 2.3$ is known as the resonance factor. (7)

Eqns. (6) and (7) are based on fundamental theory of shear waves along with the concept of multiple wave reflections within the soil medium. Relevant parameters namely the impedance ratio α, reflection coefficient *R*, and damping factor β can be calculated using the following expression:

$$\alpha = \frac{\rho_R V_R}{\rho_S V_S} \qquad (8)$$

where V_R and V_S are the equivalent shear wave velocity of bedrock and the degraded soil shear wave velocity, respectively; ρ_R and ρ_S are the weighted averages of the densities of bedrock and soil, which can be taken as 2.3 t/m^3 and 1.8 t/m^3, respectively, if more precise estimates are not available.

R is used to characterize the amplitude ratio of the upwardly propagating (reflected) shear waves and the downwardly propagating incident shear waves at the soil-bedrock interface and is obtainable using the expression:

$$R = \frac{1-\alpha}{1+\alpha} \qquad (9)$$

Damping parameter β is related to the energy dissipation within the soil layer, and is obtainable using the expression:

$$\beta = \exp(-\pi\zeta) \qquad (10)$$

where

$$\zeta = 10.8 + 6.5 \log\left(\frac{\pi}{4} \frac{RSD_{Ts}}{H_S}\right) \qquad (11)$$

ζ is the damping ratio of the soil layers and Eqn. (11) has been simplified from the original expressions developed in Tsang *et al.* (2006b).

The value of *S* can also be determined using an expression of the simplified form as shown in Figure 4. Both Figures 3 and 4 which are based on the same assumptions can be used in lieu of Eqns. (3)

through (11). There have been suggestions to express site factor as continuous function of one or two soil parameters that can readily be measured (McVerry, 2011). The *SPA* Model as presented herein is in alignment with this proposition. Alternatively, the conventional (discrete) format can also be used to present predictions by the same model (Table 1).

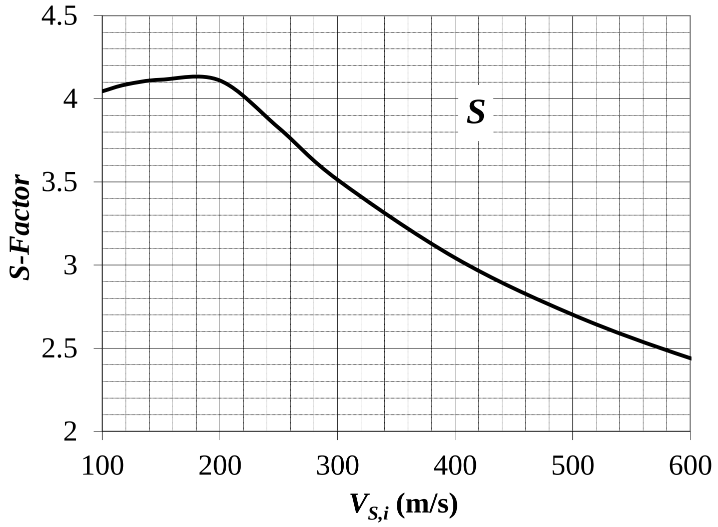


Figure 4. Chart for determination of the site amplification factor (*S*)

Table 1. Proposed site classification scheme and the associated sets of period-shift ratio T_S / T_i and site amplification factor (*S-Factor*).

Site Class	$V_{S,i}$ (m/s)	T_S / T_i	S
A	(T_i < 0.15 s)	Rock Site	
B	> 480	1.2	2.8
C	360 – 480	1.3	3.2
D	280 – 360	1.4	3.6
E	< 280	1.5	4.0

CONSTRUCTION OF SOIL DESIGN SPECTRUM

Once the shifted site natural period (T_S) and the site amplification factor (*S-Factor*) have been calculated, key parameters characterizing the response spectral properties of the modelled ground motion can be found. These key parameters are namely maximum spectral displacement $RSD_{\max}$, velocity $RSV_{\max}$ and acceleration $RSA_{\max}$ which can be obtained using Eqns. (12) - (14).

$$RSD_{\max} = RSD_{Ts} \times S \geq RSD_{LB} \quad (12)$$

$$RSV_{\max} = RSD_{Ts} \times S \times \left(\frac{2\pi}{T_S}\right) \quad (13)$$

$$RSA_{\max} = RSV_{\max} \times \left(\frac{2\pi}{T*}\right) \geq RSA_{LB} \qquad \text{where } T* = T_i \geq T_{1R} \quad (14)$$

T_{1R} is the first corner period at the upper limit of the constant spectral acceleration region of the rock response spectrum. RSD_{LB} and RSA_{LB} are the lower bound values of the two response spectral parameters, which are taken as the spectral values on the rock spectrum. In effects, there would be no deamplification anywhere on the response spectrum. It is acknowledged that deamplification at certain period range could indeed occur at some soil sites, as illustrated in Figure 1. To avoid confusion, it would be more convenient to take the rock spectrum as the minimum loading model.

The two corner periods, T_1 and T_2, of the soil response spectrum can be calculated from Equations (15) and (16). T_1 is the first corner period at the upper limit of the constant spectral acceleration region of the response spectrum, whereas T_2 is the second corner period which divides the constant velocity and constant displacement regions of the response spectrum model.

$$T_1 = 2\pi\left(\frac{RSV_{\max}}{RSA_{\max}}\right) = \frac{4\pi^2 RSD_{Ts} \times S}{RSA_{\max} \times T_S} \tag{15}$$

$$T_2 = 2\pi\left(\frac{RSD_{\max}}{RSV_{\max}}\right) = \frac{RSD_{\max} \times T_S}{RSD_{Ts} \times S} \tag{16}$$

It is noted that if the expressions on the right hand side of Equations (15) and (16) are used, it is not necessary to go through the calculation of $RSV_{\max}$ using Equation (13), in view of the fact that RSV is not commonly used in design guidelines or codes of practice.

The soil design response spectrum may then be constructed using the three parameters: $RSA_{\max}$, T_1 and T_2, using Eqn. (17), similar to those stated in major codes of practice worldwide.

$$\begin{aligned}
0 \le T \le 0.1 \quad &: \quad RSA_T(g) = \frac{RSA_{\max}}{2.5}\left(1 + 15 \times T\right) \\
0.1 \le T \le T_1 \quad &: \quad RSA_T(g) = RSA_{\max} \\
T_1 \le T \le T_2 \quad &: \quad RSA_T(g) = RSA_{\max}\left(\frac{T_1}{T}\right) \\
T_2 \le T \le 5 \quad &: \quad RSA_T(g) = RSA_{\max}\left(\frac{T_1 T_2}{T^2}\right)
\end{aligned} \tag{17}$$

The displacement response spectrum is hence obtained by direct transformation from the acceleration response spectrum using Eqn. (18).

$$RSD_T(mm) = RSA_T(g)\left(\frac{T}{2\pi}\right)^2 \times 9810 \tag{18}$$

APPLICATION

The design spectrum construction method as described in the previous sections is illustrated in the following worked example which is based on record of a real borehole in Hong Kong as given in Table 2. The exact location of the site is kept anonymous.

Table 2. Shear wave velocity values of individual soil layers at the sample site.

Type	*Thickness (m)*	*N*	*V_i (m/s)*	
Fill	3	26	270	
CDG	2	39	306	
CDG	2	33	291	
CDG	2	49	329	
CDG	2	50	331	
CDG	2	62	354	
CDG	2	71	370	
CDG	2	79	382	
HDG	1.8	387	630	
	H_S **= 18.8 m**		$V_{S,i}$ **= 337 m/s**	T_i **= 0.223 s**

The calculated values of T_S (shifted site natural period) and S (site factor) are shown along with values of various parameters involved in the calculation in Tables 3a and 3b respectively.

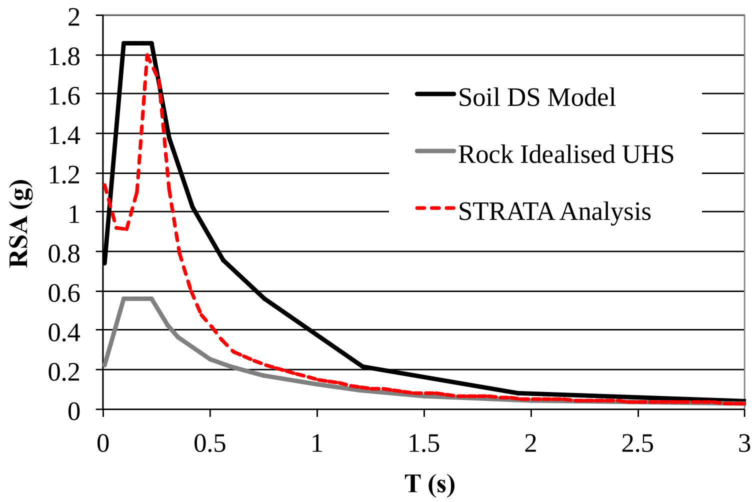
Table 3a. Calculated values of T_S and other parameters in the *SPA* model for the sample site.

T_S **(s)**	V_S (m/s)	α	R	ζ	β
0.29	261	8.95	-0.80	8.1	0.77

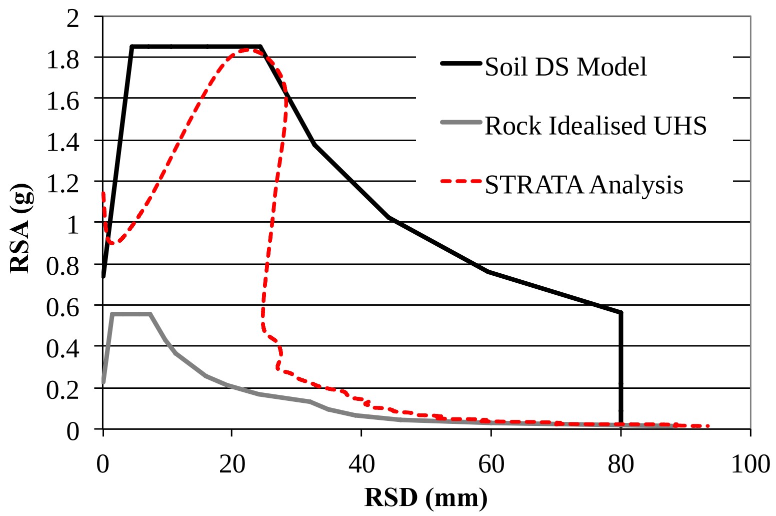
Table 3b. Calculated values of *S* and response spectrum parameters for the sample site.

S	RSD_{max} (mm)	RSV_{max} (mm/s)	RSA_{max} (g)	T_1	T_2
3.31	80	665	1.85	0.23	0.76

The design spectrum model in acceleration (*RSA*) and ADRS diagram formats are shown in Figures 5a and 5b respectively. Overlaid on the proposed design spectrum model are simulation generated by program STRATA (which has the computational algorithm essentially as the same as program SHAKE). Three input bedrock ground motions, with a probability of exceedance of 2% in 50 years (or a return period of 2475 years), each of them representing near-field (NF), medium-field (MF) and far-field (FF) earthquake scenarios for Hong Kong, have been used in the analyses. The robustness of the proposed design spectrum model is well demonstrated.



(a) Acceleration Response Spectrum.



(b) ADRS Diagram

Figure 5. Proposed design spectrum model for the sample site

SUMMARY AND CLOSING REMARKS

A heuristic site-specific design spectrum model which takes into account resonant-like amplification behaviour of soil sediments is presented in this paper. The construction of the soil design spectrum involves estimations of the site natural period (T_S) and site factor (S). The model has been well validated by comparison with results obtained from dynamic analyses of soil columns, as well as strong motion data recorded in 1994 Northridge earthquake (Tsang *et al.*, 2006a).

The initial site natural period (T_i) is first calculated from information of the shear wave velocity values (inferred from SPT-N values) and thicknesses of the individual soil layers. The effects of period-shift resulted from shear straining (nonlinearity) of the soil materials are then taken into account in the estimation of the (final) site natural period (T_S). Site factor (S) is then expressed as function of impedance ratio α, reflection coefficient R, and damping factor β. Relationships for calculation of these parameters have been simplified to waive the need of iterations. The effects of plasticity have been neglected after the simplification, as recent data and studies suggest less dependency of dynamic soil properties on plasticity index than previously thought (Darendeli, 2001; Zhang *et al.*, 2005; Vardanega and Bolton, 2011).

The calculation of the values of T_S and S has been simplified even further into two design charts which are based on assuming a constant response spectral velocity on rock of 200 mm/s or a design peak ground velocity on rock of the order of 100 – 120 mm/s. This peak ground velocity value is consistent with the hazard level of a number of major cities in Australia including Canberra, Melbourne and Sydney for a return period of 2500 years (AS1170.4).

Finally, values of response spectral parameters RSD_{max} , RSV_{max} and RSA_{max} and corner periods T_1 and T_2 are identified for construction of the soil design spectrum in different formats. The procedure is illustrated by an example which is based on analysis of information from a real borehole log. The proposed model has been verified by comparison with results generated by program STRATA (which has the computational algorithm essentially as the same as program SHAKE).

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