

# Response Analysis of a Transmission Tower-Line System to Spatial Ground Motions

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## Abstract

Collapses of transmission towers have always been observed in previous large earthquakes such as the Chi-Chi earthquake in Taiwan and the Wenchuan earthquake in Sichuan. These collapses were partially caused by the pulling forces from the transmission lines generated from out-of-phase responses of the adjacent towers owing to spatially varying earthquake ground motions. In this paper, a three-dimensional finite element model of a transmission tower-line system with geometric nonlinearity of transmission lines is established, and the nonlinear responses of the system subjected to spatially varying ground motions are analysed. The spatially varying ground motions are simulated stochastically based on an empirical coherency loss function, an assumed apparent wave velocity, and a filtered Tajimi-Kanai power spectral density function. Numerical results indicate that the assumption of uniform ground motion could not provide an accurate response estimation of the transmission tower-line system. Ground motion spatial variations increase the internal force and the displacement response of transmission towers and transmission cables. Neglecting earthquake ground motion spatial variations may lead to a substantial underestimation of the transmission tower-line system response during strong earthquake shakings.

**Keywords:** transmission tower-line system, spatially varying ground motions, geometric nonlinearity, coherency loss function

## 1. INTRODUCTION

With the rapid development of improved welding techniques, high strength steel and wire materials and the progress in structural analysis and design theory, the spans and dimensions of transmission tower-line system have been increasing dramatically. For such a structural system, it is unrealistic to assume that earthquake ground motions at multiple towers are the same because of the inevitable ground motion spatial variations. The seismic response of long span structures subjected to spatially varying earthquake ground motions has attracted the attention of many researchers. Zanardo *et al.* (2002) performed a parametrical study of the pounding effect on responses of a multi-span simply supported bridge with base isolation devices. Hao and Duan (1996) investigated the torsional responses of symmetric buildings subjected to spatially varying base excitations. Rassem *et al.* (1996) studied the effect of multiple support excitations on the response of suspension bridges. Harichandran *et al.* (1996) carried out stationary and transient response analyses of suspension and arch bridges to spatially varying ground motion and compared the results with responses computed using identical and delayed excitations. All these investigations concluded that spatially varying earthquake ground motions strongly affect the responses of long span structures. However, most of these studies focus on bridge and building structures, and only linear elastic responses are considered. Owing to complex coupled tower-line vibration and geometric nonlinearity of cable, the behaviours of transmission tower-line system under earthquake ground excitations are expected to be very different from bridge and building structures. But studies of transmission tower-line system response to earthquake ground motions are limited. Therefore, to better understand the behaviours and to achieve a more reliable design of transmission tower-line system to resist spatially varying earthquake ground motions, it is necessary to study the responses of such long and complex structures to spatial earthquake ground excitations.

Since an electric power transmission system generally covers a large area, adjacent transmission towers may locate on sites of different conditions and elevations. In this study, nonlinear responses of a transmission tower-line system at an uneven site subjected to spatially varying ground motions are analysed. The spatial ground motions are simulated stochastically based on an empirical coherency loss function and a filtered Tajimi-Kanai power spectral density function. Discussions on the ground motion spatial variation effect on the tower-line system are made and some conclusions are drawn.

## 2. TRANSMISSION TOWER-LINE SYSTEM MODELLING

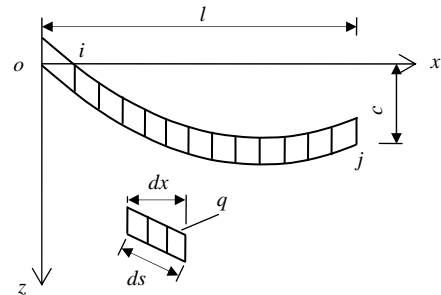
A typical three-dimensional model of coupled transmission tower-line system is established based on a real electric power transmission project in the northeast of China. Computer software SAP2000 is used to model this tower-line system in this study. The single tower model is shown in Figure 2. The weight of tower is approximately 30 t. The structural members of the tower are made of angle steel with the elastic modulus of 206 GPa. The tower is modelled by 1883 space beam members and 727 nodes, the connections of members are rigid, and the structural supports of the towers are assumed to be fixed. The first three frequencies of a standalone tower are 1.771 Hz, 1.773 Hz and 4.021 Hz. The transmission line is modelled by 300 two-node isoparametric cable elements with three translational DOF at each node. The initial axial force and large

deformation effect of cable are taken into consideration. Under self weight, the cable spatial configuration is a catenary. Based on the coordinate system illustrated in Figure 1, the mathematical expression used to define the initial geometry of the cable profile is given in the following form (Shen *et al.* 1997)

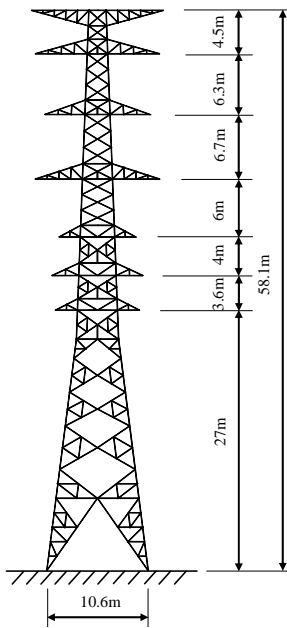
$$z = \frac{H}{q} \left| \cosh(\alpha) - \cosh\left[\frac{2\beta x}{l} - \alpha\right] \right| \quad (1)$$

$$\text{where } \alpha = \sinh^{-1} \left| \frac{\beta(c/l)}{\sin(\beta)} \right| + \beta, \quad \beta = \frac{ql}{2H} \quad (2)$$

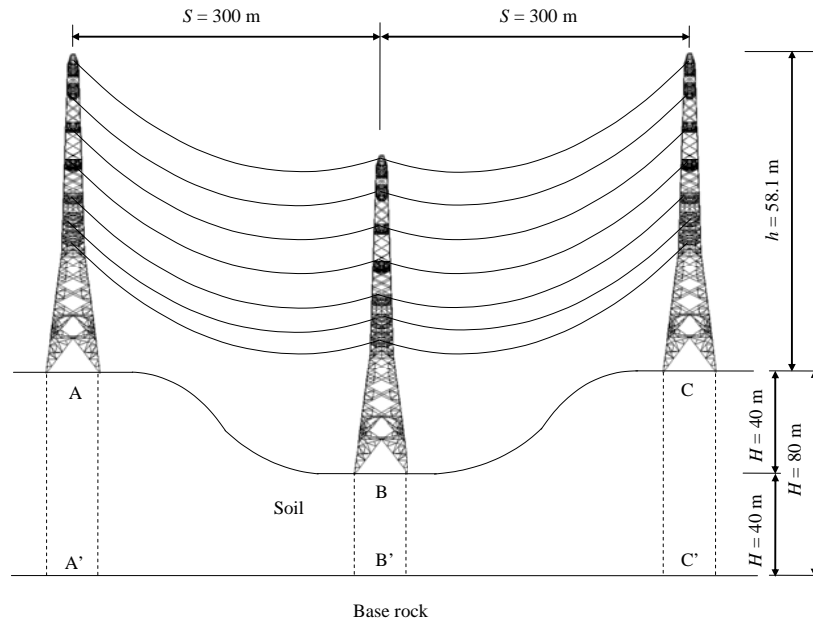
in which  $H$  represents initial horizontal tension which can be obtained from a preliminary static analysis, and  $q$  denotes uniformly distributed gravity loads along the transmission line. The structural model consists of three towers and two-span cables. The transmission lines are assumed to be pin connected to the tower cross arms. The first frequency of the coupled transmission tower-line system is 0.159 Hz, which corresponding to the vibration mode of the top layer cable.



**Figure 1. Coordinates of a single cable under self weight**



**Figure 2. Transmission tower model**



**Figure 3. Schematic view of transmission tower-line system crossing an uneven site**

Figure 3 shows the schematic view of the transmission tower-line system crossing an uneven site under consideration in this study, in which points  $A$ ,  $B$  and  $C$  are the three tower supports on the ground surface, the corresponding points on base rock are  $A'$ ,  $B'$  and  $C'$ . It should be noted that the ground motions at different supports of each tower are assumed the same due to the distances between the supports of the same tower are

small as compared with those between different towers.  $h$ ,  $S$  and  $H$  represent the height of tower, the horizontal span between the adjacent towers and the depth of soil layer under each tower, respectively, the corresponding values considered in this study are given in Figure 3.

### 3. SIMULATION OF SPATIALLY VARYING GROUND MOTIONS

#### 3.1. Power spectral density function

Ground motion intensities at  $A'$ ,  $B'$  and  $C'$  on the base rock are assumed to be the same but vary spatially. Its power spectral density is modelled by a filtered Tajimi-Kanai power spectral density function, which is expressed as (Ruiz and Penzien 1969)

$$S_g(\omega) = \frac{\omega^4}{(\omega_f^2 - \omega^2)^2 + (2\omega_f\omega\xi_f)^2} \frac{1 + 4\xi_g^2\omega^2/\omega_g^2}{(1 - \omega^2/\omega_g^2)^2 + 4\xi_g^2\omega^2/\omega_g^2} \Gamma \quad (3)$$

in which  $\omega_f$  and  $\xi_f$  are the central frequency and damping ratio of the high pass filter, respectively.  $\omega_g$  and  $\xi_g$  are the central frequency and damping ratio of the Tajimi-Kanai power spectral density function, respectively.  $\Gamma$  is a scale factor depending on the ground motion intensity, and  $\Gamma = 0.0078m^2/s^3$  is used in this study, which corresponds to a ground acceleration of duration  $T = 20s$  and peak value (PGA)  $0.3g$  estimated with the standard random vibration method (Der Kiureghian 1980). Without losing generality, in this study, it is assumed that  $f_f = \omega_f/2\pi = 0.25Hz$ ,  $\xi_f = 0.6$ ,  $f_g = \omega_g/2\pi = 5.0Hz$  and  $\xi_g = 0.6$ .

#### 3.2. Coherency loss function

An empirical coherency loss function derived from recorded strong ground motions at the SMART-1 array is used in this study to model ground motion coherency loss (Hao *et al.* 1989). The coherency loss function between two locations  $i'$  and  $j'$  on the base rock is

$$|\gamma_{ij}(i\omega, d_{ij})| = \exp(-\beta d_{ij}) \exp[-\alpha(\omega) \sqrt{d_{ij}} (\omega/2\pi)^2] \quad (4)$$

in which  $d_{ij}$  is the projected distance between points  $i'$  and  $j'$  in the wave propagation direction,  $\beta$  is a constant and  $\alpha(\omega)$  is a function with the form

$$\alpha(\omega) = \begin{cases} 2\pi a / \omega + b\omega / 2\pi + c, & 0.314rad/s \leq \omega \leq 62.83rad/s \\ 0.1a + 10b + c, & \omega > 62.83rad/s \end{cases} \quad (5)$$

where  $a$ ,  $b$  and  $c$  are constants and can be obtained by regression method. The constants of the coherency loss function used here are derived from the recorded strong motions during Event 45 at the SMART-1 array (Hao 1989), and it represents highly correlated ground motions.

The cross power spectral density function of motions at two locations  $i'$  and  $j'$  on the base rock is expressed as

$$S_{ij}(i\omega) = S_g(\omega)\gamma_{ij}(i\omega) \quad (6)$$

where

$$\gamma_{ij}(i\omega) = \left| \gamma_{ij}(i\omega) \right| \exp(-i\omega d_{ij} / v_{app}) \quad (7)$$

in which  $v_{app}$  is the apparent wave propagation velocity and the exponential function represents the influence of wave passage effect.

### 3.3. Method for generating spatially varying ground motions

The spatially varying ground motions are simulated stochastically based on the above mentioned empirical coherency loss function and filtered Tajimi-Kanai power spectral density function, which is based on the simulation method developed by Hao *et al.* (1989). The ground acceleration power spectral density function of the site with three structural supports can be expressed as

$$S(i\omega) = \begin{bmatrix} S_{11}(\omega) & S_{12}(i\omega) & S_{13}(i\omega) \\ S_{21}(i\omega) & S_{22}(\omega) & S_{23}(i\omega) \\ S_{31}(i\omega) & S_{32}(i\omega) & S_{33}(\omega) \end{bmatrix} \quad (8)$$

in which  $S_{ii}(\omega)$  and  $S_{ij}(i\omega)$  ( $i, j = 1, 2, 3$ ) are auto power spectral density function and cross power spectral density function at ground surface points, respectively. The functions can be obtained by

$$S_{ii}(\omega) = |H_i(i\omega)|^2 S_g(\omega) \quad (9)$$

$$S_{ij}(i\omega) = H_i(i\omega)H_j^*(i\omega)S_{ij}(i\omega) = H_i(i\omega)H_j^*(i\omega)\gamma_{ij}(i\omega)S_g(\omega) \quad (10)$$

where the superscript ‘\*’ represents complex conjugate,  $H_i(i\omega)$  and  $H_j(i\omega)$  are the transfer function at sites  $i$  and  $j$ , respectively, which can be estimated by one dimensional wave propagation assumption developed by Hao and Chou (2006). The transfer function can be expressed as

$$H_i(i\omega) = \frac{(1 + r_i - i\xi_i)e^{-i\omega\tau_i(1-2i\xi_i)}}{1 + (r_i - i\xi_i)e^{-2i\omega\tau_i(1-2i\xi_i)}} \quad (11)$$

in which  $\tau_i = h_i / v_i$  is the wave propagation time from point  $i'$  to  $i$ , the reflection coefficient for up-going waves  $r_i$ , which plays a significant role in the effects of the soil layer, is given by

$$r_i = \frac{\rho_R v_R - \rho_i v_i}{\rho_R v_R + \rho_i v_i} \quad (12)$$

where  $\rho$ ,  $v$  and  $\xi$  denote the density, the shear wave velocity and the damping ratio, respectively. In this study, the parameters of base rock are assumed that  $\rho_R = 3000 \text{ kg/m}^3$ ,  $v_R = 1500 \text{ m/s}$ ,  $\xi_R = 0.05$ ; the corresponding parameters of firm soil under support point  $i$  are  $\rho_i = 2000 \text{ kg/m}^3$ ,  $v_i = 450 \text{ m/s}$ ,  $\xi_i = 0.05$ .

Since the matrix  $S(i\omega)$  is Hermitian and positive definite, it can be decomposed into the multiplication of a complex lower triangular matrix  $L(i\omega)$  and its Hermitian matrix  $L^H(i\omega)$  by the Cholesky's method, which is given by

$$S(i\omega) = L(i\omega)L^H(i\omega) \quad (13)$$

The spatial ground motion time histories of the three different locations can be obtained by

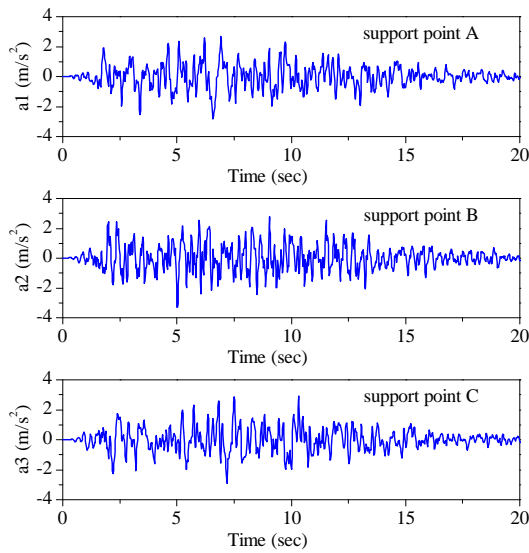
$$u_k(t) = \zeta(t) \sum_{m=1}^k \sum_{l=1}^N A_{km}(\omega_l) \cos[\omega_l t + \theta_{km}(\omega_l) + \varphi_{ml}(\omega_l)] \quad (14)$$

where  $\omega_l = l\Delta\omega$ ,  $\Delta\omega = \omega_N / N$ , and  $\omega_N$  represents an upper cut-off frequency,  $\zeta(t)$  is a shape function,  $\varphi_{ml}(\omega_l)$  is a random phase angle uniformly distributed over the range 0 to  $2\pi$ ,  $k=1,2,3$  and representing support points A, B and C in this study,  $A_{km}(\omega_l)$  and  $\theta_{km}(\omega_l)$  denote the amplitudes and phase angles of the generated time histories. In this study, the ground motion duration is assumed to be 20 sec, the simulation is carried out with the sampling frequency of 50Hz and the upper cut-off frequency is set to be 25Hz.

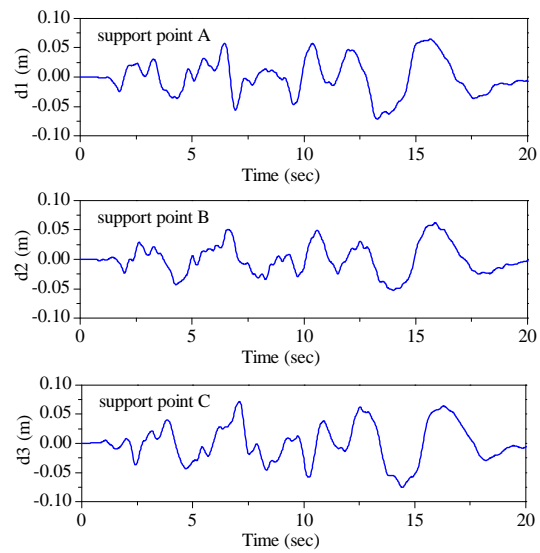
**Table 1. Seismic excitation cases**

Case	Wave Apparent Velocity	Coherency
1	infinite	perfectly
2	1000m/s	perfectly
3	infinite	highly
4	1000m/s	highly

As described in Table 1, a total of 4 cases of spatial ground motions are simulated in this study. In each case, the firm soil site is assumed and three horizontal ground motion time histories are simulated corresponding to the spatial ground motions at the three tower supports. Figures 4 and 5 show a typical set of simulated spatial horizontal acceleration and displacement time histories on uniform firm sites with apparent velocity 1000m/s and high cross correlation assumption.



**Figure 4. Generated spatial ground accelerations on uniform firm site (case 4)**



**Figure 5. Generated spatial ground displacements on uniform firm site (case 4)**

#### 4. NUMERICAL ANALYSIS

Nonlinear responses of the coupled transmission tower-line system shown in Figure 3 subjected to the above simulated spatially varying ground motions are calculated. In this

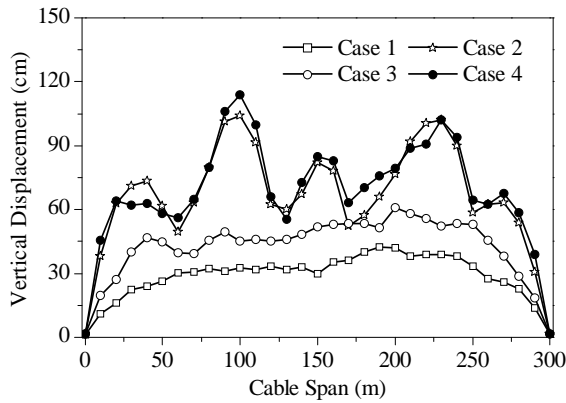
study, only the longitudinal ground excitations are considered. The damping ratio of tower and cable are assumed to be 2% and 1%, respectively. The Newmark- $\beta$  method is applied in the numerical integration, in which  $\beta$  is set to 0.25. For each ground motion case listed in Table 1, independent numerical calculations are carried out using six sets of independently simulated spatial ground motions as input. The mean response values obtained from the six numerical calculation results, corresponding to the middle tower and transmission lines of the second span, are presented and discussed in this study.

To examine the ground motion spatial variation effect, numerical results corresponding to the four spatial ground motion cases are compared. The four ground motion cases represent four different spatial variations. In particular Case 1 neglects ground motion spatial variation, which represents the traditional uniform ground motion assumption; Case 2 considers only spatial ground motions wave passage effect, or the so-called delayed input in some of the previous studies; Case 3 considers spatial ground motions with coherency loss effect only; and Case 4 is the general case which considers spatial ground motions with both wave passage effect and coherency loss effect.

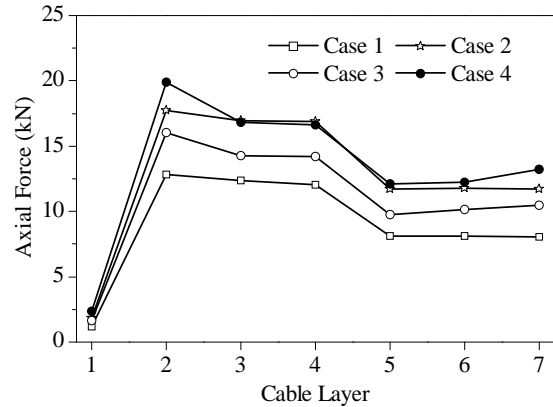
#### **4.1. Seismic response of transmission line**

Figure 6 shows the mean maximum vertical displacements of the top layer cable along the length of the second span corresponding to the 4 cases of spatial ground excitations. Mean maximum cable force in each layer of cables corresponding to the 4 cases of spatial ground excitations is plotted in Figure 7. As shown in Figure 6, the responses corresponding to uniform ground motion (Case 1) and spatial ground motion without considering phase shift are almost symmetric over the entire span, indicating the antisymmetric modes of the cable are only slightly excited. When spatial ground motions with phase shift are considered (Case 2 and Case 4), the antisymmetric vibration modes will be excited. The contributions from the antisymmetric modes are prominent. Case 4 ground motions, which have most significant spatial variations, produce the largest cable displacement equal to 113.7 cm at one-third span of the transmission line. Considering spatial ground motion wave passage effect only (Case 2) results in similar responses as those from Case 4 motions, implying the phase shift effects dominate the spatial ground motion variation effects for this structural system. A previous study (Hao 1993) revealed that spatial ground motion wave passage effect is more significant if the structure is relatively flexible as compared to the dominant ground motion frequency and when the structure responses are governed by dynamic responses; and spatial ground motion coherency loss effect becomes more pronounced if the structure is relatively stiff and its responses are governed by quasi-static responses. The coupled transmission tower-line system under consideration consists of high towers and long flexible cables, which makes it a very flexible structure. Therefore, the ground motion wave passage effect is more significant than coherency loss effect. Similar results have been observed in Figure 7, as the axial force responses in each layer of cables obtained from Case 2 and Case 4 are close to each other and are larger than those obtained from Case 1 and Case 3. It is obvious that the more significant is the ground motion spatial variation, the larger are the cable responses. The largest axial force equals 19.87 kN in the second layer cable corresponding to the response under ground motion excitation Case 4. It is noted that the axial force response in top layer cable is

minimum and the effect of ground motion spatial variation on it is least significant among all the cable layers.



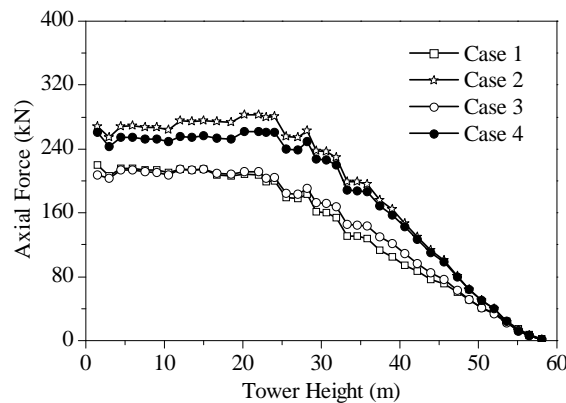
**Figure 6. Mean maximum displacements induced by spatial ground excitations**



**Figure 7. Mean maximum cable axial forces induced by spatial ground excitations**

#### 4.2. Seismic response of transmission tower

Figure 8 shows the mean maximum axial forces in tower members along the height of the middle tower corresponding to the 4 cases of spatial ground excitations. As shown, the axial force responses corresponding to Case 1 are approximately the same as those obtained from Case 3, which indicate again the coherency loss effect on the axial force responses of tower is not very significant. Different from seismic response of transmission line, the axial force responses of tower considering the wave passage effect only (Case2) are larger than those considering spatial ground motions with both wave passage effect and coherency loss effect (Case4). As can be noticed, neglecting ground motion spatial variations, the axial force responses in the tower members could be substantially underestimated, especially in those members in the lower half of the tower, where the axial forces could be underestimated by more than 25%.



**Figure 8. Mean maximum axial forces in tower members induced by spatial ground excitations**



## 5. CONCLUSIONS

This paper presents numerical simulation results of nonlinear dynamic responses of a transmission tower-line system to spatially varying ground motions. It is found that:

- (1) Ground motion spatial variations induced by wave propagation are more important than those induced by loss of coherency to the responses of transmission tower-line system.
- (2) The response values obtained from delayed excitations and multiple support excitations are similar, but delayed excitations induce smaller structural responses in most cases. Thus, the multiple support excitations considering wave passage effect and coherency loss effect simultaneously can give accurate estimation of seismic responses of transmission tower-line system.
- (3) Ground motion spatial variations have significant effect on seismic responses of transmission tower-line system. Neglecting ground motion spatial variations in predicting the transmission tower-line system responses to earthquake excitations may substantially underestimate the structural responses.

## 6. REFERENCES

- Der Kiureghian, A., (1980). Structural response to stationary excitation, *Journal of Engineering Mechanics*, Vol 106, No 6, pp 1195-1213.
- Hao, H., Duan, X.N., (1996). Multiple excitation effects on response of symmetric buildings, *Engineering Structures*, Vol 18, No 9, pp 732-740.
- Hao, H., Oliveira, C.S., Penzien, J., (1989). Multiple-station ground motion processing and simulation based on SMART-1 Array data, *Nuclear Engineering and Design*, Vol 111, pp 293-310.
- Hao, H., (1989). Effects of spatial variation of ground motions on large multiply-supported structures, Report No. UCB/EERC-89-06, University of California at Berkeley.
- Hao, H., (1993). Arch response to correlated multiple excitations, *Earthquake Engineering and Structural Dynamics*, Vol 22, pp 389-404.
- Hao, H., Chow, N., (2006). Modeling of earthquake ground motion spatial variation on uneven sites with varying soil conditions, *The 9th International Symposium on Structural Engineering for Young Experts*, Fuzhou - Xiamen, pp 79-85.
- Harichandran, R.S., Hawwari, A., Sweiden, B.N., (1996). Response of long-span bridges to spatially varying ground motion, *ASCE Journal of Structural Engineering*, Vol 122, No 5, pp 476-484.
- Rassem, M., Ghobarah, A., Heidebrecht, A.C., (1996). Site effects on the seismic response of a suspension bridge, *Engineering Structures*, Vol 18, No 5, pp 363-370.
- Ruiz, P., Penzien, J., (1969). Probabilistic study of the behaviour of structures during earthquakes, Report No. UCB/EERC-69-03, Earthquake Engineering Research Centre, University of California at Berkeley.
- Shen, S.Z., Xu, C.B., Zhao, C., (1997). *Design of Suspension Structure*, China Architecture and Building Press. Beijing; (in Chinese).
- Zanardo, G., Hao, H., Modena, C., (2002). Seismic response of multi-span simply supported bridges to spatially varying earthquake ground motion, *Earthquake Engineering and Structural Dynamics*, Vol 31, No 6, pp 1325-1345.