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Optimum Design for Passive Tuned Mass Dampers Using Viscoelastic Materials

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ABSTRACT:

High levels of vibrations can occur in floor systems due to excitation from human activities such as walking and aerobics. In building floors, excessive vibration is generally not a safety concern but a cause of annoyance and discomfort. Rectification measures for excessive vibrations in existing floor may include structural modifications to increase the floor stiffness or addition of damping. While structural stiffening can be easily designed and the corresponding effect be accurately predicted, it is often not practical due to space limitations or associated construction disruptions. The addition of mechanical dampers can be more practical and cost effectives for floors with low damping, but there are very limited proprietary systems available and they are difficult to design from first principles. This paper forms part of a research project which aims to develop an innovative cost effective Tune Mass Damper (TMD) using viscoelastic materials. Generally, a TMD consists of a mass, spring, and dashpot which is attached to a floor to form a two-degree of freedom system. TMDs are typically effective over a narrow frequency band and must be tuned to a particular natural frequency. The paper provides a detailed methodology for estimating the required parameters for an optimum TMD for a given floor system. The paper also describes the process for estimating the equivalent viscous damping of a damper made of viscoelastic material. Finally, a new innovative prototype viscoelastic damper is presented along with associated preliminary results.

Keywords: Floor vibrations, viscoelastic materials, tuned mass dampers.

1. Introduction

The application of passive Tuned Mass Damper (TMD) is an attractive option in reducing excessive floor vibrations. Generally, a TMD consists of a mass, spring, and dashpot, as shown in Figure 1, and is typically tuned to the natural frequency of the primary system (Hartog 1956). When large levels of motion occur, the TMD counteracts the movements of the structural system. The terms m_1, k_1, c_1, X_1 represent the mass, stiffness, damping and displacement of the floor, while m_2, k_2, c_2,

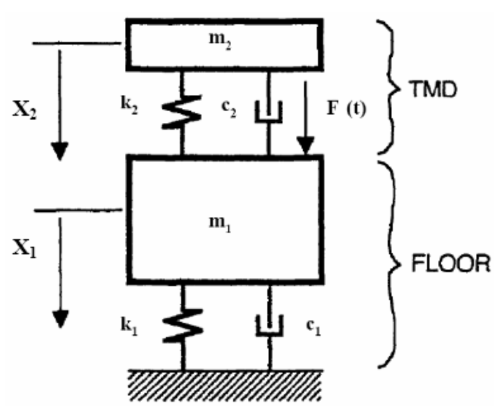

Fig. 1 Schematic Representation of a Two DOF System

X_2 represent the mass, stiffness, damping and displacement of the TMD and F(t) represents the excitation force. As the two masses move relative to each other, the passive damper is stretched and compressed, reducing the vibrations of the structure by increasing its effective damping. TMD systems are typically effective over a narrow frequency band and must be tuned to a particular natural frequency. They are not effective if the structure has several closely spaced natural frequencies and may increase the vibration if they are off-tuned (Webster and Vaicaitis 1992).

2. Optimal Design of Viscous Damper

The natural frequency of the primary system can be split into a lower (f_1) and higher (f_2) frequency by attaching a spring mass tuned to the same fundamental natural frequency (f_n) of the primary system as shown in Figure 2. The most significant design variable for the damper is the mass ratio (μ) as defined in Equation 1. When the mass ratio increases, the TMD becomes more effective and robust (Al-Hulwah 2005). In most applications the mass ratio is designed to be in the range of 1-10%.

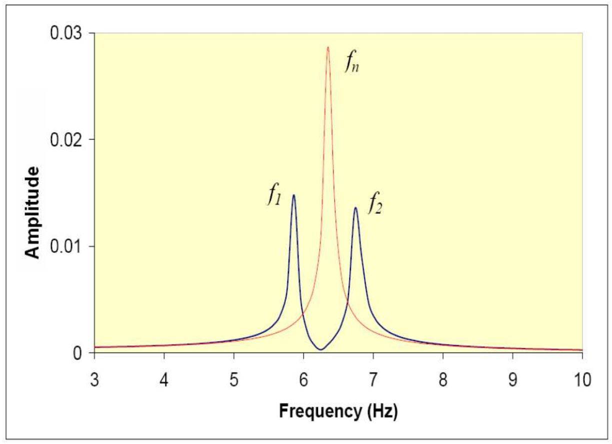

Fig. 2 Example Demonstrating the Effectiveness of a TMD

$$\mu = \frac{m_2}{m_1} \tag{1}$$

where m_2 and m_1 are the mass of TMD and mass of primary system respectively.

In the design of a TMD, the optimum natural frequency of the damper (f_d) is defined by Equation 2;

$$f_d = \frac{f_n}{1+\mu} \tag{2}$$

and, the optimum damping ratio of damper (ζ_{opt}) can be found as follows;

$$\zeta_{opt} = \sqrt{\frac{3\mu}{8(1+\mu)^3}} \tag{3}$$

If there is zero damping then resonance occurs at the two undamped resonant frequencies of the combined system (f_1 & f_2). The other extreme case occurs when there is infinite damping, which has the effect of locking the spring (k_2). In this case the system has one degree of freedom with stiffness of (k_1) and a mass of ($m_1 + m_2$).

Using an intermediate value of damping such as ζ_{opt}, somewhere between these extremes, it is possible to control the vibration of the primary system over a wider frequency range (Smith 1988).

3. Conversion of Viscous Damper to Viscoelastic Damper

A common and effective way to reduce transient and steady state vibration is to add a layer of viscoelastic material, such as rubber, to an existing structure. The combined system often has a higher damping level and thus reduces the unwanted vibration (Inman 1996). The complex form of equation of motion of a damped system is given by Equation 4;

$$m\ddot{x} + c\dot{x} + kx = F_0 e^{j\omega t} \quad (4)$$

where m, c, k, $\ddot{x}$, $\dot{x}$, x, F_0, ω and t are mass, damping, stiffness, acceleration, velocity, displacement, excitation force, excitation circular frequency and time respectively, and j is $\sqrt{-1}$. Equation 4 can be solved by assuming a solution of the form $x(t) = \mathrm{X}e^{j\omega t}$, where X is a constant. Substitution of this assumed form into Equation 4 and dividing by non-zero function $e^{j\omega t}$ yields;

$$\left[-m\omega^2 + k(1 + \frac{\omega c}{k} j)\right]\mathrm{X} = F_0 \quad (5)$$

The imaginary part of the complex stiffness represents the energy dissipation in the system, since dissipation loss factor of the system (η) has the form;

$$\eta = \frac{\omega c}{k} \quad (6)$$

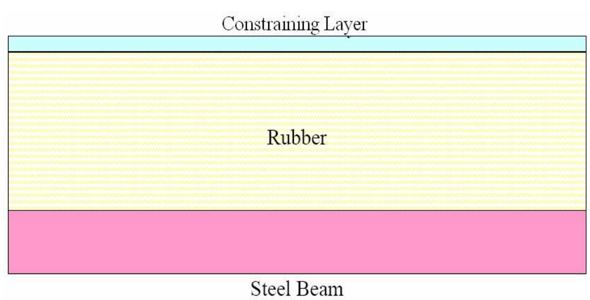

Fig. 3 Constrained Viscoelastic Beam Cross-Section

The simplest form of a viscoelastic damper is a constrained viscoelastic layer in a beam. This could be made of two constraining metal plates bonded together with high damping rubber as shown in Figure 3. This configuration can be extended to a multiple layer system, with N damping layers and $N+1$ constraining elastic layers. In this composite sandwich beam, the viscoelastic material experiences considerable shear strain as it bends, dissipating energy and attenuating vibration response (Mace 1994).

(Mead 1982) developed a detailed analytical method to estimate the overall dissipation loss factor of the composite system (η) based on the dissipation loss factor of the rubber (β), thickness of rubber, geometric parameters and Young moduli of the top and bottom plates constraining the viscoelastic material. Boundary conditions have no significant effect in this method and can be applied to any composite beam configuration such as simply supported beam, cantilever etc. The overall loss factor of the composite system can be estimated by using Equation 7;

$$\eta = \frac{\beta Y}{(2+Y)+2(1+Y)^{1/2}(1+\beta^2)^{1/2}} \tag{7}$$

where β is the dissipation loss factor of the rubber and Y is a geometric parameter calculated as;

$$Y = \frac{(E_1A_1)(E_3A_3)d^2}{(E_1A_1+E_3A_3)(E_1I_1'+E_3I_3')} \tag{8}$$

where E_1 and E_3 are the Young moduli of top and bottom constraining plates respectively, A_1 and A_3 are the cross-sectional area of the top and bottom constraining plates respectively, I_1' and I_3' are the moment of inertia of top and bottom constraining plates about their neutral axes respectively and d is the distance between the centroids of top and bottom constraining plates.

The stiffness (EI_{total}) of the composite viscoelastic system can be calculated as follows;

$$\frac{EI_{total}}{E_1I_1'+E_3I_3'} = 1+\frac{gY(1+g(1+\beta^2))}{1+2g+g^2(1+\beta^2)} \tag{9}$$

where g represents the shear parameter which can be written as;

$$g = \frac{1}{\sqrt{(1+Y)(1+\beta^2)}} \tag{10}$$

The natural frequency of a viscoelastic cantilever beam damper can be estimated as;

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{11}$$

where k and m are the effective stiffness and mass of cantilever beam respectively and can be calculated as;

$$k = \frac{3EI}{l^3} \tag{12}$$

where E, I and l are the Young's modulus, moment of inertia and length of composite viscoelastic cantilever beam respectively. The effective mass of a uniform composite cantilever beam can be calculated as;

$$m = \frac{33}{140}\rho Al \tag{13}$$

where ρ, A and l are density, cross-sectional area and length of composite cantilever beam respectively. Given the conditions presented in Equations 1 & 2, for such a cantilever beam to act as an optimum viscoelastic damper, its length and mass can be calculated from Equations 12 & 13.

4. Experimental Work

A number of experiments were conducted as part of this study to examine the use of viscoelastic material (rubber) as a mean of providing damping. These experiments are summarised as follows:
(a) Mechanical testing of commercial rubber specimens to determine the basic material properties.
(b) Dynamic testing of simply supported steel beams with and without a constrained viscoelastic layer.
(c) Dynamic testing of a steel beam with and without a TMD designed as a cantilever with a constrained viscoelastic layer.

In this paper, items (a) and (b) above are discussed in detail below. Item (c) is not reported yet, however, corresponding finite element (FE) modelling is presented in Section 5.

(a) Determination of basic mechanical properties of rubber
As most commercially available rubbers do not have adequate technical specifications about their material properties, it was also necessary to undertake specific tests on the acquired rubber to determine its elastic modulus and loss factor. This was achieved by testing small specimens using a Dynamic Mechanical Analyser (DMA) machine (TA-Instruments). The testing performed by this machine separates the viscoelastic response of a material into the two components of the complex value of modulus (E*): the real part corresponds to the elastic modulus (E') and the imaginary part refers to the damping or loss component (E"). The standard complex variable notation is E*= E'+ E". The separation of the measurement into the two components describes the two independent processes within the materials-elasticity (energy storage) and damping (energy dissipation). This is the fundamental feature of dynamic mechanical analysis that distinguishes it from other mechanical testing techniques. The loss tangent (Tan δ) is the dissipation loss factor of the rubber and is calculated using Equation 14.

$$Tan\,\delta = \frac{E''}{E'} \qquad (14)$$

Three rubber samples of 35 x 10.8 x 5.2 mm were tested and the resulting average value of β was found to be 0.12. The higher the dissipation loss factor, the higher the damping that can be achieved using this rubber.

The dissipation loss factor of the rubber can also be back calculated from vibration testing of the material and measuring the decay rate as described below.

(b) Dynamic testing of simply supported steel beams with and without constrained viscoelastic layer.

The aim of these tests was to examine the effectiveness of a constrained viscoelastic material in increasing the damping of beams. Two steel beams were made with each measuring 750mm long, 25mm wide and 4mm thick. The first beam was kept as a bare beam, while the second was bonded to a rubber layer of 12.5mm thickness which in turn was bonded to a constraining layer 1mm thick. The cross section of the second beam is identical to that shown in Figure 3. Both beams were subjected to pluck tests to obtain their dynamic properties and in particular damping. The vibrations were measured using a proximity sensor. The pluck tests were conducted several times to ensure repeatability. Typical results from the pluck tests are shown in Figure 4.

The Logarithmic Decrement Method (LDM) was used to estimate the damping ratio of both the bare and that with constrained viscoelastic layer. The damping ratio of bare beam (ζ) was calculated to be about 0.7%. One the other hand, the value of overall damping ratio for the composite system (ζ) was calculated to be 3%. This significant increase in damping is due to the constrained rubber layer. This clearly demonstrates the effectives of such a simple technique in providing damping.

As described earlier in part (a), the rubber dissipation loss factor could also be back calculated from vibration tests if access to a Dynamic Mechanical Analyser is not available. By substituting overall damping ratio of the composite beam as well as the other material and geometric properties in Equation 7, the dissipation loss factor β of the rubber can be estimated. Based on the provided details, the estimated value of β is 0.11. This is in excellent agreement with the value obtained from the DMA results.

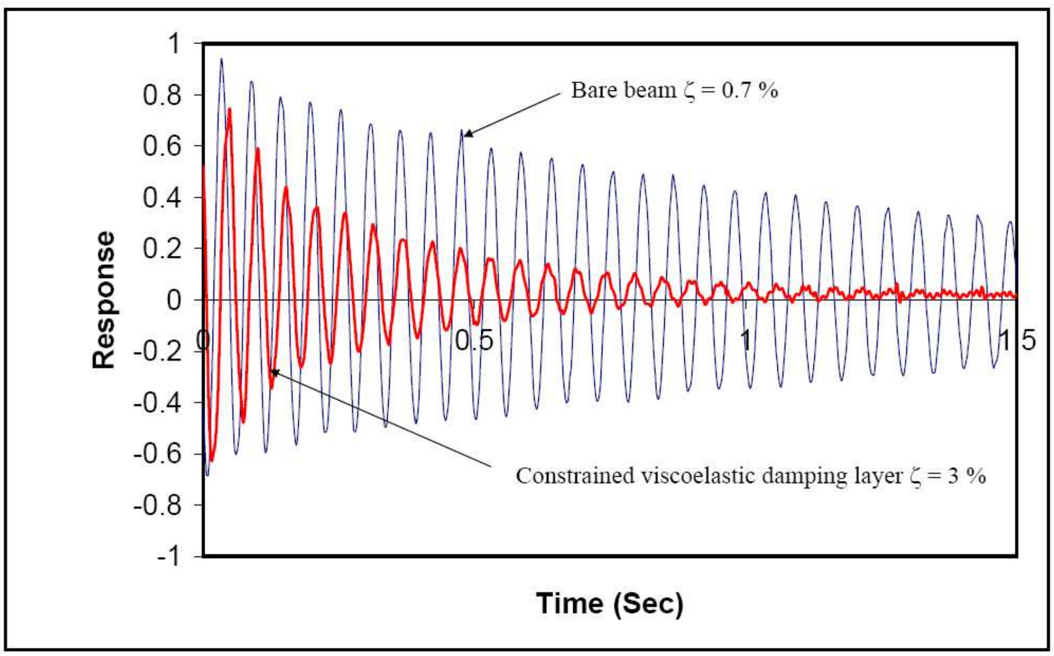

Fig. 4 Decay Rate of Bare and Constrained Viscoelastic Damping Layer

It should be noted that the dissipation loss factor of the rubber (β) is the factor that determines the upper limit of overall dissipation loss factor of the composite system. In other words, the η value ($= 2\zeta$) of the composite system can not exceed β value the rubber (Mead 1982; Nashif, Jones et al. 1985).

5. Modeling of Viscoelastic Damper

Using the constrained layer approach, a TMD was developed in the form of cantilever. This cantilever as shown in Figure 5 is attached using a rigid bracket to mid-span of beam or floor to be retrofitted. The mass, stiffness and damping properties of the cantilever could be easily tuned to obtain an optimum design. The process of tuning the cantilever damper with viscoelastic layer is identical to a conventional TMD with viscous dashpot as described using Equations 1, 2 and 3.

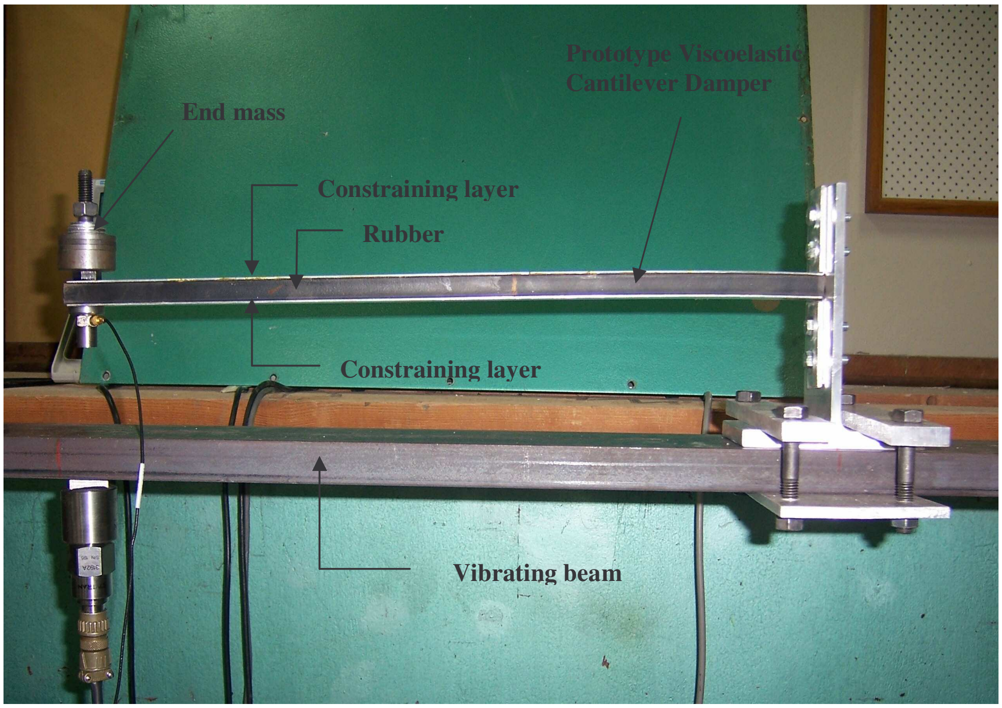

Fig. 5 Viscoelastic Damper, Attached to a Vibrating Beam

To illustrate the effectiveness of the new cantilever damper, two FE models were developed. The first model was for a simply supported bare steel beam with 3m span, 100mm width and 25mm thickness. This beam was then retrofitted with a TMD which was based on a cantilever damper with a constrained viscoelastic layer using the same rubber as reported earlier. The TMD was the optimum design with a mass ratio of 1%.

The two FE models were excited using a harmonic force. A summary of the results is shown in Figure 6. In Figure 6, the exact solution for a beam with an equivalent viscous damper is also plotted. It is clear that the new cantilever damper does perform well, and it produces almost identical reduction in vibration to a conventional viscous damper. For the optimum cantilever damper the reduction factor in response is in the order of 4. This significant reduction in response will be validated experimentally.

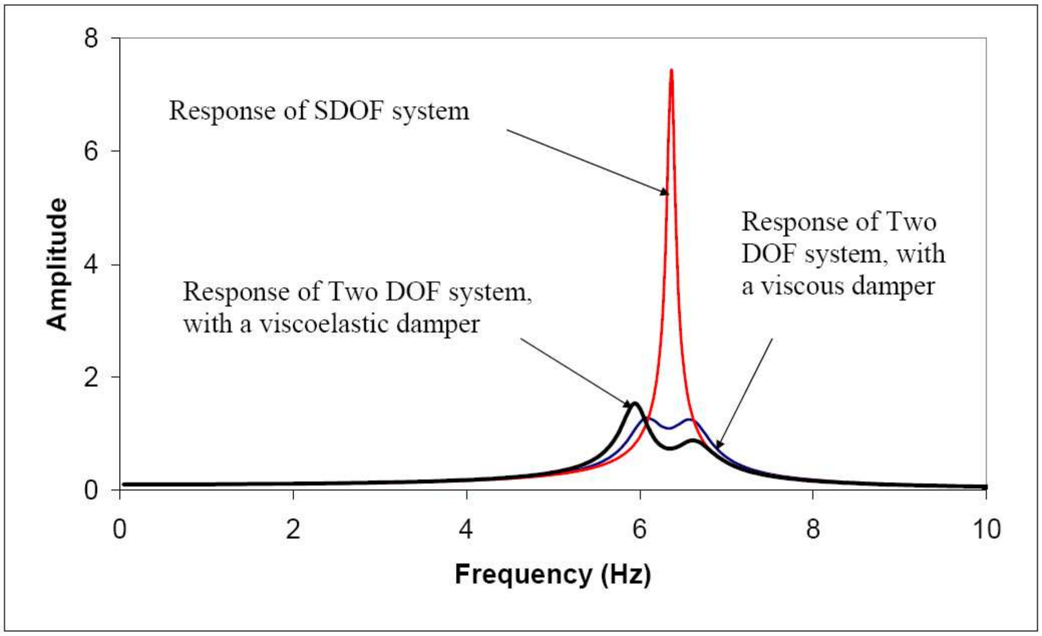

Fig. 6 Beam Response of Viscous and Viscoelastic Dampers

6. Concluding Remarks

This paper has detailed a methodology for developing an optimum tuned mass damper using viscoelastic material as a damping medium. For any given floor mass, damping and stiffness, an appropriate damper based on a constrained layer can be easily developed and tuned to reduce excessive vibrations.

An experimental cantilever damper has been developed and used to validate the analytical model. The case study demonstrates that such a damper can be an economical and simple solution for retrofitting floors with excessive vibrations.

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