Strength of Glass Panels Estimated by Fracture Mechanics

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Abstract

This paper is concerned with the strength of annealed glass panels when subject to static point loading and is a pre-cursor to further investigations on the resistance to impact by tools or flying objects. The current probabilistic models that are implicit in contemporary codes of practices for the determination of the strength of plain annealed glass is first reviewed. The alternative approach based on fracture mechanics is then introduced. With the fracture mechanics approach, factors controlling the strength of glass are resolved into the fracture toughness, which characterizes the resilience of glass as a material, and the depth of the *Griffith* flaws which control the strength of the panel. In essence, the strength of a glazing panel can be related to the depth of the critical *Griffith* flaw assuming a constant fracture toughness. The probabilistic distribution of the critical *Griffith* flaw depth is modelled by the extreme value theory based on *Gumbel*. This study is distinguished from earlier studies in that what is being modelled is the probabilistic distribution of the depth of the critical flaw in a glass panel. Results of static tests undertaken by the authors have been demonstrated to match better with the proposed model than with the popularly used *Weibull* model. Importantly, the characteristic strength of the panel has been shown to be very sensitive to its dimension.

Keywords: glass, glazing panels, fracture toughness

1. INTRODUCTION

Glass occupies the largest surface area of the protective envelope of contemporary buildings and is also widely used in internal partitions. Whilst being popular as a building material, it is also notorious for its brittle and unpredictable behaviour when disturbed and can be potentially hazardous when damaged. Although these shortcomings of glass can be mitigated by toughening and lamination, plain annealed glass that is without any extra protection is by far most widely used. The safe use of glass in exposed conditions often relies very much on design calculations to ensure that the predicted maximum flexural tensile stresses in the projected wind load scenario is exceeded by the characteristic (95 % exceedance) tensile strength capacity.

The prediction of bending moment and the associated notional stress and strain in glass is straightforward given the apparent linear elastic behaviour under normal load conditions. However, the non-linearity associated with membrane actions has been revealed (e.g. Calderone & Melbourne,1993; Vallabhan, 1983). Notwithstanding, the actual probabilistic distribution of tensile strength of glazing panels for given dimensions, load configurations and boundary conditions is still not fully understood even in the quasi-static conditions of wind pressure, let alone in transient conditions of blast and impact.

The probabilistic distribution of tensile strength of glazing panels in contemporary codes of practices is based on empirical models or a calibrated Weibull model which is found on the concept of modelling the weakest link in a chain (Section 2). Mismatch of test results with these established models, as identified by *Calderone* (Calderone *et al*., 2001) and others, have thrown doubts into the generality and robustness of the employed probabilistic distribution models. Results from static tests undertaken recently by the authors (Section 3) further demonstrate better matching of the test results by alternative models than by the *Weibull* model (Section 4). A more robust probabilistic distribution model that can be generalised to different loading and support conditions is warranted for facilitating the design of glazing panels to meet with the demand from multi-hazards including wind pressure, blast pressure and impact by projectiles.

The introduction of the fracture mechanics approach (*FMA*) in Section 5 for modelling the probabilistic strength distribution of glazing panels is the key objective of this paper. Central to the modelling methodology is the probabilistic determination of the depth of the critical *Griffith* flaw ($\boldsymbol{a_{crit}}$) based on the extreme value theory of *Gumbel*. Results from static testings obtained to date have been shown to match well by the *Gumbel* distribution. The characteristic strength of glazing panels of different dimensions have been shown to be sensitive to the panel dimensions according to predictions by *FMA*.

2. CURRENT PROBABILISTC MODELS FOR GLASS STRENGTH

Static test results show that the strength of glass can vary considerably between specimens. This is the case even when all the specimens come from the same batch of glass of identical dimensions, loading and support configurations. This variation which can be explained by the *Griffith* flaw phenomenon is often quantified by the probabilistic model of *Weibull* which is based on the concept of the weakest link in a chain of elements. This widely accepted probabilistic model is defined by equation (1).

$$P_f = F(\sigma) = 1 - e^{-\left(\frac{\sigma}{\sigma_\theta}\right)^m} \qquad (1)$$

where $F(\sigma)$ is the cumulative distribution function, σ is the strength of a specimen, m is the *Weibull* modulus.

The *Weibull* parameters in equation (1) can be obtained by applying double logarithms to the cumulative distribution function turning equation (1) into the linear form of equation (2) in order that the parameters m and σ_θ can be obtained readily by the linear regression of data.

$$\ln\left\{\ln\left(\frac{1}{1-p_f}\right)\right\} = m\{\ln(\sigma) - \ln(\sigma_\theta)\} \tag{2}$$

The *Weibull* modulus, m, characterises the spread of the distribution of specimen strengths. For example, a smaller value of m represents a wider range in the distribution of strength across the specimens. It can be shown from equation (2) that the value of $\ln\left(\ln\left(\frac{1}{1-p_f}\right)\right)$ equals zero when $\sigma=\sigma_\theta$. The value for the probability of exceedance P_f is accordingly equal to 63.2% (ASTM C1239-07, 2007).

It was observed that the value of σ_θ was dependent on the volume of the panel specimens : a lower value of σ_θ is typically observed with panels of larger dimensions, or larger volume. In view of the volume dependence, the *Weibull* distribution is presented in the modified form of equation (3).

$$P_f = F(\sigma) = 1 - e^{-\int_V \left(\frac{\sigma}{\sigma_0}\right)^m dV} \tag{3}$$

where V is the volume of the part of the glass that is subject to tension and can be described as the "tensile volume", σ_0 is the *Weibull* parameter which represents the value of σ_θ for panels of a "unit tensile volume". The solution of equation (3) can be expressed in the form shown in equation (4).

$$P_f = F(\sigma) = 1 - e^{-k.V\left(\frac{\sigma}{\sigma_0}\right)^m} \tag{4}$$

where V is volume of the panel specimens, and $k.V$ is its effective volume; k takes into account the variation of flexural tensile stress (which is proportional to the bending moment) across the specimen and σ is the maximum tensile stress (which is effectively the observed tensile strength). Note, the volume parameter can be replaced by the surface area of the panel if only surface flaws are considered and if the thickness of the panel has been held constant.

Equation 4 is the basis of the *ASTM* provisions for the determination of the probabilistic distribution of strength of the glass panels. However, σ_0 is taken as 1 and dimensionless in this provision, which turns k and m to parameters that called as surface flaw parameters. The surface flaw parameter k is recommended to be equal to $2.86.10^{-53}N^7m^{12}$ and m equal to 7 based on the empirical test data of 20 year old glazing panels (ASTM E 1300-04, 2004; Beason *et al.*, 1998). Although Beason named k and m as surface flaw parameters, the parameters can not be measured physically. Furthermore the values of k and m could vary with the dimension of the specimens and other loading parameters as noted in the literature (Calderone *et al.*, 2001). This lack of generality is a severe limitation of the existing model.

In spite of the distribution model of *Weibull* becoming widely accepted, some researchers argue that test results match better with other distribution models. *Doremus* and *Lu* (*Doremus*, 1996; *Lu et al*., 2002) suggested that *normal* distribution was better suited to the modelling of strength data for a brittle material such as glass in view of the reasoning that the variations in strength associated with a standard manufacturing process should naturally behave in a manner consistent with that of a normal distribution relationship.

Calderone (Calderone *et al*., 2001) proposed the use of the *log-normal* distribution model. Calderone tested some full-scale rectangular window panels and concluded that the test results matched better with a calibrated *log-normal* distribution model than that of a *Weibull* distribution model. It was also revealed by Calderone that older glass specimens exhibited less variation in strength than new specimens.

Log-normal distribution is suited to the modelling of a physical process in which the logarithmic of the test data is consistent with the trend of *normal* distribution. Its probability density function is defined by equation (5).

$$f(\sigma) = \frac{1}{\sqrt{2\pi}\beta}\sigma^{-1}.e^{-\frac{(Ln(\sigma)-\alpha)^2}{2\beta^2}} \tag{5}$$

where σ is the measured strength, and α and β is the *mean* and *standard deviation* of *Ln*(σ) respectively.

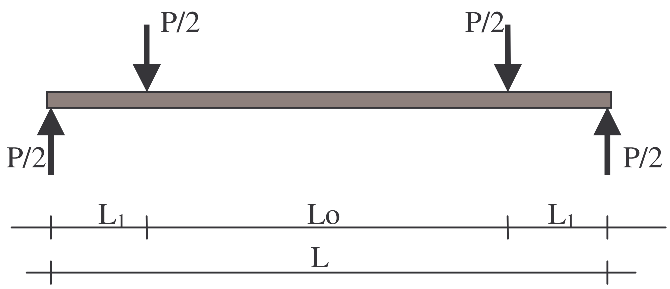
3. EXPERIMENTAL STATIC TESTINGS

Static tests were undertaken recently by the authors in a pilot investigation to calibrate the probabilistic models of different forms as introduced earlier in the paper. A total of 33 specimens were tested. The specimen dimensions are listed in Table 1 and the test results in Table 2. The performance of the various models was then compared by matching the calibrated models with the individual test results.

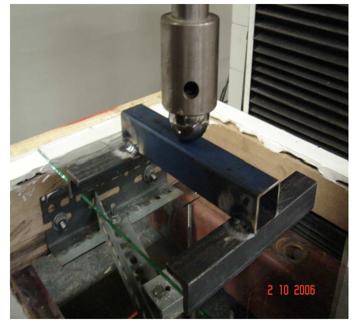
It is known that the strength distribution is dependent on many factors including the type of glass (eg. annealed versus toughened), load duration, age of glass, size of specimen (tensile volume) and loading configuration. In this pilot study, many of these variables have been held constant in order that random variability which is associated directly with the distribution of the *Griffith* flaw can be studied. For example, all the specimens tested were plain (soda-lime silicate) annealed glass and were all newly manufactured. The loading rate was held constant at 1mm/minute. All specimens had the same notional glass thickness (5 mm) but of different dimensions as shown in Table 1. A simple one-way four-point loading was applied in the static testing. Distance between the supports (*L*) was 350mm, and distance between the twin loads (*Lo*) was 175mm, as shown in Figure. 1.

Table 1 Details of specimens

Glass Type	Specimen size(mm)	Number of samples
Annealed	400 x 150 x 5	13
Annealed	400 x 200 x 5	20



(a) Schema of loading configuration



(b) Photograph of test set-up

Figure 1 Flexural Test Set-up

Table 2 Test results

Specimen No	Specimen size (mm)	Ultimate Load, P (N)	Strength (MPa)	Specimen No	Specimen size (mm)	Ultimate Load, P (N)	Strength (MPa)
1	400x150x5	592	41.44	18	400x200x5	683	35.84
2	400x150x5	597	41.79	19	400x200x5	686	36.02
3	400x150x5	635	44.45	20	400x200x5	730	38.33
4	400x150x5	673	47.11	21	400x200x5	747	39.21
5	400x150x5	685	47.95	22	400x200x5	748	39.29
6	400x150x5	707	49.49	23	400x200x5	763	40.08
7	400x150x5	798	55.86	24	400x200x5	774	40.61
8	400x150x5	876	61.32	25	400x200x5	776	40.72
9	400x150x5	945	66.15	26	400x200x5	777	40.78
10	400x150x5	962	67.34	27	400x200x5	783	41.09
11	400x150x5	983	68.81	28	400x200x5	800	42.01
12	400x150x5	993	69.51	29	400x200x5	833	43.74
13	400x150x5	1443	101.01	30	400x200x5	834	43.76
14	400x200x5	532	27.91	31	400x200x5	870	45.67
15	400x200x5	607	31.85	32	400x200x5	879	46.15
16	400x200x5	680	35.69	33	400x200x5	890	46.72
17	400x200x5	681	35.73				

4. EVALUATION OF THE DISTRIBUTION MODELS

The cumulative probabilistic distribution of data with N number of samples was calculated by ranking the data from the lowest value to the highest value using equation (6).

$$P_f^{\,i} = \frac{i - 0.5}{N} \tag{6}$$

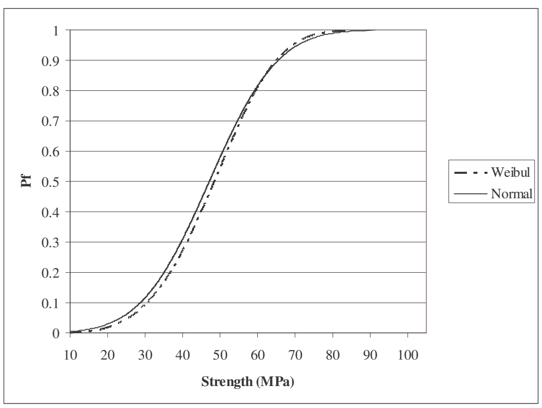
where *i* is the ranking of the data.

Meanwhile, test results were regressed in accordance with the functional form of the (i) *Weibull* distribution model, (ii) *Normal* distribution model and (iii) *Log-normal* distribution model to obtain the "best match" theoretical distribution functions and their associated modelling parameters as listed in Table 3.

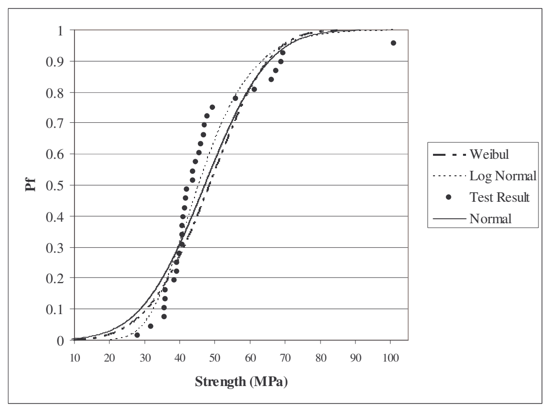
Figures 2a & 2b show the theoretical distribution relationships (as obtained by regression) together with the ranked test data for comparison. It is shown that the calibrated *Weibull* and *Normal* distribution models are very similar but neither of these models matches with the individual (ranked) test results better than the calibrated log-normal distribution model. A comparison of the three calibrated models in the range of low probability of exceedance is shown in Figure 2(c). Clearly both the *Weibull* and the *normal* distribution models overstate the probability of failure significantly. In comparison, the log-normal distribution is less conservative and matches best with the ranked test results.

Table 3 Statistical parameters of the *Weibull*, *Normal*, and *Log-normal* distributions

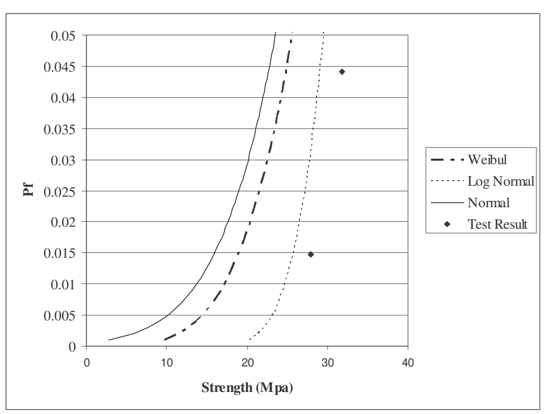
Distribution	Parameter	
Weibull	m	4.1
	σ_θ	53.0
Normal	mean	47.1
	standard deviation	14.3
Log normal	mean	3.8
	standard deviation	0.3



(a) *Weibull* and *Normal* distributions



(b) *Weibull* and *Log-normal* distribution



(c)Model predictions at low probability of failure

Figure 2 Model predictions for probability of failure

5. FRACTURE MECHANICS APPROACH OF MODELLING DISTRIBUTION OF STRENGTH

By basic fracture mechanics, the flexural tensile strength (σ_f) of the glass panel can be expressed in the form of equation (7) which can be re-written as equation (8) for back calculation for the depth of the critical *Griffith* flaw a_{crit}.

$$\sigma_f = \frac{K_C}{Y\sqrt{\pi . a_{crit}}} \tag{7}$$

$$a_{crit} = \frac{K_C^2}{Y^2 . \pi . \sigma^2} \tag{8}$$

where Y is the shape factor, and K_C is the fracture toughness of glass as a material.

For surface flaws the shape parameter, Y, is taken to be 0.713. Fracture toughness is taken to be 0.78 MPa.m$^{1/2}$ and is assumed to be uniform across all the specimens. Thus, the distribution of strength is effectively mirroring the distribution of a_{crit}. The value of a_{crit} that was back calculated from the measured values of σ_f using equation (8) is listed in Table 4. Given that a_{crit} is an extreme value within the specimen, the functional form of *Gumbel* (1958) as shown by equation (9) is employed for the modelling.

$$F(a_{crit}) = e^{-e^{-\left(\frac{a-\delta}{\gamma}\right)}} \tag{9}$$

where γ and δ are *Gumbel* parameters which are dependent on a number of factors including the effective tensile volume of the specimen.

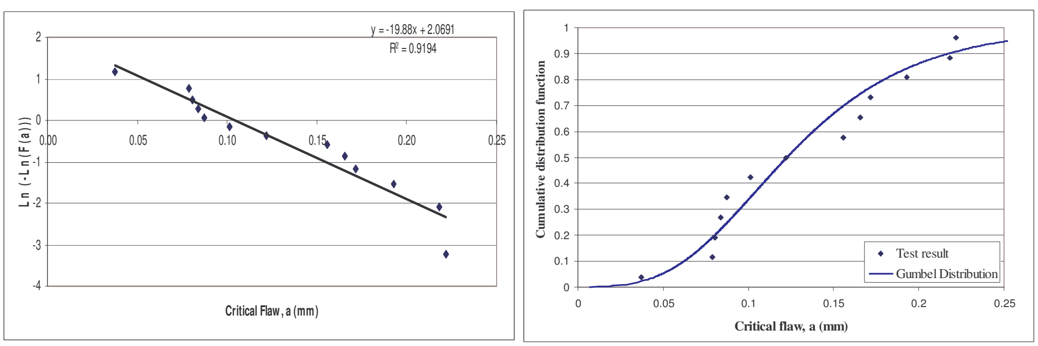
Table 4 Calculated critical flaw from test results.

Specimen No	Strength (MPa)	Critical Flaw (mm)	Specimen No	Strength (MPa)	Critical Flaw (mm)
1	41.44	0.222	18	35.84	0.297
2	41.79	0.218	19	36.02	0.294
3	44.45	0.193	20	38.33	0.259
4	47.11	0.172	21	39.21	0.248
5	47.95	0.166	22	39.29	0.247
6	49.49	0.156	23	40.08	0.237
7	55.86	0.122	24	40.61	0.231
8	61.32	0.101	25	40.72	0.230
9	66.15	0.087	26	40.78	0.229
10	67.34	0.084	27	41.09	0.226
11	68.81	0.080	28	42.01	0.216
12	69.51	0.079	29	43.74	0.199
13	101.01	0.037	30	43.76	0.199
14	27.91	0.489	31	45.67	0.183
15	31.85	0.376	32	46.15	0.179
16	35.69	0.299	33	46.72	0.175
17	35.73	0.298			

The probability of having larger flaws in a specimen increases with the dimension of the specimen (ie. its tensile volume). Thus, panels of 150 mm and 200 mm width have been separated into two groups for the modelling of a_{crit} . The calculated model parameters (γ and δ) are summarised in Table 5. The modelled distribution of a_{crit} for both groups is shown in Figure 3 and 4 along with results inferred from the individual test data. The inferred data has been well correlated with the calibrated model (with R^2 greater than 90% for both groups of specimens). The distribution of σ_f based on substituting equation (9) into equation (7) is shown in Figure 5 for comparison with the (conventional) calibrated *Weibull* model. Significant differences between predictions from the conventional model of *Weibull* and the *Fracture Mechanics Approach* (FMA) are well demonstrated. The model of *FMA* in particular was consistent with the general understanding that the probability of failure will increase with increasing tensile volume of the specimen.

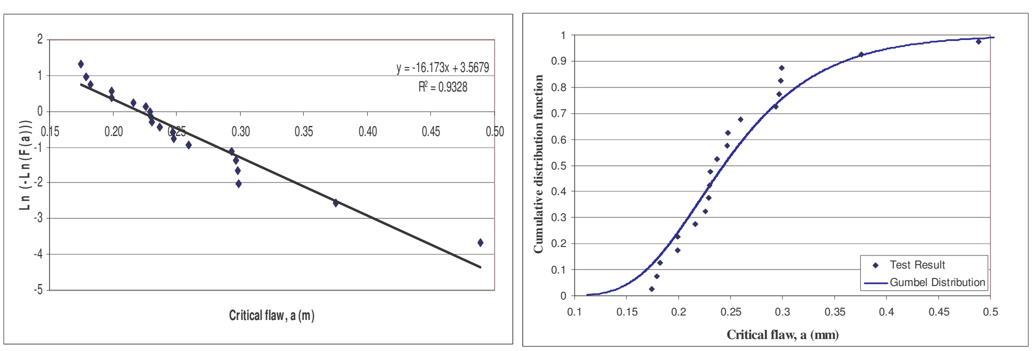
Table 5 Statistical parameters

Panel Size	Distribution	Parameter	
400x150x5mm	Weibull distribution of strength	m	4.3
		σ_θ	64.6
	Gumbel distribution of critical flaw	γ	5.03×10^{-5}
		δ	1.04×10^{-4}
400x200x5mm	Weibull distribution of strength	m	9.7
		σ_θ	41.6
	Gumbel distribution of critical flaw	γ	6.18×10^{-5}
		δ	2.21×10^{-4}



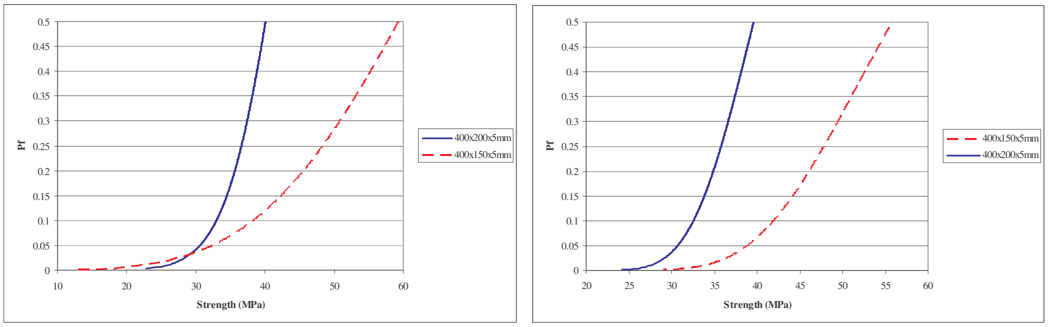
(a) Analysis of Gumbel parameter (b) Cumulative distribution function

Figure 3 Analysis of critical flaw distribution for 150mm width panels



(a) Analysis of Gumbel parameter (b) Cumulative distribution function

Figure 4 Analysis of critical flaw distribution for 200mm width panel



(a) P_f from Weibull (b) P_f from *FMA*

Figure 5 Comparison between models of *Weibull* and *FMA*

6. FURTHER STUDIES

Results from this pilot investigation provide some evidences in support of the use of *FMA* (equations 7 - 9) as presented in Section 5 for modelling the strength distribution of glass. A larger quantity of experimental data will be sourced to further examine the suitability and robustness of the proposed model. Further work is required for developing a stochastic model that can be generalised for different conditions (including the dimension of the specimens) in order that the model parameters need not be calibrated from test results undertaken for specific conditions.

Research will be progressed in the future to address the following considerations:

- Spatial distribution of the stress field which is associated with the shape of the bending moment diagram and boundary conditions.
- Bi-axial stress states associated with two-way bending.
- Modifications of the bending moment diagram by inertia forces generated in transient conditions.
- The dependence of strengths on the duration of loading in transient conditions.
- The duration of contact between the glass and the impacting object which can be modelled by contact mechanics.

7. CONCLUSIONS

- Test data on the flexural tensile strength of glass has a better match with the *Log-normal* distribution than the *Weibull* and *Normal* distributions, which are in support of earlier observations by Calderone.
- The alternative approach for modelling the strength distribution of glass based on fracture mechanics and the *Gumbel* distribution for a_{crit} has been introduced.
- Strength predictions from the model of *Weibull* and that of *FMA* were very different, and the dependence of the strength distribution on the tensile volume of the specimen was well demonstrated.
- Scope for further studies has been outlined.

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