Lateral Dynamic Response of a Single Pile Model

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Abstract

The dynamic response of a single pile with an attached mass giving a single degree of freedom (SDOF) system subjected to lateral pile-head loading is described. Two types of dynamic tests were performed, primarily free-vibration and dynamic forced-vibration tests. The main aims of this paper were to provide a basic understanding of the single pile head action and to evaluate the applicability of the elastic continuum method (ECM) in estimating the dynamic parameters of the pile-soil system. The single pile model was constructed using an instrumented steel pipe with an outside diameter of 50.65 mm and a wall thickness of 1.05 mm. The measured data from the frequency domain analysis were analysed to determine the natural frequencies f_n, damping ratios ζ, and lateral pile-mass connection displacement u_{Lat} of the pile-soil system. The experimental results obtained were then compared with those calculated from the analytical calculation based on the ECM. The analytical prediction was based on a perfect pile-soil bonding with the pile embedded in a homogeneous soil layer with a constant Young's modulus with depth. The natural frequency obtained from the free-vibration test was greater than that from forced-vibration test. This was found to be in general agreement with other related published works. The experimental and analytical results were shown to be reasonably matched, thus, the ECM demonstrates a good potential application in estimating the dynamic parameters and the lateral displacement.

Keywords: dynamic response, lateral pile head loading**,** single pile, single degree of freedom.

1 INTRODUCTION

Piles are used to support structure in a variety of situations by transmitting the loads to the strata with adequate bearing capacity. The behaviour of single piles under lateral dynamic loading is crucial and has received considerable attention from designers and researchers for more than three decades. It was observed that there is a significant difference between an earthquake loading of the ground and foundation, and cyclic pile head excitation. During an earthquake, the entire soil foundation is excited and the loads transmitted to the pile and the frequency of pile response, are all dependent on the soil conditions. In contrast, the dynamic loading applied to a pile head creates disturbance in the soil adjacent to the pile only (Finn and Gohl, 1992). However, Pender (1993) stated that this technique is useful not only for an earthquake loading but also for wind, wave and any other type of cyclic loading of pile foundations. Researchers have performed a variety of small-scale model tests and several analytical methods were developed to simulate the effect of soil and pile interaction on the static and dynamic lateral response of piles. These approaches are typically characterised by the treatment of the soil medium. Despite the numerous sophisticated linear and nonlinear, theoretical, semi-analytical and numerical models proposed, there are still limited experimental data available to validate the reliability of the methods. The main aims of this paper were to provide a basic understanding of the single pile head action and to evaluate the applicability of the elastic continuum method (ECM) in estimating the dynamic parameters of the pile-soil system. In this paper, response data on a single pile under free-vibration and dynamic forced-vibration by using a shaker are presented and analysed.

2 SINGLE PILE MODEL

The single steel pipe model used in the tests has an outside diameter of 50.65 mm and a wall thickness of 1.05 mm, as shown presented in Figure 1(a). The pile was installed in a soil to a depth of 700 mm. Figure 1(b) shows the data acquisition box connected to a computer for recording the output and input data. MATLAB 2007 software was used to control the shaker and data acquisition operations. The flexural rigidity (*EI*) of the pile was computed based on the properties provided by the manufacturer with Young's modulus of pile (E_p) and Poisson ratio (ν) of 200 GPa and 0.27, respectively.

(a) Experimental setup for a single model pile

(b) Data acquisition box and laptop for operation and collection of data

Figure 1: Experimental setup and instrumentation

A rigid pile head mass (steel-cap and shaker) with a weight of 0.83 kN was clamped to the head of the pile to replicate the effect of a superstructure on the pile-soil system. The distance at the connection between the pile and the mass is 400 mm. A total of five accelerometers with a maximum range of ±2g and one displacement transducer (LVDT) were used throughout the tests to measure accelerations and displacement of the lateral dynamic response at the pile head (output) produced by the shaker (input). One accelerometer was mounted on top of the shaker, two at the pile-mass connection and the remaining two accelerometers were attached to the pile shaft following the mode of excitation. The accelerometers were placed at the centres of gravity for convenience in calculation of the shear force applied to the pile. The free-vibration and forced-vibration tests were conducted after a prolonged period of wet weather, so that the soil can be assumed to be saturated to the ground surface.

3 ANALYTICAL ANALYSIS

An analytical research study for determining the behaviour of laterally loaded piles was performed by using the elastic continuum model (ECM). Mathcad 11 was used as a tool to envisage both static and dynamic response of the single pile. The assumption of the study is that the pile is embedded in a homogeneous soil layer with a constant Young's modulus with depth. Both the piles and the soil are assumed to behave elastically and the foundation response is assumed to be adequately modelled by an equivalent single-degree-of freedom (SDOF) model. In this paper, only a brief explanation of the theory used for these analyses will be presented. A detailed explanation of the theory on the elastic continuum model can be found in Pender (1993).

The pile-soil response is determined by assuming the pile to be cantilevered out from the ground and the mass attached to the pile head to be perfectly rigid. Thus, the lateral displacement of the pile at the connection between the pile and the mass is calculated using the following equation:

$$u_{pile} = \frac{He_1^3}{3EI} + \frac{M_{pile}e_1^2}{2EI} \tag{1}$$

where: H is the applied lateral load, M_{pile} ($=eH$) is the applied moment, I is the second moment of inertia of the pile and e_1 is the location at the pile-mass connection from the ground surface.

Next, the pile head stiffness is determined using the following equation:

$$\begin{pmatrix} K_{HH} & K_{HM} \\ K_{MH} & K_{MM} \end{pmatrix} = \frac{1}{\left(f_{uH}f_{\theta M} - f_{uM}^2\right)} \begin{pmatrix} f_{\theta M} & -f_{uM} \\ -f_{\theta H} & f_{uH} \end{pmatrix} \tag{2}$$

where: K_{HH}, K_{HH}, K_{HH}, K_{HH} are the pile head stiffness coefficients and f_{uH}, f_{uM}, $f_{\theta H}$, $f_{\theta M}$ are the flexibility coefficients derived as follows:

$$f_{uH} = \frac{1.3K^{-0.18}}{E_sD};\ f_{uM} = f_{\theta H} = \frac{2.2K^{-0.45}}{E_sD^2};\ \text{and}\ f_{\theta M} = \frac{9.2K^{-0.73}}{E_sD^3}$$

(D = pile diameter, and K = ratio of Young's modulus of the pile to that of the soil, i.e. E_p/E_s).

Then, the equivalent unrestrained pile head lateral stiffness and rotational stiffness are obtained by expanding equation (2) to produce the following:

$$K_h = \frac{K_{HH}K_{MM} - K_{HM}^2}{K_{MM} - e_1K_{HM}} \quad \text{and} \quad K_\theta = \frac{K_{HH}K_{MM} - K_{HM}^2}{K_{HH} - K_{HM}/e_1} \tag{3}$$

where: K_h is the horizontal stiffness, and K_θ is the rotational stiffness of a free head pile.

For the dynamic response, Wolf (1985) gives an equivalent SDOF model, which essentially has three degree of freedom (assuming the structure has only one degree of freedom). Thus, the equivalent natural frequency of the equivalent SDOF (ω_{SDOF}) model is determined as follows:

$$\omega_{SDOF} = \frac{\omega_s}{\left(1+\frac{k_s}{K_h}+\frac{e_1^2 k_s}{K_\theta}\right)^{0.5}} \tag{4}$$

where: ω_s is the natural frequency of the structure supported by the foundation and k_s is the lateral stiffness of the extension of the pile shaft above the ground surface (i.e., $k_s = \frac{3E_p I_p}{e_1^3}$).

Further, there are a number of component damping values (i.e., the structure, the horizontal and rotational stiffnesses of the pile foundation) and these are combined into a single equivalent value given by:

$$\zeta = \frac{\zeta_s + \zeta_h \frac{k_s}{K_h} + \zeta_\theta \frac{k_s e_1^2}{K_h}}{1+\frac{k_s}{K_h}+\frac{k_s e_1^2}{K_\theta}} \tag{5}$$

where: ζ is the damping value for the equivalent SDOF model, ζ_s is the damping for the structure, ζ_h is the damping for the horizontal motion of the foundation, and ζ_θ is the damping for the rotational stiffness of the foundation.

4 DYNAMIC TESTING OF SINGLE PILE

4.1 Free Vibration (Plucking) Tests

Two free vibration tests were carried out on the single pile model by simply giving the pile head mass a small displacement and then letting it vibrate freely. The single pile head was given a small tap with the fist of the operator before and after the dynamic forced vibration tests. In all cases, the operator attempted to use a similar force for all the 'plucks'. The response from this test is expected to be that of a free and damped single degree of freedom system as illustrated in Figure 2.

4.2 Forced Vibration Tests

An APS Dynamic Model 113 electrodynamic shaker was used to generate steady-state excitation (i.e., stepped-sine) to the mass in the lateral modes of excitation. The model pile was subjected to lateral forced vibration with frequency range from 1 Hz to 5 Hz with a step increment of 0.1 Hz. After each frequency increment, a sufficient delay is applied to allow the pile vibration to become stationary.

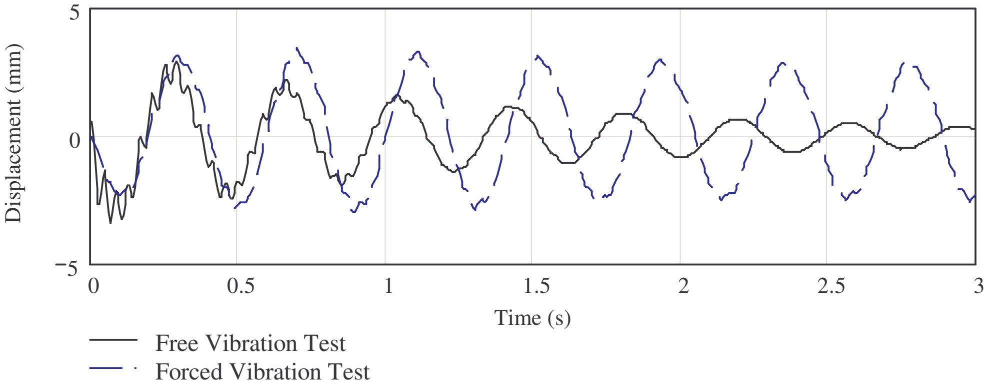


Figure 2 Dynamic single pile test records

A total of three forced-vibration tests were conducted with the stepped-sine input motion. The first 0.125g forcing amplitude was applied to the single pile and a further increment of 0.5g forcing amplitude in the second test, before reducing it back to 0.125g forcing amplitude in the final test. Figure 2 shows one of the steady-state vibration records obtained from the forced-vibration test. From these vibration records, the frequency and the corresponding vibration amplitudes were then calculated.

5 RESULTS AND DISCUSSION

5.1 Free Vibration Response

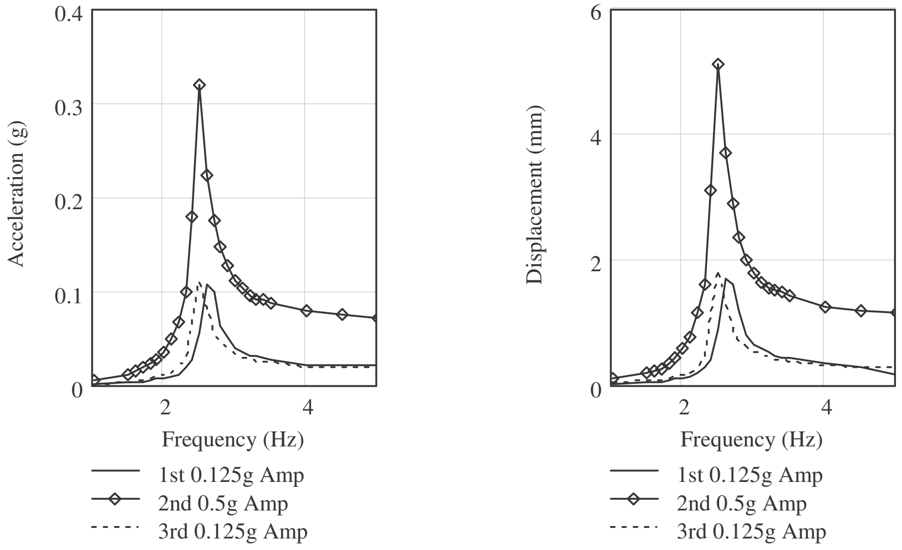
The natural frequency and damping ratio of the pile-soil system determined from the free-vibration records are presented in Table 1. The average signals of three taps were employed in order to reduce the random noise which may be present during the tests. The damping ratio, ζ_{Free}, was computed from the time history of acceleration by using the logarithmic decrement method, as given in the following equation:

$$\varsigma_{Free} = \frac{1}{2\pi} \ln \frac{a_j}{a_{j+1}} \tag{6}$$

where: a_j and a_{j+1} are the maximum amplitudes of vibration in two successive cycles.

5.2 Forced Vibration Response

For the dynamic forced-vibration response, the natural frequency and damping ratio of the single pile model determined are tabulated in Table 1. The frequency response curves for the acceleration and displacement amplitudes were plotted directly from the measured data obtained from the frequency domain analysis as shown in Figure 3. From the figure, the estimated frequency range used clearly identified the natural frequency of the pile and the damping ratios were calculated from the shape of the frequency response curve using the half-power bandwidth method. Also, the figure illustrated that the natural frequency decreases as the amplitude of the acceleration and/or displacement increases, indicating a non-linear response of the pile-soil system due to the degradation of the soil stiffness. This observation was found to be similar with those obtained by Puri and Prakash (1992) and Boominathan and Ayothiraman (2006) based on forced-vibration tests on a single pile embedded in clay.



(a) Accelerometer input data (b) LVDT input data

Figure 3 Single pile model responses

5.3 Natural Frequency and Damping Ratio

An analytical approach to the dynamic response of a single pile was initially performed to predict the natural frequency of the pile-soil system. It is very important to estimate the frequency range of interest that needs to be applied on the actual dynamic forced vibration tests. In this analysis, the soil shear strength (s_u) value of 3.8 kPa was used, which was obtained from vane shear tests conducted at the vicinity. The Young's modulus of the soil (E_s) was then calculated using the recommended relationship of $300s_u$ (Pender, 1993; Poulos and Davies, 1980). The analytical natural frequency and damping ratio are presented in Table 1. In general, it can be seen that the analytical natural frequency obtained was slightly higher than the natural frequency obtained from the free-vibration and forced-vibration tests. This may be due to the softening of the soil adjacent to the pile shaft during dynamic loading applied by the actual testing (Pender, 1993).

Table 1 also provides the natural frequencies and damping ratios for the single pile-soil system obtained from free-vibration and forced-vibration tests. The natural frequency of 2.633 Hz determined from the first forced-vibration test decreased to 2.477 Hz during the third forced-vibration test at the same level of 0.125g forcing amplitude after experiencing a higher level of 0.5 forcing amplitude. The reduction in natural frequency may be due to the formation of a gap of approximately 1.5 mm at the pile-soil interface measured on the surface at the end of the test, as shown in Figure 4. This finding is in good agreement with the results reported by Blaney and O'Neill (1986). Also, the natural frequency identified from the initial free-vibration test was approximately 6% higher than the second forced-vibration test. This is because the initial free-vibration test was performed before conducting the forced-vibration tests and therefore no degradation in the soil stiffness had occurred as yet. The natural frequency obtained from the final free-vibration test was found to be higher than that of the third forced-vibration tests even after the application of the higher level of 0.5g forcing amplitude in the second forced-vibration test. This may be due to the very low force applied on the final free-vibration test, which was done by a small tap at the pile head with the fist of the operator. Thus, it can be concluded that the natural frequency obtained from the forced-vibration test is always less than that obtained from the free-vibration test. This demonstrates similar results with those obtained by Boominathan and Ayothiraman (2006). Further, the reduction of the natural frequency from 2.633 Hz to 2.515 Hz was evident between the initial and final free-vibration tests. This result may also indicate the occurrences of a non-linear response of the pile-soil system due to the degradation of soil stiffness as mentioned earlier. The soil stiffness degradation was mainly due to the formation of a gap between the pile shaft and the surrounding soil, possibly caused by the high level of forcing amplitude applied during the dynamic test.

The damping ratios determined from the 0.5g forcing amplitude was greater than that of the applied 0.125g forcing amplitude with about 6.7% and 12.3% differences in the first and third forced-vibration tests, respectively. Both the initial and final free-vibration tests show a decrease in the damping ratios from 4.53% to 4.30%. The reduction of the damping ratios may be caused by additional hysteretic damping due to the non-linear behaviour of the soil layer present induced by the high magnitude of dynamic forces applied to the pile in the second forced-vibration test.

Figure 4 Pile-soil gap at 1.5 mm

Table 1 Measured and computed natural frequencies and damping ratios

Approach	Natural Frequency (f_n) (Hz)	Damping Ratio (ζ) (%)
Elastic Continuum Model (ECM)	3.987	5.83
Free Vibration – Initial Test	2.633	4.53
Forced Vibration – 1st 0.125g Amplitude	2.633	3.84
Forced Vibration – 2nd 0.5g Amplitude	2.452	4.12
Forced Vibration – 3rd 0.125g Amplitude	2.477	3.61
Free Vibration – Final Test	2.515	4.30

5.4 Dynamic Lateral Displacement

The experimental dynamic lateral displacement (u_{Lat}) determined at the pile-mass connection shows very good correlation with those computed from the analytically derived values using linear elastic continuum solutions as presented in Table 2. Figure 5 shows the deflection curves for the single pile model evaluated using ECM along with the comparison of the measured displacements at the pile-mass connection during the forced-vibration tests.

Table 2 Lateral dynamic displacement of at the pile-mass connection

Approach	ECM Lateral Displacement (u_{pile}) (mm)	Lateral Displacement (u_{Lat}) (mm)
Forced Vibration – 1st 0.125g Amplitude	1.764	1.753
Forced Vibration – 2nd 0.5g Amplitude	5.298	5.299
Forced Vibration – 3rd 0.125g Amplitude	–	1.895

6 CONCLUSIONS

In conclusion, the lateral response of a single pile subjected to dynamic loading causes reduction in the natural frequency of the pile-soil system. This was clearly influenced by the non-linear soil response and the pile-soil gap formation due to the degradation of the soil stiffness. In general, the natural frequency determined from the free-vibration test was greater than that of the forced-vibration test. This was found to be in general agreement with other related published works. The ECM provides acceptable estimation on the natural frequency, damping ratio and lateral displacements at the pile-mass connection of the pile-soil system. The recommended relationship of 300s_u was found to be reliable in estimating the E_s, which was very important in the application of ECM. In the future, full-scale testing on a single pile subjected to lateral dynamic loads is being planned in order to validate the applicability of the ECM in predicting the dynamic parameters (i.e. natural frequency and damping ratio) of the pile-soil system. Therefore, the dynamic parameters estimated from the ECM can be strongly recommended for the design of piles subjected to lateral dynamic loads.

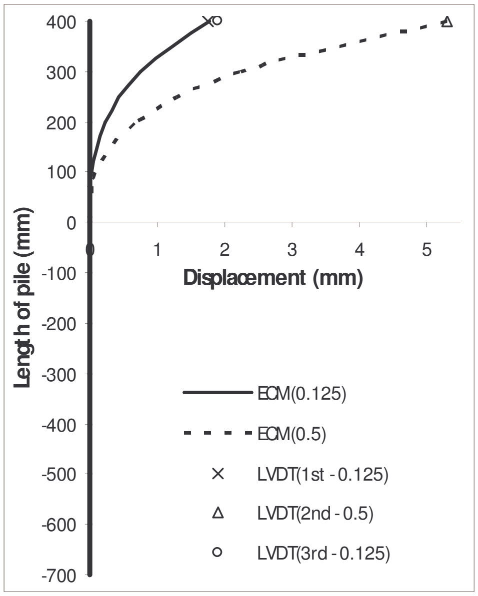


Figure 5 Deflection curves for a single pile

7 ACKNOWLEDGEMENTS

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