Prediction of Sloshing and Seismic Response of a Floating Roof in a Cylindrical Liquid Storage Tank by the Response Spectrum Method

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Abstract

A response spectrum method is proposed to predict the sloshing response of a floating roof in a cylindrical liquid storage tank under seismic excitation. The liquid is assumed to be inviscid, incompressible, and irrotational, while the floating roof is considered as an orthotropic elastic plate of variable thickness. The dynamic interaction between the liquid and the floating roof is taken into account exactly within the framework of linear potential theory. The analysis is based on expanding the response of the floating roof into the free-vibration modes *in liquid* to derive the diagonalised equations of motion for the coupled liquid-floating roof system. The modal analysis based on the response spectrum is then carried out to predict the sloshing response. Numerical results are presented to illustrate the applicability of the proposed approach.

Keywords: liquid storage tank, floating roof, sloshing, long-period ground motion, potential theory, modal analysis, response spectrum.

1. INTRODUCTION

Sloshing of contained liquid is one of the major considerations in the design of liquid storage tanks. In the past major earthquakes many tanks have been subjected to serious damages which may be attributed to liquid sloshing. Especially during the September 2003 Tokachi-oki Earthquake, Japan, seven oil storage tanks with a single-deck type floating roof located at Tomakomai were seriously damaged (Hatayama, 2005). Damages include the sinking of the floating roof which led one of the seven tanks to the whole surface fire, as observed also in the 1999 Kocaeli Earthquake, Turkey (The Japan Society of Civil Engineers, 1999). Although the failure of the floating roof and the fire of oil storage tanks have been observed frequently, e.g. during the 1964 Niigata Earthquake and the 1983 Nihonkai-chubu Earthquake, the sinking of the floating roof caused by sloshing has never been experienced so far in Japan. This was a very dangerous situation that the oil surface was directly exposed to the air. Around the pontoons of the sunken roofs the damages due to local buckling can be observed, from which it is presumed that the damaged pontoon was progressively filled with oil, lost its buoyancy, and was led to sinking. However, the detail of the mechanism of sinking of the floating roof is not fully understood.

After the 2003 Tokachi-oki Earthquake, the Fire and Disaster Management Agency of Japan (2005) has issued the amended Notification of the Fire Defense Law in which the design spectrum for sloshing was increased to almost twice the spectrum in the past Notification. In addition, the standard for the seismic design of floating roofs under long-period ground motion, which has never been included in the past Notification, has been newly regulated. This requires the evaluation of earthquake-resistance capacity of the floating roof of many existing as well as newly designed tanks, and raising it up to the level requested by the amended Notification. Thus there is a rapidly increasing demand for predicting the sloshing response of floating roofs under long-period seismic excitation.

In the earlier papers by the author, analytical solutions have been presented for the sloshing of a floating roof in a cylindrical liquid storage tank under seismic excitation. The solutions which are exact within the framework of linear potential theory have been derived in explicit forms for a floating roof idealized as a uniform isotropic elastic plate (Matsui, 2006), and a single-deck type floating roof composed of an inner deck and an outer pontoon (Matsui, 2007a). A computational method based on coupled finite element and analytical solutions has also been proposed to deal with a floating roof composed of an orthotropic elastic plate of variable thickness (Matsui, 2007b). These earlier studies have been based on expanding the response of the floating roof into the free vibration modes *in air* (dry modes). Due to the presence of the coupling terms, the resulting equations of motion for the coupled liquid-floating roof system cannot be diagonalised. In the present paper, with the purpose of applying the response spectrum method, the approach based on expanding the response into the free vibration modes *in liquid* (wet modes) is employed to derive the diagonalised equations of motion. The modal analysis based on the response spectrum is then carried out to predict the sloshing response. Numerical results are presented to illustrate the applicability of the proposed approach.

2. BOUNDARY-VALUE PROBLEM

The sloshing in a cylindrical liquid storage tank of radius R with flat bottom is considered here. The tank is partially filled with liquid to a height H. The liquid surface is covered by a floating roof which is assumed to move always in contact to the liquid. The tank wall may be assumed to be rigid with reasonable accuracy because the natural periods of shell vibration modes are much shorter than the natural periods of sloshing modes. A cylindrical coordinate system (r,θ,z) is defined, as shown in Figure 1, with the origin at the center of the tank bottom and the z -axis vertically upwards. The tank is subjected to a horizontal ground acceleration $\ddot{x}_g(t)$ in the direction $\theta = 0$.

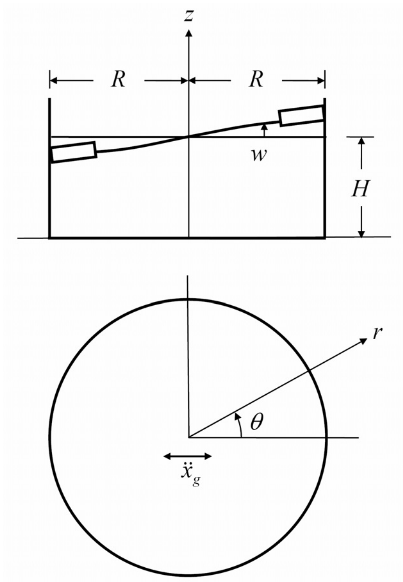

Figure 1 Tank geometry and coordinate system

The liquid is assumed to be inviscid, incompressible and irrotational. Then the liquid motion may be completely defined by a velocity potential function ϕ, for which the boundary-value problem can be defined as

$$\nabla^2\phi = 0 \quad \text{in liquid} \tag{1a}$$

$$\frac{\partial\phi}{\partial r} = \dot{x}_g(t)\cos\theta \quad \text{at the tank wall } r = R \tag{1b}$$

$$\frac{\partial\phi}{\partial z} = 0 \quad \text{at the tank bottom } z = 0 \tag{1c}$$

$$\frac{\partial\phi}{\partial z} = \dot{w} \quad \text{at the liquid surface } z = H \tag{1d}$$

where w denotes the vertical displacement of the floating roof, and the dot denotes the derivative with respect to time t.

The validity of the superposition principle is assumed for both the displacement of the floating roof and the liquid motion. Then the displacement of the floating roof w can be represented as a linear superposition of the free vibration modes $Z_n(r)\cos\theta$ *in air*

$$w(r,\theta,t) = \sum_{n=0}^{N} \xi_n(t) Z_n(r)\cos\theta \tag{2}$$

where $\xi_n(t)$ denotes the modal displacement, and N is the adopted number of modes.

The free vibration modes *in air* can be evaluated analytically for a uniform isotropic plate (Matsui, 2006), and a single-deck type floating roof (Matsui, 2007a). For more general cases such as an orthotropic elastic plate of variable thickness, the finite element method can be employed to evaluate the free vibration modes (Matsui, 2007b).

Herein the floating roof is assumed to be an orthotropic elastic plate obeying the following constitutive laws (Timoshenko and Woinowsky-Krieger, 1959)

$$M_r = -\left[D_r \frac{\partial^2 w}{\partial r^2} + D_1 \left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} \right) \right]$$
$$M_\theta = -\left[D_\theta \left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} \right) + D_1 \frac{\partial^2 w}{\partial r^2} \right] \quad (3)$$
$$M_{r\theta} = -D_{r\theta} \left(\frac{1}{r}\frac{\partial^2 w}{\partial r \partial \theta} - \frac{1}{r^2}\frac{\partial w}{\partial \theta} \right)$$

where M_r, M_θ and $M_{r\theta}$ denote the bending and twisting moments, and D_r, D_θ, D_1 and $D_{r\theta}$ denote the bending and twisting rigidities.

Neglecting the damping term*, the equation of motion of the floating roof can be expressed in the form

$$m \sum_{n=0}^{N} \left[\ddot{\xi}_n(t) + \omega_n{}^2 \xi_n(t) \right] Z_n \cos\theta = p \quad (4)$$

where m denotes the mass density of the floating roof per unit surface area, and ω_n the natural circular frequency of the floating roof in air. The hydrodynamic pressure p on the floating roof can be evaluated from the linearised Belnoulli's equation

$$p = -\rho \left(\frac{\partial \phi}{\partial t} \right)_{z=H} - \rho g w \quad (5)$$

where ρ denotes the mass density of liquid, and g denotes the acceleration of gravity.

The equations (1)-(5) constitute the complete set of equations which determines the response of the coupled liquid-floating roof system in a rigid cylindrical tank.

3. LIQUID-FLOATING ROOF COUPLING ANALYSIS BASED ON DRY-MODE EXPANTION

The solution for the potential ϕ that satisfies the Laplace equation (1a), the wall condition (1b), the bottom condition (1c), and the liquid-surface kinematic condition (1d) is given by (Matsui, 2007b)

$$\phi = \left[\dot{x}_g(t) r + \sum_{i=1}^{I} \frac{g}{\Omega_i^2} \frac{2}{\varepsilon_i^2 - 1} \frac{\cosh(\varepsilon_i z/R)}{\cosh(\varepsilon_i H/R)} \frac{J_1(\varepsilon_i r/R)}{J_1(\varepsilon_i)} \sum_{n=0}^{N} a_{in} \dot{\xi}_n(t) \right] \cos\theta \quad (6)$$

where I is the adopted number of free surface modes, J_1 denotes the Bessel function of the first kind of order one, ε_i denotes the ith positive root of $J_1'(\varepsilon_i) = 0$, Ω_i denotes the natural circular frequency of free surface mode of ith order given by

* The damping term is included later.

$$\Omega_i = \sqrt{\frac{g}{R}\varepsilon_i \tanh\left(\varepsilon_i H/R\right)} \tag{7}$$

and

$$a_{in} = \frac{\varepsilon_i^{\,2}}{J_1\left(\varepsilon_i\right)}\int_0^1 Z_n J_1(\varepsilon_i u)u du \tag{8}$$

The unknown modal displacements $\xi_n(t)$ involved in (6) can be determined by solving the equation of motion (4) of the floating roof. On substitution of (2) and (6) into (5) and then into (4), the equation of motion of the floating roof can be written as

$$m\sum_{n=0}^{N} Z_n\left[\ddot{\xi}_n(t) + \omega_n^2\xi_n(t)\right]$$
$$= -\rho\left[\ddot{x}_g(t)r + \sum_{i=1}^{I}\frac{g}{\Omega_i^2}\frac{2}{\varepsilon_i^2-1}\frac{J_1\left(\varepsilon_i r/R\right)}{J_1\left(\varepsilon_i\right)}\sum_{n=0}^{N} a_{in}\ddot{\xi}_n(t)\right] - \rho g\sum_{n=0}^{N} Z_n\xi_n(t) \tag{9}$$

On multiplying rZ_n on the both sides of (9), integrating with respect to r over $0 \le r \le R$, and making use of the orthogonality relation of the free vibration modes lead to the equation of motion in terms of the modal displacement $\xi_n(t)$

$$\sum_{l=0}^{\infty}\left[\left(\delta_{nl}M_n + \mu_{nl}\right)\ddot{\xi}_l(t) + \left(\delta_{nl}M_n\omega_n^2 + K_{nl}\right)\xi_l(t)\right] = -\gamma_n\ddot{x}_g(t) \quad (n = 0, 1, \cdots, N) \tag{10}$$

where δ_{nl} denotes the Kronecker delta defined by $\delta_{nl} = 0\ (n \ne l), \delta_{nl} = 1\ (n = l)$, and

$$M_n = m\int_0^R Z_n^{\,2} r dr \tag{11}$$

$$\mu_{nl} = \rho R^2\sum_{i=1}^{I}\frac{g}{\Omega_i^2}\frac{2}{\varepsilon_i^2\left(\varepsilon_i^2-1\right)}a_{in}a_{il} \tag{12}$$

$$K_{nl} = \rho g\int_0^R Z_n Z_l r dr \tag{13}$$

$$\gamma_n = \rho\int_0^R Z_n r^2 dr \tag{14}$$

In the equation of motion (10), M_n denotes the effective modal mass, μ_{nl} the added mass coefficient due to liquid, K_{nl} the hydrostatic restoring force coefficient due to change of buoyancy, and γ_n the participation factor. The integrations in (8), (11), (13) and (14) are evaluated analytically for a uniform isotropic plate (Matsui, 2006) and a single-deck type floating roof (Matsui, 2007a). For more general cases these can be evaluated numerically, e.g., by applying Gaussian quadrature formula (Matsui, 2007b).

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4. LIQUID-FLOATING ROOF COUPLING ANALYSIS BASED ON WET-MODE EXPANTION

As discussed in the preceding section, the approach based on the dry-mode expansion leads to the non-diagonalised equations of motion (10) for the coupled liquid-floating roof system. In order to apply the response spectrum method, it is preferable to employ the wet-mode expansion to derive the diagonalised equations of motion.

Let denote the natural circular frequency of n th order of the floating roof *in liquid* by $\hat{\omega}_n$ and the corresponding natural modes by Λ_{nl}, which are evaluated by solving the homogeneous equation of motion (10) with vanishing the forcing term. Then the modal displacement $\xi_n(t)$ associated with the dry-mode expansion can be expressed as a superposition of the wet mode Λ_{nl}

$$\xi_n(t) = \sum_{l=1}^{L} \Lambda_{nl} \zeta_l(t) \tag{15}$$

where $\zeta_l(t)$ denotes the modal displacement associated with the wet-mode expansion, and $L(\leq N)$ is the adopted number of wet modes. Substituting (15) into (10), pre-multiplying Λ_{ln} on the both sides, and making use of the orthogonality relation of the free vibration modes lead to the diagonalised equations of motion in terms of the modal displacement $\zeta_l(t)$ associated with the wet-mode expansion

$$\ddot{\zeta}_l(t) + 2\hat{h}_l \hat{\omega}_l \dot{\zeta}_l(t) + \hat{\omega}_l^2 \zeta_l(t) = -\hat{\gamma}_l \ddot{x}_g(t) \quad (l = 1, 2, \cdots, L) \tag{16}$$

where

$$\hat{\gamma}_l = \frac{\sum_{n=0}^{N} \Lambda_{ln} \gamma_n}{\hat{M}_l + \hat{\mu}_l}, \quad \hat{M}_l = \sum_{n=0}^{N} \Lambda_{ln} M_n \Lambda_{nl}, \quad \hat{\mu}_l = \sum_{n=0}^{N} \sum_{m=0}^{N} \Lambda_{ln} \mu_{nm} \Lambda_{ml} \tag{17}$$

In the above equations, $\hat{\gamma}_l$ denotes the participation factor, $\hat{M}_l$ the effective modal mass, and $\hat{\mu}_l$ the added mass coefficient, respectively, associated with the wet-mode expansion, and $\hat{h}_l$ denotes the damping ratio of l th mode added to consider the damping effects. Once the modal displacements $\zeta_n(t)$ have been obtained by solving the equation of motion (16), these can be substituted into (15) and then into (2) and (3) to evaluate the roof displacement w, and the bending and twisting moments M_r, M_θ, $M_{r\theta}$ on the floating roof

$$w = \sum_{l=1}^{L} \zeta_l(t) \hat{Z}_l(r) \cos\theta \tag{18}$$

$$\begin{aligned} M_r &= -\sum_{l=1}^{L} \zeta_l(t) \left[D_r \frac{d^2 \hat{Z}_l}{dr^2} + D_1 \left(\frac{1}{r} \frac{d\hat{Z}_l}{dr} - \frac{\hat{Z}_l}{r^2} \right) \right] \cos\theta \\ M_\theta &= -\sum_{l=1}^{L} \zeta_l(t) \left[D_\theta \left(\frac{1}{r} \frac{d\hat{Z}_l}{dr} - \frac{\hat{Z}_l}{r^2} \right) + D_1 \frac{d^2 \hat{Z}_l}{dr^2} \right] \cos\theta \\ M_{r\theta} &= \sum_{l=1}^{L} \zeta_l(t) D_{r\theta} \left(\frac{1}{r} \frac{d\hat{Z}_l}{dr} - \frac{\hat{Z}_l}{r^2} \right) \sin\theta \end{aligned} \tag{19}$$

where

$$\hat{Z}_l(r) = \sum_{n=0}^{N} \Lambda_{nl} Z_n(r) \tag{20}$$

5. MODAL ANALYSIS BASED ON RESPONSE SPECTRUM

With the diagonalised equations of motion (16) in hand, the modal analysis based on the response spectrum can now be carried out to predict the maximum responses.

The maximum modal responses of the roof displacement w, and the bending and twisting moments M_r, M_θ, $M_{r\theta}$ on the floating roof in lth mode can be predicted from

$$(w)_l = \hat{\gamma}_l \hat{Z}_l \cos\theta \cdot S_D(\hat{T}_l, \hat{h}_l) \tag{21}$$

$$\begin{aligned}
(M_r)_l &= \hat{\gamma}_l \left[D_r \frac{d^2 \hat{Z}_l}{dr^2} + D_1 \left(\frac{1}{r}\frac{d\hat{Z}_l}{dr} - \frac{\hat{Z}_l}{r^2} \right) \right] \cos\theta \cdot S_D(\hat{T}_l, \hat{h}_l) \\
(M_\theta)_l &= \hat{\gamma}_l \left[D_\theta \left(\frac{1}{r}\frac{d\hat{Z}_l}{dr} - \frac{\hat{Z}_l}{r^2} \right) + D_1 \frac{d^2 \hat{Z}_l}{dr^2} \right] \cos\theta \cdot S_D(\hat{T}_l, \hat{h}_l) \\
(M_{r\theta})_l &= \hat{\gamma}_l D_{r\theta} \left(\frac{1}{r}\frac{d\hat{Z}_l}{dr} - \frac{\hat{Z}_l}{r^2} \right) \sin\theta \cdot S_D(\hat{T}_l, \hat{h}_l)
\end{aligned} \tag{22}$$

where $S_D(T,h)$ denotes the displacement response spectrum of the input ground motion, and $\hat{T}_l = 2\pi/\hat{\omega}_l$ denotes the natural period of the floating roof in liquid.

There are two methods most commonly-used for superposing the maximum modal responses to predict the maximum total response. One is the square-root-of-sum-of-squares (SRSS) method, and the other is the complete-quadratic-combination (CQC) method, which is effective for the system with closely-spaced natural periods. For example, the maximum total response of roof displacement w can be predicted from

$$w|_{\max} = \sqrt{\sum_{l=1}^{L} (w)_l^{\,2}} \tag{23}$$

by the SRSS method, or from

$$w|_{\max} = \sqrt{\sum_{l=1}^{L} \sum_{n=1}^{L} c_{ln} (w)_l (w)_n} \tag{24}$$

by the CQC method, where c_{ln} denotes the correlation coefficient between the lth and nth modes, given by

$$c_{ln} = \frac{8\sqrt{h_l h_n}(h_l + \beta_{ln} h_n)\beta_{ln}^{3/2}}{(1-\beta_{ln}^2)^2 + 4h_l h_n \beta_{ln}(1+\beta_{ln}^2) + 4(h_l^2 + h_n^2)\beta_{ln}^2} \tag{25}$$

with $\beta_{ln} = \hat{\omega}_l / \hat{\omega}_n$ (Der Kiureghian, 1981). The similar expressions can be obtained for the maximum total responses of bending and twisting moments.

6. RESULTS AND DISCUSSION

Numerical studies have been performed to illustrate the applicability of the proposed method. Table 1 shows the principal parameters of the tank model analyzed here. This is the model of typical oil-storage tank of 100,000m^3 capacity with a single-deck type floating roof, for which an analytical solution has been presented (Matsui, 2007a). The EW component of the 2003 Tokachi-oki earthquake recorded at Tomakomai K-NET station (HKD1290309260450EW) was used as an input ground motion, of which the time history of acceleration and the velocity response spectrum are shown in Figure 2.

Figure 3 shows the maximum responses predicted with varying the adopted number of modes. The stiffness-proportional damping was assumed with the damping ratio shown in Table 1 for the fundamental mode. Both the results predicted by the SRSS and CQC methods are presented. It can be noted that only 3 modes are sufficient to obtain the converged solutions for the roof displacement, while more than 30 modes are required to obtain the reasonable predictions for the bending stresses. This is because a larger number of modes are needed to express precisely the bending stresses in the single-deck type floating roof, which change rapidly in the neighbourhood of the connection between the deck and the pontoon (Matsui, 2007a). Figure 4 shows the predicted maximum responses along $\theta = 0$ (which is parallel to the direction of ground motion). The predictions based on the response spectrum method are compared with the results of time history analysis. Both the SRSS and CQC methods are found to provide

Table 1 Principal parameters of tank models

Radius	40 m
Liquid depth	20 m
Mass density of liquid	850 kg/m^3
Thickness of deck	4.5 mm
Width of pontoon section	5 m
Height of pontoon section	800 mm
Thickness of upper and lower decks of pontoon	4.5 mm
Thickness of inner and outer rims of pontoon	12 mm
Bending rigidity of deck	1.719 kN-m
Bending rigidity of pontoon	1.671×10^6 kN-m^2
Twisting rigidity of pontoon	2.259×10^6 kN-m^2
Effective cross-sectional coefficient of pontoon	5717 mm^3
Mass density of inner deck (per unit area)	58 kg/m^2
Mass of pontoon (per unit length)	819 kg/m
Mass moment of inertia of pontoon (per unit length)	2815 kg-m^2/m
Young's modulus	206 GPa
Poisson's ratio	0.3
Damping ratio (for the fundamental mode)	0.005

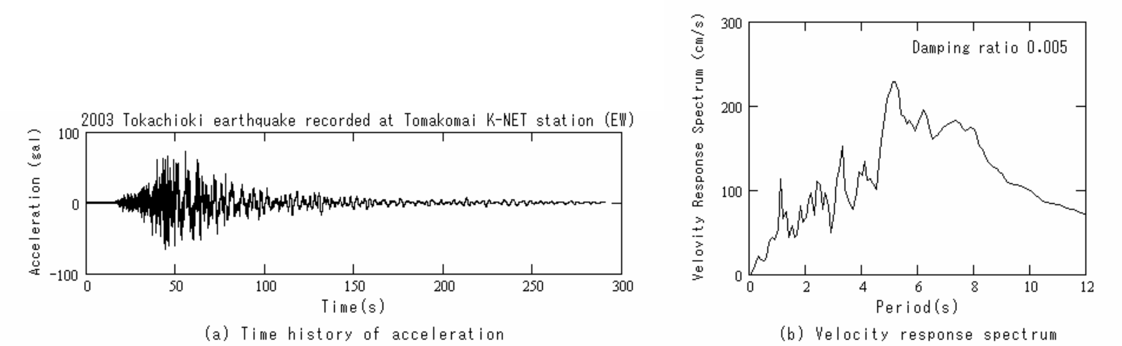

Figure 2 (a) Time history of acceleration, and (b) velocity response spectrum of EW component of the 2003 Tokachi-oki earthquake recorded at Tomakomai K-NET station

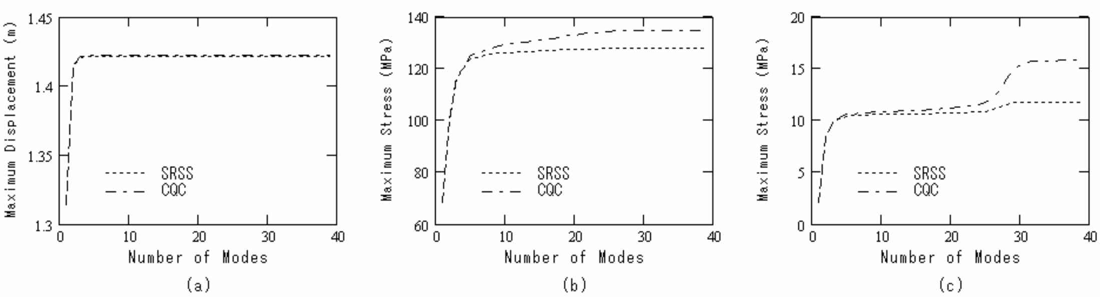

Figure 3 (a) Maximum roof displacement along the tank wall, (b) maximum radial bending stress in the deck along the connection with the pontoon, and (c) maximum circumferential bending stress on the pontoon predicted with varying number of modes

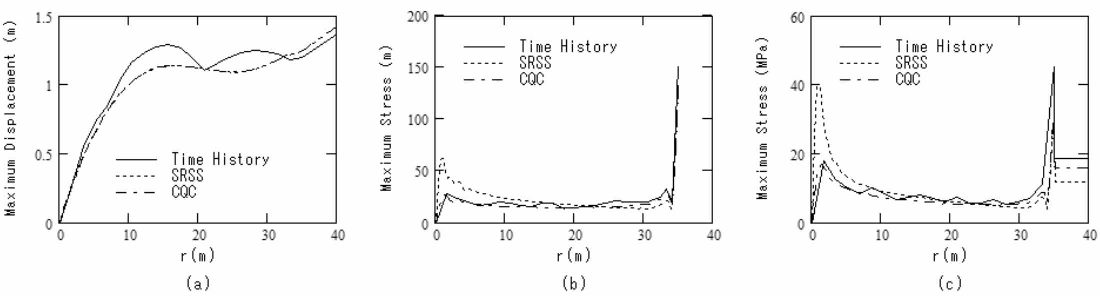

Figure 4 (a) Maximum roof displacement, (b) maximum radial bending stress, and (c) maximum circumferential bending stress along $\theta=0$ compared with time history analysis

reasonable predictions for the roof displacement, whereas the CQC method gives better predictions for the bending stresses than the SRSS method, which are close to the results of time history analysis.

7. CONCLUSIONS

A response spectrum method has been proposed to predict the sloshing response of a floating roof in a cylindrical liquid storage tank under seismic excitation. The validity of the proposed method has been confirmed by comparison with the results of time history analysis. It can be concluded that the CQC method is preferable than the SRSS method because the former gives better predictions for both the roof displacement and the bending stresses, which are close to the results of time history analysis.

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