# CYCLIC STRUCTURAL CAPACITY OF R/C FRAMES FOR MAXIMUM DUCTILITY AND INTER-STORY DRIFT

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## **ABSTRACT**

Experimental evidence suggests that the linear cumulative damage index can estimate, with good accuracy, the structural damage to account for cumulative plastic deformation demands. This index is formulated in terms of the cyclic structural demand and capacity, and it can be evaluated with experimental tests for structural elements. Nevertheless, there is a challenge to extrapolate the use of this index for practical application in the design of multi degree of freedom (MDOF) structures. The aim of this paper is the assessment of the cyclic structural capacity of R/C frames in terms of maximum ductility and inter-story drift based on experimental results performed on reinforced concrete elements and steel bars of several diameters. Three standard occupation buildings with different seismic coefficients and fundamental periods of vibration, designed according to the new Indonesian Building Code are analyzed. Modal push-over analyses is performed on those structures in order to correlate the maximum curvature ductility of the structural cross section, maximum rotation ductility of member elements with the global displacement ductility, and inter-story drift capacity of the frames. A high correlation between the curvature, rotation, global displacement ductility and inter-story drift is observed. This high correlation let the assessment of the cyclic capacity of R/C frames. Finally, an expression to evaluate the cyclic capacity of regular R/C structures as a function of their global mechanic characteristics is proposed, which is a fundamental piece to evaluate the linear cumulative damage index in actual structures.

#### 1. INTRODUCTION

Performance-based seismic design requires the use of reasonable tools to estimate structural damage within numerical design methodologies. Usually, the maximum interstory drift and ductility have been used as performance parameters to estimate the structural damage. However, in certain cases, the effect of cumulative plastic deformation demands should be accounted. The cumulative demands can be considered by means of energy concepts, especially through the plastic hysteretic energy, due to the clear relation between this parameter and the structural damage. Several studies have shown the importance of hysteretic energy dissipation in structures with low cycle capacity and/or fundamental periods similar to the dominant period of the soil, especially when they are subjected to long duration seismic motions, such as those that occur in soft soils (Fajfar and Krawinkler, 1997; Terán-Gilmore and Jirsa, 2007; Bojorquez et al., 2008). Nevertheless, through the use of the dissipated hysteretic energy to evaluate the structural damage, the number and magnitude of the cycles of plastic behaviour are not taken into account. The linear cumulative damage index can be used to evaluate the structural damage produced by cumulative plastic deformation demands., in such index, information about the number and magnitude of plastic cycles is required. Two parts are necessary to evaluate this index. The first is the cyclic demand curve, which can be obtained with nonlinear dynamic time history analysis; and the second part corresponds to the cyclic capacity curve. While it is relatively easy to evaluate the cyclic capacity through experimental tests for reinforcing steel and structural elements, the principal challenge is try to obtain the cyclic capacity curve for MDOF structures. In this study, a procedure to evaluate the cyclic structural capacity of ductile R/C frames is proposed. Since the maximum ductility and interstory drifts are the principal parameters for most of seismic design codes, the procedure is used to evaluate the cyclic capacity curves in terms of these parameters.

#### 2. CYCLIC STRUCTURAL CAPACITY OF R/C MEMBERS

A cyclic capacity curve represents the capacity that a structure or element has to accommodate up to failure, where the failure is defined as the number of plastic behavior cycles with constant amplitude (e.g. ductility demand) supported by the structure. Figure 1 shows a typical cyclic capacity curve of an R/C member. It is clear that, as the maximum constant ductility demand increases, the number of half plastic cycles  $(2N_f)$  that the structure can develop before the failure decreases. The capacity curve can be obtained through experimental tests or by means of analytic models. Based on the use of capacity curves, it is possible to estimate structural damage in elements or structural systems through the use of the linear cumulative damage index  $(I_{DL})$  described by equation 1, which is based in such way in past studies developed by Miner (1945).

$$I_{DL} = \sum_{i=1}^{Ndif} \frac{n_i}{N_i} \tag{1}$$

For equation 1  $N_i$  is given by the cyclic capacity curve, and represents the number of plastic excursions the structure can undergo before failure when cycled to excursions with amplitude  $\delta_{ci}$ ;  $n_i$  the number of plastic excursions of amplitude  $\delta_{ci}$  resulting from the ground motion demands on the structure; Ndif the number of different intervals into which all plastic excursions are classified according to their amplitude; and  $\delta_{ci}$ , the cyclic displacement (amplitude) associated with the  $i^{th}$  interval.  $I_{DL}$  equal to one implies incipient failure. Because the importance of the cyclic capacity curves for damage evaluation, the present paper is focusing to obtain such curve for R/C frames.

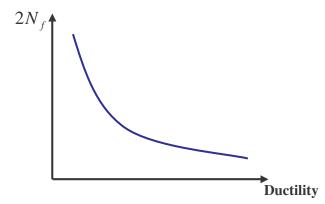


Figure 1. Typical cyclic capacity curve in R/C members.

## 2.1 Relation of plastic strain and number of plastic cycles, and cyclic capacity of R/C members

Three potential failure modes were identified for R/C elements from tests results (Brown and Kunnath, 2004). The first is essentially a precursor to the ultimate failure mode and consist of global buckling of longitudinal bars that occurs over a length corresponding to several hoop spaces.; the second failure mode is a result of confinement failure following the rupture of the transverse hoop steel.; and the third on, typically associated with large displacements, is a low cyclic fatigue fracture of the longitudinal bars. If proper detailing methods, such as the provision of closely spaced transverse reinforcement, are employed in plastic hinge regions, confinement failure can be controlled and bar buckling can be delayed. When a reinforced concrete member, particularly a column, is subjected to reverse cyclic loading, the concrete cover will typically spall at small strains (below the yield strain of reinforcing steel). When the spalling progresses, the reinforcing steel is exposed to air. The cyclic response of exposed reinforcing bars in the inelastic range can be reasonably captured in a fatigue test of individual bars without the presence of

concrete. For this case, the Coffin-Manson (1954) equation formulates the fatigue behavior of longitudinal bars under the reversed cyclic loading as following:

$$\varepsilon_p = \varepsilon_f' (2N_f)^c \tag{2}$$

where,  $\mathcal{E}_p$  is the ultimate plastic strain amplitude,  $\mathcal{E}_f$  is a material constant to be determined from fatigue testing and  $2N_f$  is the number of half cycles before the failure. Kunnath et al. (1997) used the strain curvature relationship described by equation 3 in their tests analyses assuming that the section strains vary linearly.

$$\varepsilon_p = \phi_p \overline{d} / 2 \tag{3}$$

In equation 3,  $\phi_n$  is plastic curvature, and  $\overline{d}$  is distance between centers of longitudinal bars. During actual tests the neutral axis of the section does not always stays at the center of the section. However, it is supposed that the total plastic strain amplitude of a main bar becomes equal to twice  $\phi_n \overline{d}/2$  after one completed loading cycle with the same displacement or curvature to both of the opposite lateral directions, as long as the section strain vary linearly and the main bar, of which the strain amplitude is examined, is located outside the neutral axis. Assuming the plastic rotation  $\theta_p$  is at the center of plastic hinge of vertical length  $L_p$  and neglecting shear (Priestley and Paulay, 1992), plastic curvature  $\phi_p$  is expressed as:

$$\phi_p = \frac{\theta_p}{L_p} = \frac{\delta_p / (L_s - 0.5L_p)}{L_p} \tag{4}$$

where  $\delta_p$  is the plastic displacement and  $L_s$  is the member length. Brown and Kunnath (2004) by experimental tests on steel bars, demonstrated how the low cyclic fatigue plastic strain depends on the diameter of the bar. From a low cycle fatigue failure test of reinforcing steel bars, the following set of fatigue life equations were obtained by Brown and Kunnath (2004) (where  $\mathcal{E}_p$  is the plastic strain amplitude):

$$\varepsilon_p = 0.16(2N_f)^{-0.57}$$
 for No. 6 bars (19.05mm) (5)

$$\begin{split} \mathcal{E}_{p} &= 0.16(2N_{f})^{-0.57} & \text{for No. 6 bars (19.05mm)} \\ \mathcal{E}_{p} &= 0.13(2N_{f})^{-0.51} & \text{for No. 7 bars (22.23mm)} \\ \mathcal{E}_{p} &= 0.09(2N_{f})^{-0.42} & \text{for No. 8 bars (25.4mm)} \\ \mathcal{E}_{p} &= 0.07(2N_{f})^{-0.37} & \text{for No. 9 bars (28.58 mm)} \end{split} \tag{8}$$

$$\varepsilon_p = 0.09(2N_f)^{-0.42}$$
 for No. 8 bars (25.4mm) (7)

$$\varepsilon_p = 0.07(2N_f)^{-0.37}$$
 for No. 9 bars (28.58 mm) (8)

Substituting the plastic strain amplitude in equation 3 multiplied by 2 and equation 4, the followings equations for the number of plastic cycles of capacity for R/C members are obtained.

$$2N_f = \left(0.16L_p / \theta_p \bar{d}\right)^{1/0.57} = \left(0.16L_p / (\theta_y (\mu_\theta - 1)\bar{d})\right)^{1/0.57} \quad \text{no. 6 bar (19.05mm)}$$
(9)

$$2N_f = \left(0.13L_p / \theta_p \bar{d}\right)^{1/0.51} = \left(0.13L_p / (\theta_y (\mu_\theta - 1)\bar{d})\right)^{1/0.51} \quad \text{no. 7 bar (22.23mm)}$$
 (10)

$$2N_f = \left(0.09L_p / \theta_p \bar{d}\right)^{1/0.42} = \left(0.09L_p / (\theta_y (\mu_\theta - 1)\bar{d})\right)^{1/0.42} \quad \text{no. 8 bar (25.4 mm)}$$
 (11)

$$2N_f = \left(0.07L_p / \theta_p \bar{d}\right)^{1/0..37} = \left(0.07L_p / (\theta_y (\mu_\theta - 1)\bar{d})\right)^{1/0..37} \quad \text{no. 9 bar (28.58mm)}$$
 (12)

The results of the above equations are plotted in Figure 2. The horizontal axis represents the rotation ductility demand, and the vertical axis the number of plastic cycles that the members are able to undergo before the failure.

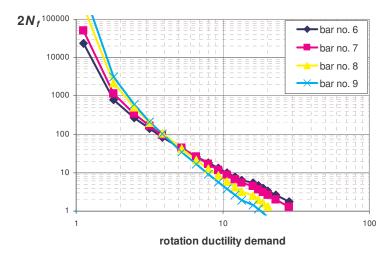


Figure 2. Cyclic capacity curve of R/C members for different diameter of the reinforcing steel bars.

#### 3. CYCLIC STRUCTURAL CAPACITY OF R/C FRAMES: METHODOLOGY

In this part, the procedure to obtain the cyclic capacity of R/C frames is described. This procedure will be applying to three standard occupation buildings with 5, 10, and 15 story levels (modeled as structural frames), different seismic yielding coefficients and different fundamental periods of vibration, which were designed in accordance to the Indonesian Building Code (Lumantarna, 2001). The global mechanical characteristics of the frames are summarized in Table 1. The seismic coefficients  $(C_v)$  and the yield displacements ( $\delta_v$ ) were obtained by means of "modal push-over" analyses carried out with the computer program RUAUMOKO (Carr, 2000). In table 1, 5st 1mode represents the frame with 5 stories and the characteristics for the first mode of vibration. The same is for the other cases. The structural members of the frames were assumed to have bilinear behaviour with post-yielding stiffness equal to 4% of the initial stiffness. (It should be mentioned that actual R/C members have a behaviour with strength and moderate stiffness degradation, the bilinear behavior captures some important characteristics of the structural behavior of R/C frames with enough accuracy.) It was assumed 2% of critical damping. All analyses considered mean values for loads and material strength (Lumantarna, 2001). Capacity design philosophy has been used according to the Indonesian code and further details regarding the design of the structural ductile R/C frames can be found in (Lumantarna, 2001).

FRAME	Number of stories	Seismic yielding Coefficient $(C_v)$	Period (T)	Yield Displacement δ <sub>y</sub> (m)
5st_1mode	5	0.265	0.836 s	0.065
5st_2mode	5	0.280	0.263 s	0.02
10st_1mode	10	0.0994	1.565 s	0.088
10st_2mode	10	0.119	0.524 s	0.031
15st_1mode	15	0.0686	2.278 s	0.126
15st_2mode	15	0.0625	0.79 s	0.062

## 3.1 Hypotheses

The next hypotheses were considered to obtain the cyclic structural capacity of the R/C frames:

- i) The response of the structure is dominated by the first two modes of vibration.
- ii) Failure of the frames is due to failure of the beams, and the beams are the only elements with capacity to dissipate energy through plastic behaviour (weak beam-strong column design philosophy).
- iii) The damage level is equal in all the beams of the same story.
- iv) Failure of the system corresponds to failure of the critical story.

# 3.2 Proposed procedure

The procedure to obtain the cyclic structural capacity is similar than the procedure proposed by (Bojórquez et al., (2006), but in this case, two modes of vibrations are included, instead of just one. The methodology is the following:

- 1. In first place, a modal push-over analysis of the frame is performed (for cyclic capacity curve of displacement and drift, considering the first two modes are quite accurate) (Chopra and Goel, 2002).
- 2. Using the modal push-over analysis results, the history of the local rotation or curvature ductility demand  $(\mu_{\theta}, \mu_{\phi})$  in all the beams of the frame is obtained up to the failure of the structure (which occurs when the beams of the critical story fail). Bojórquez et al. (2006) demonstrated that the beams exhibit a similar level of local ductility in the same story, and that the damage level in the critical and neighbouring stories is quite similar. Also they concluded that the failure of the critical story, followed by seismic demand redistribution in the frame will cause the failure of other stories and, consequently, the failure of the structure. Due to this, hypothesis iii and iv can be accepted.
- 3. Next, the maximum global ductility  $\mu_{\Delta}$  and maximum interstory drift of the frame are obtained for different stages of the modal push-over analysis. The maximum global ductility is defined as the ratio between the maximum roof displacement at a particular stage and the displacement at yielding (obtained also from the modal push-over analysis).
- 4. A maximum global ductility value is associated to each local ductility value of interest (rotation and curvature ductility), and the number of cycles the structure can undergo for a given level of maximum global ductility is evaluated. In this way, the cyclic structural capacity of the ductile R/C frames is obtained.
- 5. Finally, the cyclic structural capacity of the frame in terms of local ductility  $\mu_{\theta}$  and  $\mu_{\phi}$ , the global ductility  $\mu_{\Delta}$  and the interstory drift ( $\delta$ ) are obtained.

#### 3.3 Results

An approach to obtain the cyclic structural capacity curve of R/C frames has been proposed. In this section the procedure is applied to the three R/C frames before described. The results obtained are illustrated in Figure 3 in log-log scale, where local (curvature and rotation ductilities), global displacement ductility and interstory drift cyclic capacities are compared. A reasonable correlation between the four curves is observed. A linear trend in log-log scale is observed for all the capacity curves and for all the frames studied, especially for the interstory drift and global ductility cyclic capacity curves.

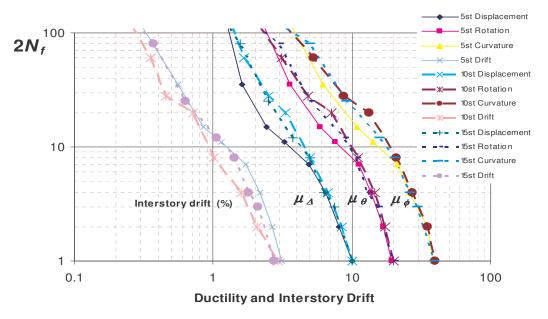


Figure 3. Comparison between local (rotation and curvature), global (top displacement) ductility and interstory drift cyclic capacity curves for the three R/C frames analyzed.

#### 4. SIMPLIFY EVALUATION OF THE CYCLIC CAPACITY CURVE FOR R/C FRAMES

Figure 3 suggests, that a linear behaviour in log-log scale is observed for the drift and global ductility cyclic capacity curves. In this part of the study an equation to obtain the cyclic capacity curve for ductile R/C frames in terms of the global characteristics is proposed. The equation can be used to evaluate the structural damage by means of the linear cumulative damage index for structures with similar characteristics to those studied here. The following relation, that correlates the number of plastic cycles  $2N_f$  and the maximum global ductility  $\mu_{\Delta}$  and maximum interstory drift ( $\delta$ ), is used to fit the results obtained by each of the frames:

$$2N_f = \frac{a}{\mu_{\Lambda}^b} \tag{13}$$

The values of *a*, *b* resulted for global ductility and interstory drift corresponding to each frame were obtained. Then, the parameters *a* and *b* were related to the global mechanic characteristics of the frames (fundamental period of vibration and seismic yield coefficient). Based on this, a final expression (Equation 14 and 15) are proposed to evaluate the cyclic capacity for ductile R/C frames in terms of global ductility and maximum interstory drift. Figure 4 illustrates the comparison between the equation for interstor drift and the results obtained from modal push-over analysis. It can be observed that the equation proposed result in very good approximation for all the frames considered. In general the buildings analyzed only can resist 1 cycle for a maximum interstory drift equals to 3%, since the Indonesian Code considers the revision of just one maximum interstory drift of 2%. This implies that they will be very susceptible to due to the incursion of several plastic hysteretic cycles, which demonstrates the importance to account for cumulative plastic deformation demands.

$$2N_f = \frac{174.4 + 77.448T}{(\mu_g)^{2.4005}} \qquad 2N_f = \frac{15.185 - 1.2809T}{(\delta)^{1.8972}}$$
(14, 15)

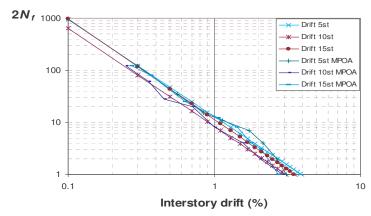


Figure 4. Comparison between the cyclic capacity curve obtained from the modal push-over analysis of the frames and the equation proposed.

#### 5. CONCLUSIONS

A procedure for the assessment of the cyclic capacity curve of R/C frames by modal push-over analysis was proposed. The procedure was applying to obtain the cyclic capacity of three R/C frames designed with the Indonesian Code in terms of maximum ductility and maximum interstory drift. It was shown that, the local, global ductility and maximum interstory drift capacity curves are very well correlated. In fact, there is a linear relation for all the parameters in a log scale with the number of plastic cycles the structure can develop before the failure, which was used to propose an expression to evaluate the ductility and interstory drift capacity curves for R/C frames with similar characteristics to the frames here analyzed. This equation can be used to evaluate the linear cumulative damage index in reinforced concrete structures. Finally, due to the limitations in the cyclic structural capacity of R/C frames, it was observed the importance to account for parameters related with cumulative demands for design purposes.

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