Dynamic Response Characteristics of Urban Trees

N. Haritos$^1$ & K. James$^2$

$^1$Civil & Environmental Engineering, $^2$Land & Food Resources, The University of Melbourne, Parkville, Victoria 3101

E-Mail: nharitos@unimelb.edu.au

ABSTRACT: Currently there is little data on wind loading available to assess dynamic excitation effects by wind on trees. Wind forces are the largest forces that trees must endure, and global climate changes are likely to generate more extreme weather events that will increase the likelihood of wind damage to trees and property. Wind induced dynamic excitation in two urban trees (a She-oak and a spotted gum located on the main campuses of The University of Melbourne and Monash University, respectively) has been assessed using tri-axial accelerometers and (separately) a purpose-designed digital strain meter that measures the outer fibre movement of a trunk or branch as it bends in the wind. The measurements on these two trees were taken on the 2nd April, 2008 when an approximately 1 in 20 year wind storm was experienced in Melbourne. Pluck tests were also performed on She-oak on separate near still wind occasions as a follow-up to the wind measurements. Results from a spectral based model applied to the wind measurements and a fitting procedure exercised on the pluck test measurements are presented in this paper that assist in providing insight into the dynamic characteristics of these two tree species to turbulent wind in an urban environment.

Keywords: wind induced vibrations, spectral analysis, damping, resonant frequency

1. INTRODUCTION

When trees in urban areas fail they can cause serious damage and issues of liability may result in costly litigation – this is especially so in the United States (Mortimer & Kane 2004). Wind loading effects exert the largest actions on trees and as trees reach their maturity in height and age, the combined wind and dead load effects can cause the wood to approach its failure point.

Trees have adapted to their load environment through the process of evolution (Gardiner & Quine 2000, Mattheck & Breloer 1994) via a number of mechanisms that include:

1. adaptive growth in which sections under high stress can thicken and so provide added strength
2. tapering of the trunk and branches
3. streamlining of their canopy and leaves to reduce the drag area exposed to the wind,
4. shedding leaves, and even branches, under high winds to reduce wind exposure
5. energy dissipation from several complex damping mechanisms including high material damping, tuned mass-damping of the branches and aerodynamic damping from the interaction with the wind itself

Static analysis methods have generally been used by botanists and tree scientists to study the mechanical properties of trees and to evaluate wind forces, (Brudi 2002, Moore 2000). However, there’s a growing body of evidence that static analysis over-estimates tree strength (Hassinen et al. 1998, Gardiner 1995) presumably because wind forces are not static and tree response requires a dynamic analysis to evaluate the effects of wind.
Whilst recent dynamic studies have modeled trees as single columns (Wood 1995, Guitard & Castera 1995, Gaffrey & Kniemeyer 2002), there is growing recognition of the complexity of tree dynamics in these studies, with the introduction of more complex models (James et al. 2006, James & Haritos 2008) and dynamic methods including modal analysis and finite element analysis (de Langre 2007).

The paper being presented here highlights recent investigations on two trees in the urban environment, and their dynamic response to wind excitation. The studies have separately involved experimental measurements of the trunk acceleration response and inferred base bending moment (using purpose built strain meters developed by the second author specifically for such investigations (James & Kane 2008)), from excitation by wind. Emphasis has been placed in these experiments in the identification of the basic dynamic properties (in-line natural frequency and associated damping values), of these trees under the strong wind conditions experienced on 2nd April, 2008. In addition, basic “pluck testing” has also been conducted to identify natural frequency and damping values under still conditions for one of these trees. It is hoped that better understanding of the dynamics of trees and their response to wind loading will eventually lead to design guidelines for trees and winds in urban environments.

2. DYNAMIC RESPONSE OF TREES TO WIND

2.1 Nature of Wind Forces on Trees

Wind forces are extremely variable both spatially and temporally, so are considered to be dynamic in character. Consequently, trees respond dynamically to wind excitation with their trunks and branches swaying in a complex fashion requiring a dynamic system of measurement and analysis to be able to model, (Hassinen et al 1998).

The dynamic response of trees to wind excitation has been studied using a variety of methods and instrumentation, including accelerometers (Peltola 1996), displacement transducers, (Kerzenmacher & Gardiner 1998), extensometers or strain gauge devices on the trunk or roots (Guitard & Castera 1995), lasers (Baker 1997), tilt sensors (Rudnicki et al 2001) and video based techniques (Peltola 1996).

The dynamic character of the response of trees to wind, lends itself to a spectral modelling approach, (James & Haritos 2008).

2.2 Spectral Based Model for Wind Loading on Trees

A spectral based model for investigating the dynamic response of trees to wind loading is outlined here. (The approach follows that of Davenport as exercised in the Gust Factor method found in most wind codes, (including AS1170 Part 2) for modelling the in-line response of buildings to wind).

Consider an equivalent single degree of freedom (SDOF) modelling of the response to wind excitation of an urban tree, where the in-line displacement response at a suitable reference point (eg the centroid of exposed area of the tree canopy) is given by \( x(t) \), then:

\[
m\ddot{x}(t) + c\dot{x}(t) + kx(t) = F(t)
\]

in which \( m \) is the equivalent mass, \( c \) the equivalent structural damping and \( k \) the equivalent stiffness, at the reference point and \( x(t) \) is the displacement response.

The in-line wind force, \( F(t) \), acting on a tree is considered to be drag dependent, so would be related to the relative along wind speed, \( (V(t) - \dot{x}(t)) \), where \( V(t) \) and \( \dot{x}(t) \) are the wind speed and tree velocity, respectively via:

\[
F(t) = \alpha (V(t) - \dot{x}(t))^2 \quad (\alpha = \frac{1}{2}\rho C_D A)
\]
in which \( \rho \) is air density (~1.2 kg/m\(^3\)), \( C_D \) is the total effective drag coefficient for the tree, branches and leaves and \( A \) is the orthogonal area of exposure to the wind of these elements.

Because the tree canopy is normally dominated by leaves and twigs (as opposed to more “rigid” elements such as the trunk and branches), and these deform and “streamline” in the wind, it is assumed that the \( \alpha \) value will be wind speed dependent, so that

\[
\alpha = \alpha_o \left( \frac{\bar{V}^n}{V^2} \right) = \alpha_o \bar{V}^{n-2}
\]

(3)

\[
F(t) = \alpha_o \bar{V}^{n-2} \left( V(t) - \dot{x}(t) \right)^2 \left( \alpha_o = \frac{1}{2} \rho C_D A_o \right)
\]

(4)
in which \( C_{D_o} \) and \( A_o \) represents the drag coefficient and exposed area of the canopy under still wind conditions, respectively, and exponent \( n \), is less than 2.

For \( F(t) \) acting at an effective height above the ground of \( h_m \), (which could be chosen to correspond to the reference point location), the moment acting on the tree trunk at its base, (the base moment), is given by:

\[
M(t) = h_m F(t) = \beta \bar{V}^2(t) \quad (\beta = h_m \alpha)
\]

(5)

Simply multiplying both sides of Eqn. (1) by \( h_m \), we have:

\[
h_m \left( m\ddot{x}(t) + c\dot{x}(t) + kx(t) \right) = h_m F(t) = M(t)
\]

(6)

Now for along wind speed consisting of a mean, \( \bar{V} \), and turbulent component, \( v(t) \), then

\[
V(t) = \bar{V} + v(t) = \bar{V} \left( 1 + \frac{v(t)}{\bar{V}} \right)
\]

(7)

After considering Eqn. (2), the base moment of the tree at its base is therefore given by:

\[
M(t) = \beta \bar{V}^2 \left( 1 + \left( \frac{v(t)}{\bar{V}} \right)^2 \right) - 2 \beta \bar{V} \dot{x}(t)
\]

\[
= \bar{M} + m(t) - h_m c_a \ddot{x}(t)
\]

in which \( \bar{M} = \beta \bar{V}^2 \left( 1 + I^2 \right) \); \( m(t) = 2 \beta \bar{V} v(t) \); \( c_a = 2 \alpha \bar{V} \); \( I \) is the turbulence intensity, \( v_{RMS}/\bar{V} \).

The assumption made here is that both the response \( \ddot{x}(t) \) and the wind speed fluctuation \( v(t) \) are small compared to the mean wind speed, \( \bar{V} \). The term \( c_a \) in Eqn. (8) can be considered to be an “aerodynamic damping” contribution term and can be taken to the left hand side of Eqn. (6) to enhance the overall damping, so that:

\[
h_m \left( m\ddot{x}(t) + (c + c_a)\dot{x}(t) + kx(t) \right) = \bar{M} + 2 \beta \bar{V} v(t)
\]

(9)

Considering the above modelling approach, the spectral description for the base bending moment of a tree under wind excitation, \( S_M(f) \), can therefore be obtained by considering fluctuating terms, viz:

\[
S_M(f) = \left( 2 \beta \bar{V} \right)^2 \chi_m^2(f) \chi_a^2(f) S_v(f) = T^2(f) S_v(f)
\]

(10)
in which \( \chi_m^2(f) \) is the structure magnification function, \( \chi_a^2(f) \) is the “aerodynamic admittance” function – a tree size dependent/frequency dependent reduction factor, \( S_v(f) \) is the along wind speed spectrum, and \( T^2(f) \) represents the overall transfer function given by:
\[ \chi_m^2(f) = \frac{1}{\left(1 - \left(\frac{f}{f_o}\right)^2\right)^2 + \left(2\zeta\left(\frac{f}{f_o}\right)\right)^2} \]  
\[ T^2(f) = (2\beta\bar{V})^2 \chi_m^2(f) \chi_o^2(f) \]

in which \( \zeta \) represents the effective damping ratio (inclusive of all damping contributions) and \( f_o \), the primary mode frequency of the tree.

The above modelling approach can be further refined by introducing the concept of modal response fixed at the primary mode shape, \( \phi(z) \), with amplitude \( \eta(t) \) at the reference point – the effect of which is considered to be small in the case of typical tree structural forms.

### 2.3 Estimating Dynamic Characteristics \((f_o \text{ and } \zeta)\) from Wind Response Measurements

The spectral modelling approach above can be used to estimate \( f_o \), the primary mode frequency of the tree and \( \zeta \), its effective damping ratio (inclusive of all damping contributions), from response measurements conducted under wind excitation.

Assuming the wind speed spectrum to be in the inertial sub-range, where it takes on a theoretical \(-5/3\) power law variation with frequency, \( f \), with scaling coefficient, \( S_v \), then

\[ S_M(f) = \frac{C_{f_o} \cdot f^{\frac{5}{3}}}{\left(1 - \left(\frac{f}{f_o}\right)^2\right)^2 + \left(2\zeta\left(\frac{f}{f_o}\right)\right)^2} \] where \( C_{f_o} = (2\beta\bar{V})^2 \chi_o^2(f_o) S_v \)

represents an approximation that can be applied over the frequency band encompassing the resonant peak in the response for the purpose of fitting \( f_o \) and \( \zeta \) to the observed \( S_M(f) \). A similar expression to Eqn. (13) is obtained if instead of base moment, the along wind acceleration spectrum of tree response to wind is measured via accelerometers placed at a suitable reference point, viz

\[ S_x^2(f) = \frac{C_{f_o'} \cdot f^{\frac{7}{3}}}{\left(1 - \left(\frac{f}{f_o}\right)^2\right)^2 + \left(2\zeta\left(\frac{f}{f_o}\right)\right)^2} \] where \( C_{f_o'} = (2\pi)^4 (2\alpha\bar{V})^2 \chi_o^2(f_o) S_v \)

The expression for Eqn. (14) can then be applied over the frequency band encompassing the resonant peak in the response for the purpose of fitting \( f_o \) and \( \zeta \) to the observed \( S_x^2(f) \).

### 2.4 Estimating Dynamic Characteristics \((f_o \text{ and } \zeta)\) from “Pluck Tests”

Response of the SDOF oscillator of Eqn. (1) to a pluck test of initial amplitude \( a \), becomes:

\[ x(t) = a e^{-2\pi f_o t} \left( \cos(2\pi f_o t) + \frac{\zeta}{\sqrt{1 - \zeta^2}} \sin(2\pi f_o t) \right) \] for small \( \zeta \)

Equation (15) can be fitted to the observed time history of response (whether from an accelerometer or from the base bending moment) for a pluck test to extract parameters \( a, f_o \) and \( \zeta \) in still conditions, which would exclude the presence of aerodynamic damping contributions.
3. APPLICATION OF SPECTRAL FITTING MODELS TO TWO URBAN TREES

3.1 She-oak on Main Campus at The University of Melbourne

Figure 1(a) below depicts the 23m tall She-oak outside of the Civil and Environmental Engineering (CEE) building that was instrumented using an tri-axial accelerometer data-logging package strapped to the trunk at approximately 21m from ground level (Fig. 1(b)), on the 2nd April, 2008. On this day, Melbourne city experienced 1 in 20 year return period high winds for most of the late morning and into the evening. Over 4½ hours of tri-axial acceleration response recordings from approximately 3:20 pm to ~8 pm were recorded in a continuous file at a 64Hz sampling rate using this logger pack. Figure 1(c) depicts a portion of record in the early part of the data capture where an impact event of the tree on the building was observed. (Individual trees in the row of She-oaks outside the CEE building are known to have frequent audible collision events with the parapet wall at the top of the building in high winds, as is evidenced by clearly visible scaring of their trunks at that level).

![Figure 1(a) She-oak Instrumented outside Civil Engineering Building at Melbourne University](image)

![Figure 1(b) Tri-axial Accelerometer Data-logging pack](image)

![Figure 1(c) Sample response plot in high wind](image)

3.1.1 Analysis of damping properties from response records under wind excitation

The single file of ~ 4.5 hours of useful recorded acceleration response records was split into 18 separate contiguous files of 960 second (16 minute) long records (ie 61,440 data points per record). These files were treated as 15 contiguous records of 4096 data points (64 seconds
long each at the 64Hz sampling rate) for the purpose of evaluating ensemble averaged acceleration response spectra.

Figure 2 depicts a typical curve-fit using the procedure of Eqn (14) by way of illustration of this fit against the ensemble-averaged spectral data using both a linear-linear and a log-log plot to emphasize some of the detail. It is clear that the fit is quite good over the resonant peak (important to the estimation of $f_o$ and $\zeta$) and also the “tail” of the spectrum, at least to a 2Hz frequency beyond which the response spectrum appears to decay at a faster rate than the fit, possibly due to the influence of Aerodynamic Admittance (here treated as being unity).

![Figure 2. Fit to Acceleration Response Spectrum ((a) Linear-linear and (b) log-log plots)](image)

Figure 3 depicts the damping levels obtained using the curve-fitting procedure of Eqn (14) to fit to the sequence of ensemble averaged acceleration response records obtained at 16-minute intervals as a function of time over the recorded period. Also shown on the same plot, are the reported mean wind speed values at Melbourne in kph against time, and the mean value of critical damping ratio for this tree obtained from pluck tests performed on a still day later that month (30th April, 2008). It is evident from these time variations that there is a reasonable correlation of the damping value with mean wind speed, so that values of $\zeta$ were re-plotted against interpolated values of wind speed from Fig. 3 to produce Fig. 4.

Also plotted in Fig. 4 is the straight line variation of critical damping ratio implied by the theoretical model for total critical damping ratio given by:

$$\zeta_{tot} = \frac{c + c_a}{2 \sqrt{km}} = \frac{c + c_a}{2k} \omega_o = \zeta_{V=0} + \frac{c a \omega_o}{k} \sqrt{V}$$

in which $\omega_o = \sqrt{k/m} = 2\pi f_o$.

Now for $k = 900$ N/m (determined by rope pull tests using a load cell and applying a load at that level on the tree), at the height of the instrumentation (21 m) and allowing for the centre of tree exposed area (estimated at 25 m$^2$) to be ~10m from the ground in Category 3 conditions, converting to wind speed in kph from m/s units (conversion factor 1 kph $\approx 0.278$ m/s) yields the straight line variation depicted in Fig. 4 for a $C_D$ value of 0.8 and a $\zeta_{V=0}$ value corresponding to the pluck test mean in still conditions of 7.8%.

It can be observed in Fig. 4 that this straight line variation against mean wind speed appears to model the aerodynamic damping contribution reasonably well in this wind speed range.
3.1.2 Analysis of damping properties from pluck tests in still wind

Figure 5 depicts the fitted segments from a typical pluck test using data captured from the Z axis in the acceleration response measurements and from the trunk bending moment as measured by the N-S strain meter near the base of the tree, respectively. The agreement between the two is considered to be rather good, both for frequency and damping estimates. Typical $\zeta$ obtained values from these alternative measurements corresponded rather closely one to the other from repeat test record to repeat test record ranging from 5.9% – 9.9% and averaging 7.9% in the case of the accelerometer measurements and ranging from 5.2% – 12.1% and averaging 7.6% in the case of the strain meter measurements.

It should be noted that the natural frequency obtained from the fitting procedure was consistently stable at $0.25 \pm 0.01$ Hz using records from either of the two measurement systems.
3.2 Spotted Gum on Main Campus at Monash University

Figure 6(a) depicts the approximately 24m tall spotted gum outside the administration building on the Clayton campus of Monash University whilst Fig. 6(b) depicts a view of the orthogonally disposed strain meters positioned on the trunk approximately 1.2m above ground level, used to monitor the response of the tree at a 20Hz sampling rate on the 2nd April, 2008 and on several other windy days. The instrumentation is set to record continuously for a designated period once a threshold level is exceeded, which level and recording length can be adjusted by the user.

Data from the strain meter orthogonal to the building orientation was analysed for its spectral variation in a similar way to that of the accelerometer response data of the She-oak on the University of Melbourne campus. The spectra were ensemble averaged in batches of eight from contiguous response record segments of 4096 data points (204.8 secs each) but only strong wind data was recorded for the 2nd April wind conditions. Consequently data from an-
other windy day on the 18th March, 2008 was also analysed in this way to extend the wind speed range investigated. A plot of the damping versus mean wind speed values at a height $h_m$ corresponding to the centroid location of the canopy (taken to be 19m) was then produced as depicted in Fig. 6. Also shown on this plot is the straight line variation for aerodynamic damping inferred using identical conditions as for the She-oak but with a modified value for the canopy area of 150 m$^2$. Unfortunately pluck tests to enhance damping estimates in still air conditions, (mean wind = 0) have yet to be performed on this tree. Consequently this straight line has been positioned by eye so that at least the low wind results for critical damping $\zeta$ straddle this line reasonably well.

Whilst this line is indeed a good fit to the damping results for low wind speeds (say less than about 15 kph), the observed damping levels appear to progressively taper off with higher wind speeds, so that the dotted curve appears as a better overall fit. This would imply a systematic reduction in aerodynamic damping with increasing wind speed which could be accounted for by a systematic reduction in canopy area with wind speed for this type of tree, ie a reduction in the $\alpha$ value with wind speed in the model of Eqn. (16).

4. CONCLUDING REMARKS

An accelerometer datalogger pack and purpose-built strain meters have been respectively used to obtain response measurements under strong wind conditions of two mature urban trees located on the respective main campuses of the University of Melbourne (a She-oak) and Monash University (a Spotted gum). In addition, pluck tests have been conducted under still wind conditions using both of these instrumentation systems on the She-oak at the University of Melbourne.

Results obtained provide clear evidence of a wind-speed dependent aerodynamic damping contribution to overall damping in the response of these trees consistent with a relative flow assumption. In the case of the Spotted gum, a progressive departure from linear aerodynamic damping was observed in the damping estimates with increasing wind speed which could be logically attributed to a reduction in canopy area of this tree with these conditions.

More systematic testing of the dynamic response of a range of urban tree types in strong wind conditions in conjunction with the performance of pluck tests in still air conditions on these trees would be necessary to improve on these limited observations and before any firm conclusions can be drawn on the complex damping characteristics of urban trees.

![Figure 6. Variation of critical damping ratio with wind speed for Spotted Gum (Monash University)](image-url)
REFERENCES


