# Concrete damage assessment for blast load using pressure-impulse diagrams

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## *Abstract*

The duration of blast pressure is significantly important along with its magnitude for dynamic response of concrete elements. Pressure-Impulse (P-I) diagrams which include both blast pressure magnitude and duration are often used in concrete damage assessment. Available design guidelines and manuals for protective design are mostly based on the Single degree of freedom (SDOF) approach. Types of resistance function used in SDOF analysis of concrete elements influence the ultimate response and eventually the amount of blast damage. Representation of concrete damage, relating only to the blast pressure magnitude and the over simplification of resistance function, can sometimes be misleading in obtaining the structural responses. This paper explores different methods of obtaining P-I diagrams using SDOF model. Development of nonlinear resistance function using nonlinear material models has also been discussed. Both bilinear and nonlinear resistance functions have been used in SDOF analysis to obtain the P-I diagrams to correlate the blast pressure and the corresponding concrete flexural damage. Realistic combination of pressures and impulses were chosen during analysis to simulate the effect of both the near and far-field blast scenarios. Variation in the post peak response of SDOF models due to use of simplified resistance function has also been presented. Field test result was compared to the analytical result to assess the effectiveness of P-I diagrams in blast damage assessment.

Keywords: SDOF, Pressure-Impulse (P-I) diagrams, Blast load, Concrete damage

## *Introduction*

Single degree of freedom (SDOF) models have been widely used for predicting dynamic response of concrete structures subjected to blast and impact loading. The popularity of the SDOF method in blast-resistant design lies in its simplicity and cost-effective approach that requires limited input data and less computational effort. SDOF model gives reasonable good results if the response mode shape is representative of the real behavior. Accuracy of the dynamic response calculations significantly depends on whether the adopted resistance function resembles the actual hysteretic behavior of the structure. Explosion is an extreme event with a low probability of occurrence, design guidelines and manuals (Task Committee on Blast Resistant Design, 1997, TM 5-1300, 1990, Task Committee, 1999) often use over simplified elastic-perfectly plastic resistance functions to obtain the response of concrete elements. Simplified elastic-perfectly plastic resistance function for concrete elements ignores the nonlinear behavior of concrete. Elastic-plastic-hardening and elastic-plastic-softening resistance functions can model the nonlinear behavior of concrete better than the elastic-perfectly plastic resistance function. The pressure-impulse (P-I) diagrams or isodamage curves are used to correlate the blast load to the corresponding damage where the flexural mode of failure dominates damage of the element. These diagrams incorporate both the magnitude and duration of blast loading to correlate blast load and corresponding damage which can be readily used for quick damage assessment of concrete structures under different blast scenarios.

In this paper different methods of deriving the P-I diagrams for blast damage assessment using the SDOF model are discussed. Comparison between the P-I diagrams obtained using nonlinear resistance functions and elastic-perfectly plastic resistance functions are shown. Use of elastic-plastic-hardening and elastic plastic softening resistance functions in SDOF analysis for obtaining the P-I diagrams is discussed. Post peak response of high ductility concrete elements is also important for blast damage assessment. Variation in post peak response due to the use of simplified resistance functions instead of nonlinear resistance function is presented.

## SDOF analysis models for blast design

Protective design manuals (eg.USACE manuals), which are in the public domain, use two common SDOF modeling approaches: the modal method and the equivalent SDOF method. In the modal method, the elastic forced response of a member is approximated by the first mode of free vibration. The natural period of the SDOF model is taken as the period of the first mode of vibration of the element with distributed mass. The main drawback of this method is its lack of versatility as it can only be used with the aid of charts and can not be used for obtaining generalized numerical solutions of SDOF systems involving complex loading histories and resistance functions. In the Equivalent SDOF method, before analyzing the response of a structural element with distributed mass and loading; the mass, resistance and loading are replaced in Newton's equation of motion with the equivalent values for a lumped mass-spring system. The equivalency is based on energy; with the equivalent mass calculated using principles of equal kinetic energy; the equivalent resistance having equal internal strain energy and the equivalent loading having equal external work to the distributed system. The transformation factors that are applied to the distributed values for calculating the equivalent lumped mass values are functions of the distribution of mass and loading over the element and the shape function of the deflected shape (Morision, 2006). The equivalent SDOF method is widely used in protective design practices. This method is also the basis of factors available in Biggs (1964) and TM5-1300 for SDOF calculation for dynamic response. Recent publication "Explosion-Resistant Buildings by Bangash and Bangash also referred to this method for protective design (Bangash and Bangash, 2006).

## Resistance functions

The resistance of concrete elements under blast load is highly nonlinear. In practice, an idealised resistance function(R-Δ) is used which is a prediction of the resistance that the element would offer in a quasi-static test. Bilinear elastic-perfectly plastic R-Δ determination ignores some nonlinear effects such as softening due to cracking, tension stiffening effect, initial yielding and strain hardening of reinforcing steel. In the attempt to develop the Nonlinear R-Δ relationship including all these effects, fully nonlinear stress-strain relationship of concrete and steel are used for the analysis. The high strain-rate effect on materials is taken into account by applying DIF factors whilst Bond-slip is considered through the tension stiffening effect. Typical resistance functions are simplified by the bi- or tri-linear curves as show in Fig.1.

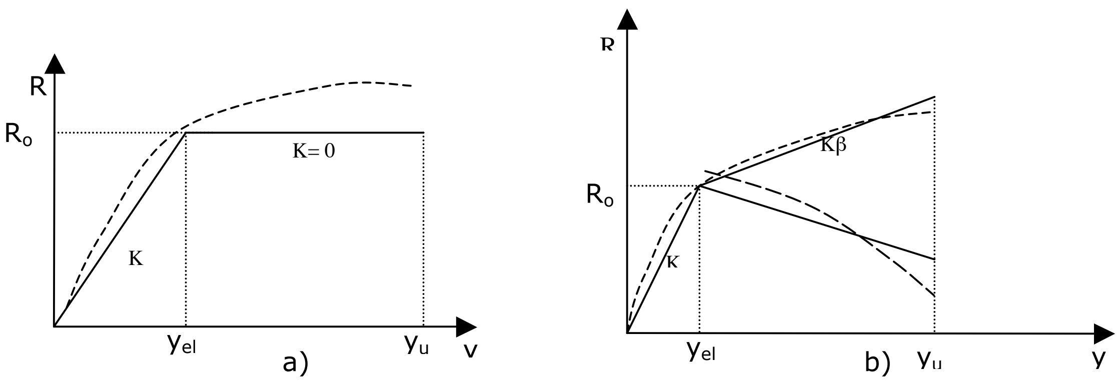


Fig.1 Nonlinear resistance functions and there idealization: a) elastic-perfectly plastic b) elastic-plastic-hardening/softening

## Bilinear resistance function

The bilinear elastic-perfectly plastic resistance function has been well-defined in many materials, such as in those by Biggs(1964), Task Committee on Blast Resistant Design(1997). In obtaining bilinear resistance function, resistance, $R_o$ of a structure is chosen as the smaller value of $R_b$ and $R_s$. Where $R_b$ is the bending resistance and $R_s$ is the shear resistance. The bending resistance behavior of an element can be expressed as a function of the ultimate moment capacity, Mp and the length of the element. The effective stiffness, $K_E$, is dependent on whether shear deformation is included or not. If only flexural deformation is under considerations, $K_E$ can be calculated as follows for the condition of uniform loading.

$$K_E = \frac{384 EI_e}{5L^3} \qquad \text{......(1)}$$

where E is the Young's modulus of concrete, $I_e$ the equivalent moment of inertia, and L is the length of the element.

Krauthammer et al. (1986), from experimental observations, argued that for a SDOF system analysis for blast loading, the flexural and shear effects on the elements can be uncoupled and analysed as independent of each other. When the observed failure mode is the direct shear, sufficient time is not allowed for the specimen to develop any type of flexural response. Similarly, when flexural failures occur, the failure is usually controlled by the fracture of the reinforcing bars which occurs much later than when the slab exhibits significant shear deformation. In the development of the resistance-deflection function presented hereafter, only the flexural effect is considered in the modelling.

The ultimate resistance, $R_o$ of the Elastic-perfectly-plastic model for uniformly distributed blast loading has been calculated from the plastic moment capacity of the concrete section using the following equation.

$$R_o = \frac{8M_{pc}}{L} \qquad \text{.....(2)}$$

where $M_{pc}$ is the plastic moment capacity calculated using modular ratio theory and L is the member length.

## Nonlinear resistance function development

The deflection can be obtained by a double integration of the curvature, $\varphi_{(x)}$ along an element as shown by equation (3).

$$\Delta_{(x)} = \iint \varphi_{(x)} dx dx \qquad \text{......(3)}$$

In many cases the variation of curvature cannot be expressed as a continuous relationship (Warner et al., 1998). Hence, the deflection must be calculated from the curvatures using numerical methods. With known material parameters, a theoretical moment-curvature curve model for the section has been derived using fiber sectional method for concrete element. For a given concrete strain in the extreme compression fiber, $\varepsilon_c$, and neutral axis depth, $d_n$, the steel strains $\varepsilon_s$, and $\varepsilon'_s$ was determined from the properties of similar triangles in the strain diagram. The stresses $f_s$ and $f'_s$, corresponding to the strain $\varepsilon_s$ and $\varepsilon'_s$, were obtained from the stress-strain curves. Then, the reinforcing steel forces, T, T', may be calculated from the steel stresses and areas. The distribution of concrete stress, over the compressed and tensioned parts of the cross-section, was obtained from the concrete stress-strain curves. For any given extreme compression fiber concrete strain, $\varepsilon_c$, the resultant concrete compression and tension forces, C and C', were determined by numerically integrating the stresses over their respective areas. In order to do so, the cross-section was divided into rectangular trips along its height. The concrete stress at the middle of each trip was calculated based on considerations of the strain. The stress so obtained was multiplied by the area of the strip to derive the force.

The position of these forces is measured from the extreme compressive fiber. They are calculated by applying the method of geometry with respect to the stress diagram of concrete in the cross-section. The complete moment-curvature relationship was determined by incrementally adjusting the concrete strain, $\varepsilon_c$, at the extreme compression fiber.

### Material models and strain-rate effect

The theoretical derivation of moment-curvature relationship was based on the concrete model by Hognestad (1951)and the reinforcement model by Park and Paulay (1975). The formula to calculate elastic modulus of concrete, $E_c$, specified in ACI 318-95 (1995), is adopted in the study. When carrying out the SDOF system analysis, the effect of high strain-rate is taken into account through the use of the DIF factors as proposed in ASCE task committee report (Task Committee on Blast Resistant Design, 1997). The values of the ultimate and rupture strength of concrete, and the yield and ultimate strength of steel were multiplied by the corresponding DIF factors.

## *Pressure-impulse diagrams*

P-I diagrams are isodamage curves based on the maximum deflection criteria as represented in the space of pressure and impulse of the pulse loading (Li and Meng, 2002a).These curves are equal energy curves which predicts the degree of damage as a function of the physical parameters. These curves are similar to the characteristic curves suggested by Abrahamson and Lindberg (1976). Vaziri et al (1987) produced isoresponse curves which are similar to the characteristics curves. Mays and Smith (1995) and Krauthammer (1998) used P-I diagrams based on elastic SDOF model for damage assessment. P-I diagrams are generally load-shape dependent but Youngdahl (1970) introduced two effective loading parameters in order to omit the load-shape effect on structures of rigid-plastic material and Li and Meng (2002b) extended that work to eliminate pulse load shape effect in both the elastic and elastic-plastic structures. Li and Meng (2002a) also studied P-I diagrams of a SDOF model using dimensional analysis and concluded that P-I diagrams for an elastic system is unique in nature and can be derived from dimensionless parameters as shown equations 4a and b.

$$i = \frac{I}{y_m\sqrt{KM}} \qquad \text{.......(4a)}$$

$$p = \frac{F_m}{Ky_m} \qquad \text{.......(4b)}$$

where, i and p are scaled impulse and scaled pressure. I is the total impulse, $y_m$ maximum structural deflection, K and M are elastic stiffness and lump mass of the SDOF system. $F_m$ is the maximum force on the system.

The calculated value, from equation (4a) for any given elastic SDOF system under a specified blast load, gives the value of the impulsive asymptote on the P-I plot. Similarly, the value calculated from equation (4b) gives the value of the quasi-static asymptote. Using these two values a p-I curve can be plotted for a specific damage level.

In a recent publication, Fallah and Louca (2006) introduced ways of deriving the P-I diagrams using elastic-plastic-hardening and elastic-plastic-softening resistance functions under explosive loading. Some dimensionless parameters to establish the analytical models for elastic-plastic-hardening and elastic-plastic-softening SDOF systems have also been proposed. The following equations to derive quasi-static and impulsive asymptote respectively have been proposed.

Quasi-static asymptote: $$\frac{F_m}{Ky_m} = \alpha(1-\theta\psi^2) + \frac{\theta}{2}(\psi^2 - \theta\alpha^2 + \alpha^2\psi^2) \qquad \text{......(5a)}$$

Impulsive asymptote: $$\frac{I}{y_m\sqrt{KM}} = \sqrt{2\alpha(1-\theta\psi^2)+\theta(\psi^2-\theta\alpha^2+\alpha^2\psi^2)} \qquad .......(5b)$$

where $F_m$, K, $y_m$, and I are as defined when introducing equation (4).

$\alpha$, $\psi$ and $\theta$ are dimensionless parameters, defined as $\alpha = \frac{y_{el}}{y_c}, \psi^2 = \frac{K\beta}{K}, \theta$ = +1 for elastic-plastic hardening and -1 for elastic-plastic softening

The P-I diagrams of an element subjected to blast loading was established based on certain discrete points. These points are determined by repeated analysis of the equivalent SDOF system. Each point represents the limit state of the structure with respect to the specified damage criterion, which is defined by the ratio of given deflection, $\Delta$, and the span of the element, L. At each point, the time duration td of an idealized triangular blast pressure is firstly determined. Blast pressure is then increased from zero to the ultimate value P, until the maximum response of the equivalent SDOF system reaches the given deflection value, according to the specified damage criterion. Impulse is the area under the pressure-time curve. The time duration td ranges from 5ms to 100ms with 5ms increments. In the present study, quasi-static and impulsive asymptotes were also calculated for elastic-plastic-hardening SDOF model using equations given in Fallah and Louca (2006). Damage criteria for deriving P-I diagrams are based on the damage criteria given in technical manual TM5-1300. Support rotation of simply-supported members has been taken to define damage. Damages have been classified into light, moderate and severe. Support rotation of $2^\circ$ causes light damage, $5^\circ$ support rotation causes moderate damage and $12^\circ$ rotation causes severe damage.

## *Analysis and results*

A normal strength concrete panel and a singly reinforced beam have been used for developing nonlinear and bilinear resistance functions and to obtain the pressure-impulse diagrams for different blast pressure-impulse combinations. The panel, modeled here, was placed under open-air blast trial conducted in Woomera, South Australia in 2004. Details of the panel and other test data can be found in Ngo (2005). Dimensions (in mm) and the properties of the panel and the beam are given in Fig.2 and in Table 1.

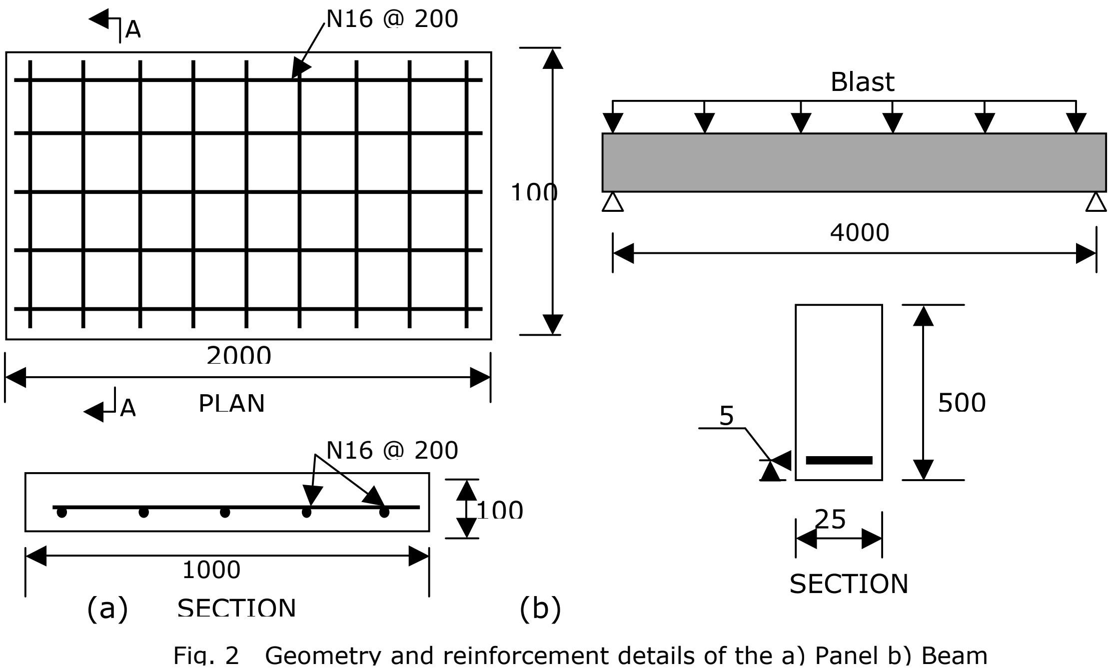


Fig. 2 Geometry and reinforcement details of the a) Panel b) Beam

In the present study both the panel and the beam was analyzed with different reinforcement ratios keeping the physical dimensions the same to obtain resistance functions for different shapes. Different pressure-impulse combinations have also been applied to the panel to get the pressure-impulse points for both the near and far-field conditions. In the present analysis, in-house computer codes have been used for developing the nonlinear resistance function from moment-curvature results by numerical integration techniques. The repeated analysis of equivalent SDOF systems under different combinations of pressure and impulse has also been conducted using the same computer codes. These codes were developed and verified as part of research work at the University of Melbourne (Thuong, 2006).

Table 1- Material properties of the concrete panel and beam

| Properties | Notation | Panel material | Beam material |
|---|---|---|---|
| Static compressive strength of concrete | $f'_{cs}$ | 39.8 MPa | 40 MPa |
| Static tensile strength of concrete | $f_t$ | 3.7 MPa | 3.5 MPa |
| Static elastic modulus of concrete | $E_{cs}$ | 31.5 GPa | 32 GPa |
| Density of concrete | $\rho$ | 2430 kg/m3 | 2400 kg/m3 |
| Static yield strength of steel | $f_y$ | 575 MPa | 400 MPa |
| Elastic modulus of steel reinforcement | $E_s$ | 201 GPa | 200 GPa |


Fig 3. NSC panel after explosion (after Ngo 2005)

The structural response of the panel with different reinforcement ratios and blast loading are shown in Fig.4. Use of bilinear resistance function produces higher panel response under different blast loading. Increased value of the reinforcement ratio reduces the variation in shape of the bilinear and nonlinear functions thus reducing the variation of the peak response of the structural member. Generally, the responses of both the beam and the panel are fairly close in terms of response time. The peak deflection from the nonlinear R-Δ model appears to be smaller than that from the elastic-perfectly plastic R-Δ model. Post-yield behavior of both the beam and the panel under different blast loading was greatly influenced by the shape of the resistance function.

As the R-Δ function influences the peak response of a SDOF system subjected to blast loading, it has a significant effect on the P-I diagram which is related to the maximum response of the system. P-I obtained for different scenarios have been given in Fig.5 to7. The variation in P-I diagrams due to different damage levels are more prominent in the region where damage is dominated by pressure rather than the impulse value.

In the field test, the panel was placed 40m from the ground zero or the centre of blast. The panel experienced a peak reflected pressure of 735kPa with a duration of 33ms. Panel failure due to concrete breach was observed.

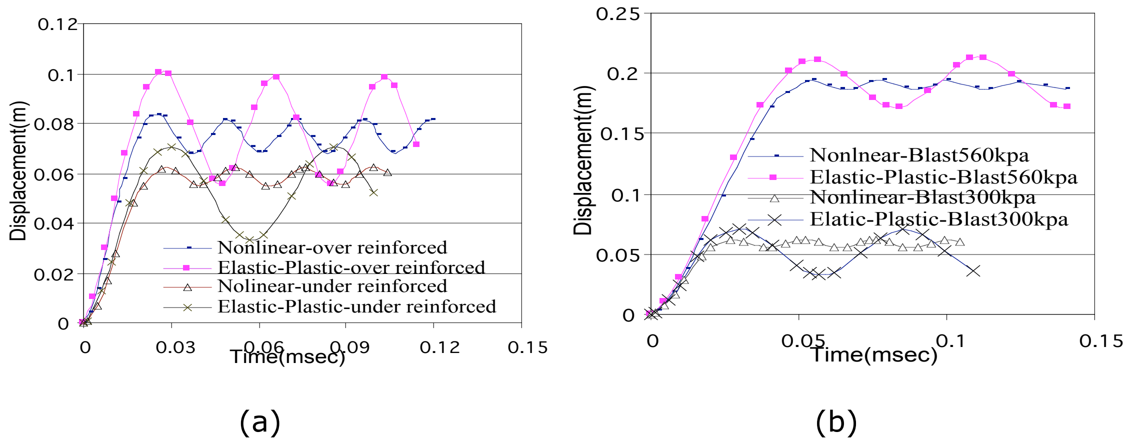


Fig.4 Time history response of the panel a) with different reinforcement ratios b) under different blast loads.

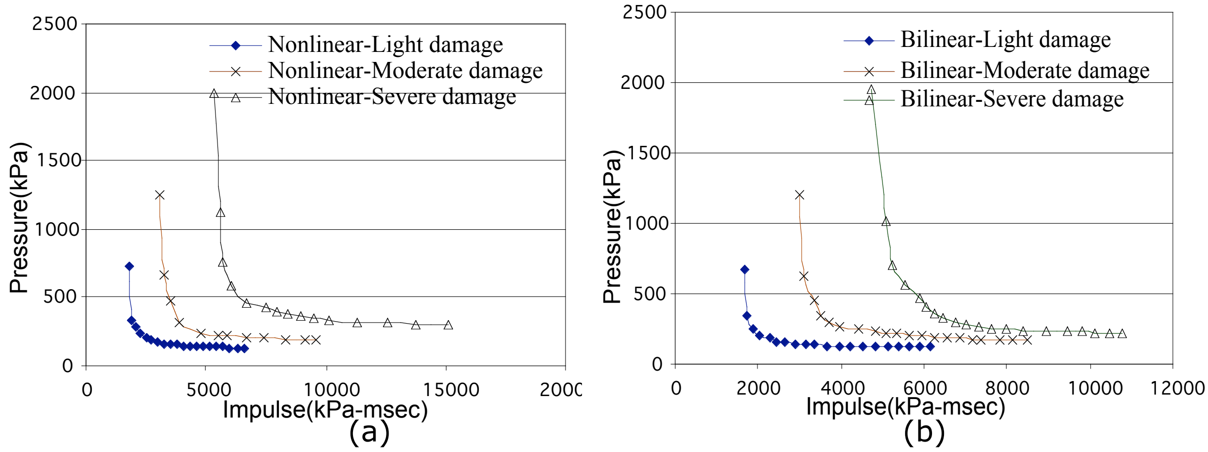


Fig. 5 P-I diagrams of the panel with 1% reinforcement for different damage levels using a) Nonlinear b) Bilinear resistance functions

FE analysis of the panel under similar pressure-impulse to the blast trial gives a peak inward deflection of 186mm at time t = 28.9msec. In the field test a permanent deformation of 142mm was measured along with an approximately 8-mm wide crack at the mid-span. The failure mode of the panel is shown in Fig.3. From P-I diagrams in Fig.5 shows that the combination of pressure and impulse experienced by the panel in the field trial falls well above and right side of the P-I curve obtained for severe damage condition. So the P-I curve also predicts severe damage of the panel. It also can be seen from the P-I curve that a blast load of same magnitude of 735kPa is not sufficient to cause serious damage if the duration of the pressure is much lower than the field value, i e 10ms. Peak response of concrete panel obtained by SDOF analysis with nonlinear resistance function gives close results to the experiment values. Peak response obtained using nonlinear resistance function gives nearly 10% higher value than that obtained by using bilinear function. The strain energies dissipated into the system are different when the bilinear or nonlinear R-Δ relation is incorporated into the SDOF system. Bilinear R-Δ functions found to have under estimated the capacity of concrete element by exhibiting higher peak response than nonlinear resistance when subjected to the same blast.

Figure 6 shows that the parameters given in equation (5a) and (5b) gives P-I asymptotes close to that obtained using bilinear resistance function. If the response of a structural system under blast pressure is not in the dynamic regime, then the asymptotes (by equation 5a and b) can be used for quick estimate of damage under a given combination of pressure and impulse. Higher amount of reinforcement in concrete member causes stiffer structure and produce close bilinear and nonlinear resistance functions. Fig.7a shows that for stiffer structure whose idealized resistance function is close to the real resistance function, produced less variation in the P-I diagrams.

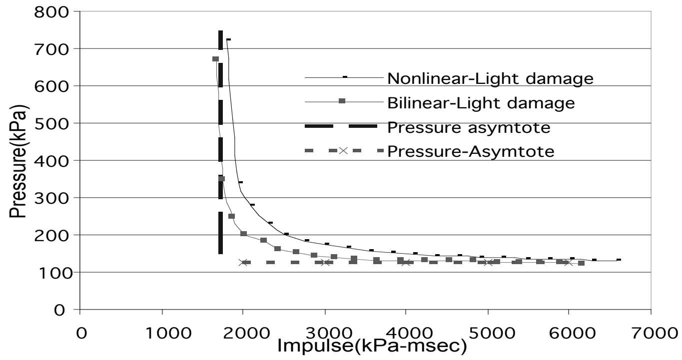


Fig.6 P-I diagrams using nonlinear and bilinear resistance functions along with pressure and impulse asymptotes obtained using method proposed by Fallah and Louca(2006)

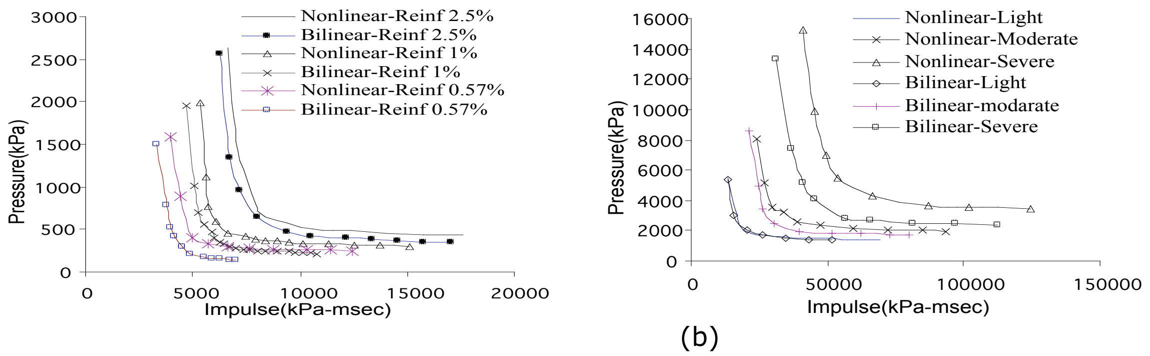


Fig. 7 P-I diagrams: a) for the panel with different reinforcement ratios for severe damage level b) for the beam for different damage levels

P-I diagrams for light damage are less sensitive to the choice of the nonlinear or idealised resistance functions. Fig.7b shows that variations in P-I diagrams are significantly higher for case of severe damage. For lightly reinforced structures where the plastic part of the response is dominant, the sensitivity to the difference in shape of the nonlinear (or bilinear) resistance functions could be significant. Hence, there is a higher variation in the P-I diagrams.

## *Conclusion*

P-I diagram is a useful tool for the preliminary (or fast-track) damage assessment of concrete elements subjected to blast loading. Steps to develop nonlinear resistance function using nonlinear material models have been discussed. Difference in response of SDOF system to the nonlinear, or bilinear, R-$\Delta$ models has been discussed. The post peak response behavior is found to be significantly different when bilinear resistance function is used instead of the full nonlinear resistance function. Use of bilinear resistance function in dynamic analysis of SDOF model found to produce higher peak response which causes variation in the P-I diagrams obtained using those peak responses. Dynamic response of SDOF system also significantly depends on the structural characteristics and loading parameters. For severe damage conditions, the variation in the P-I curves derived using nonlinear and bilinear functions are significantly different. Use of simplified elastic-perfectly plastic model can be misleading in damage estimates when the load is expected to produce severe deformation of the member. The variation in the amount of strain energy level associated with an equivalent bilinear resistance function can cause significant difference in the P-I diagrams. Nonlinear R-$\Delta$ model can help establish a better P-I diagram than the common Bilinear R-$\Delta$ model, especially in the case of high level damage criterion.

## References


Abrahamson, G. R. and Lindberg, H. E. (1976) 'Peak Load-Impulse Characterization of Critical Pulse Loads in Structural Dynamics', Nuclear Engineering and Design, 37, 35-46.

Bangash, M. Y. H. and Bangash, T. (2006) Explosion-Resistant Buildings-Design,Analysis and Case Studies, Springer, Berlin.

Biggs, J. M. (1964) Introduction to Structural Dynamics, McGraw-Hill, New York.

Fallah, A. S. and Louca, L. A. (2006) 'Pressure-impulse diagrams for elastic-plastic-hardening and softening single-degree-of-freedom models subjected to blast loading', International Journal of Impact Engineering.

Hognestad, E. (1951) 'A Study of Combined Bending and Axial Loads in Reinforced Concrete Members',In Bulletin Series No. 399University of Illinois Engineering Experimental Station.

Krauthammer, T. (1998) 'Blast mitigation technologies: developments and numerical considerations for behaviour assessment and design',In Proceedings of International conference on Structures under shock and Impact, Vol. SUSI Computational Mechanics Inc., pp. 3-12.

Krauthammer, T., Bazeos, N. and Holmquist, T. J. (1986) 'Modified SDOF Analysis of R.C Box Type Structures', Journal of Structural Engineering, 112, 726-744.

Li, Q. M. and Meng, H. (2002a) 'Pressure-Impulse Diagram for Blast Loads Based on Dimensional Analysis and Single-Degree-of-Freedom Model', Journal of Engineering Mechanics, 128.

Li, Q. M. and Meng, H. (2002b) 'Pulse loading shape effects on pressure-impulse diagram of an elastic-plastic, single-degree-of-freedom structural model', International Journal of Mechanical Sciences, 44, 1985-1998.

Mays, G. and Smith, P. D. (1995) Blast effects on buildings : design of buildings to optimize resistance to blast loading, T. Telford, London.

Morision, C. M. (2006) 'Dynamic response of walls and slabs by single-degree-of-freedom analysis-a critical review and revision', International journal of Impact Engineering, 32, 1214-1247.

Ngo, T. D. (2005) 'Behaviour of high strength concrete subject to impulsive loading',PhD thesis, In Department of Civil and Environmental Engg The University of Melbourne,

Park, R. and Paulay, T. (1975) Reinforced Concrete Structures, John Wiley & Sons, New York.

Task Committee (1999) 'Structural Design for Physical Security: State of the Practice',In American Society of Civil Engineers.

Task Committee on Blast Resistant Design (1997) 'Design of Blast Resistant Buildings in Petrochemical Facilities',In Petrochemical Committee of the Energy Division, ASCE.

Thuong, V. X. (2006) 'Nonlinear SDOF System Analysis of Elements Sublected to Blast Loading',Maters Thesis, In Dept of Civil and Environmental Engg The University of Melbourne,

TM 5-1300 (1990) 'The Design of Structures to Resist the Effects of Accidental Explosions',In Technical Manual Army TM 5-1300/Navy NAVFAC P-397/Air Force AFR 88-22Department of the Army, Navy, and Air Force, Washington DC.

Vaziri, R., Olson, M. D. and Anderson, D. L. (1987) 'Dynamic Response of Axially Constrained Plastic Beams to Blast Loads', International Journal of Solids and Structures, 23, 153-174.

Warner, R. F., Rangan, B. V., Hall, A. S. and Faulkes, K. A. (1998) Concrete Structures, Addison Wesley Longman Pty Limited, Australia.

Youngdahl, C. K. (1970) 'Correlation Parameters for Eliminating the Effect of Pulse Shape on Dynamic Plastic Deformation', Journal of Applied Mechanics, ASME, 37, 744-752.