CORE COUPLING-BEAMS IN TALL BUILDINGS

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ABSTRACT

Cores in tall buildings subdivide and fire-separate vertical riser services: lifts, stairs, air-conditioning, utilities and rooms requiring access to plumbing. This paper addresses the structural design of load-bearing concrete cores as distinct from skeletal framed cores fire-protected by gypsum plasterboard. Concrete core-walls may be more resistant to abnormal events such as terrorist attack and also provide more damping. A decision for a concrete core leaves open the decision to use concrete or structural steel for floor and façade framing.

Concrete core-walls are penetrated by vertical families of openings for doors to stairs, lifts and other spaces. These openings separate the core as a whole into a number of sub-cores linked by coupling-beams being the residual strips of concrete core-wall above and below openings.

Coupling-beams can be thought of as large-scale shear connectors providing composite action between distinct sub-cores. First yield will often occur in coupling-beams and spread vertically. The span/depth ratio of these beams is determined by non-structural considerations and usually well into the ‘deep beam’ range prone to brittle behavior.

New Zealand and American seismic design practice, based on experimental research at the University of Canterbury, requires complete ‘X’-reinforcement consisting of tied rebar cages (as for columns) on both diagonals of coupling beams. X-reinforcement can definitely improve the ductility of otherwise brittle elements. It does, however, create practical construction problems: ties are closely spaced (for Bauschinger buckling) and the crossing X-cages together with ‘basketing’ reinforcement can amount to 6 to 8 or 10 layers of reinforcement across the thickness of a wall which may also require 40 mm cover for fire-rating. 250 walls are barely possible and 300 walls present difficulties.

For regions of lower earthquake-risk, such as Australia, one wonders whether there are reasonable alternatives to X-reinforcement. Australia uses advanced climbing formwork systems which do not easily combine with X-reinforcement.

Theoretical understanding of coupling-beams has been impeded because the usual lateral-load collapse mechanism for normal slender beams in moment-resisting frames is inappropriate for coupling-beams. Indeed the usual mechanism predicts compression yield of the main top and bottom flange rebars at ‘compression’ corners where tests often show horizontal tensile strains.

This paper will describe rigid-plastic plane-stress mechanisms relevant to the analysis of coupling-beams and consider just horizontal/vertical reinforcement.

INTRODUCTION

Fig 1 shows the skeletal ‘weak-beams/strong-columns’ mechanism usually used for the design of moment-resisting seismic frames. Columns rotate about plastic hinges at the base of each column. Beams remain straight and horizontal by rotating through equal but opposite angles at plastic hinges at each column face.

This skeletal mechanism provides a straightforward analysis for bending strength and that analysis can easily be generalized to include the effect of coincident gravity load when the sagging hinge may move from the column face towards mid-span.

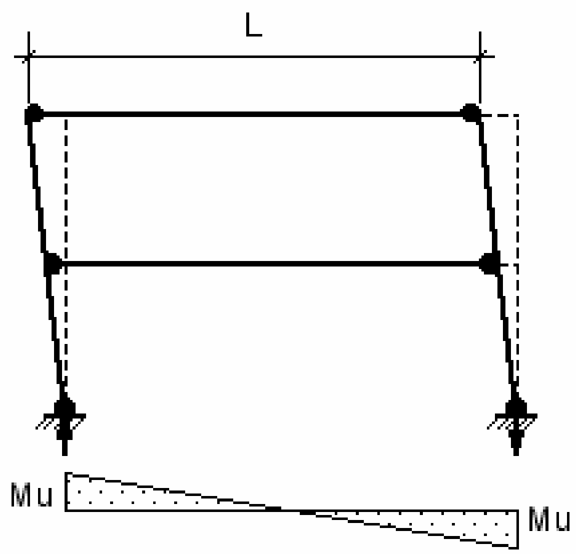

Fig 1 Skeletal mechanism for moment-resisting frames

Alas design against premature brittle failure in shear inherits the empirical state of the art for general (non-seismic) concrete design in shear which has long been unsatisfactory. There are also the inherent limiting assumptions of skeletal theories:

- Plane sections remain plane and
- Cross-section dimensions are ‘small’ compared to the span.

These limitations preclude any simple skeletal approach to concrete beams in shear generally and more so in the case of core coupling-beams.

Coupling-beams typically have clear spans *L* of about 1000 mm (Fig 2) although openings of 2000 to 3000 are occasionally required for goods lifts. Door-head heights are usually slightly less than 2000 leaving coupling-beam depths *D* around 600 to 1800. Rebar content varies from minimal to heavy depending on lateral load demands.

ACI 318-05 c21.7.7 considers the skeletal mechanism of Fig 1 inappropriate for span/depth (L/D) ratios < 4 hence inappropriate for most coupling-beams.

If skeletal theories are inadequate then the next simplest theoretical approach would seem to be simple rigid-plastic plane-stress (two-dimensions 2D in-plane) analysis. See ‘Acknowledgements’ below.

A recent report (Collins et al 2007) on long-term research into shear in concrete at the University of Toronto, Canada, describes ‘an adequate theory for shear strength’ and notes that a minimum content of closely spaced (200 – 300 mm) mid-depth horizontal web-reinforcement is sufficient to suppress the ‘depth effect’ which otherwise reduces the shear strength of deep concrete beams.

One notes that the mid-depth half of a coupling-beam is always in horizontal tension over the full span and beyond regardless of the direction of reversing earthquake loads. One wonders whether the photograph of ‘sliding-shear failure’ in Park and Paulay 1975 Fig 12.2.8 also indicates a need for stronger mid-depth horizontal web-reinforcement.

Perhaps stronger web-reinforcement would improve the ductility of shear failure in deep beams and provide an incentive to reconcile the Toronto, Canterbury and other research with simple rigid-plastic plane-stress analysis for use as a design model. Rigid-plastic analysis can correctly address the detailed boundary and equilibrium conditions of each individual case.

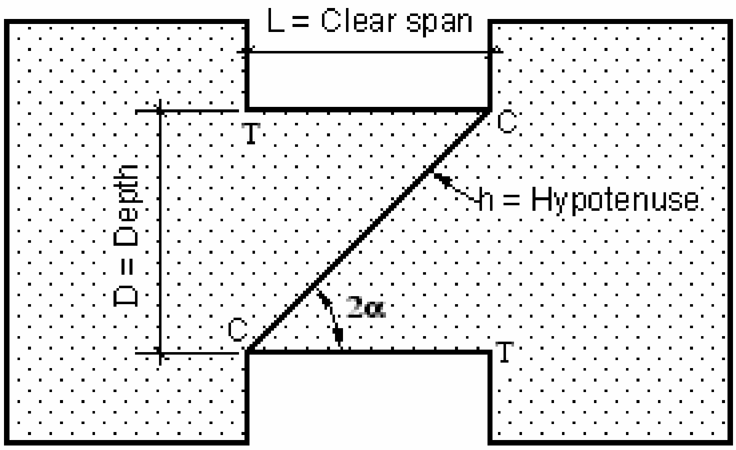

Fig 2: Basic geometry of coupling-beams
T/C indicate corners of horizontal tension/compression respectively

The minimum horizontal web content mentioned in the Canadian code is $0.003b_ws_h$ (0.0015 per face) corresponding to a smeared tensile yield-strength of 1.5 MPa (500 MPa rebar) as compared to a figure of 0.35 MPa used in AS3600 to determine minimum vertical shear reinforcement. One suspects that the Australian figure may be too low to ensure ductile performance. Perhaps the Canadian figure was influenced by Nielsen's 1999 book where he recommends a minimum content of 0.0015 (0.75 MPa) in both directions for structures subject to monotonic/light loads structures but 0.003 (p284) for 'initially cracked' structures presumably including hysteretic loading.

COLLAPSE MECHANISMS

One cannot expect that a single plane-stress mechanism will adequately describe behavior over the range of variables mentioned above. There is a need for an inventory of plane-stress mechanisms to capture a range of behaviors. I am, so far, aware of 3 groups totalling 5 mechanisms each of which seems likely to be the correct mechanism for some range of variables:

- **2 'shear' mechanisms** each characterized by yield of vertical shear reinforcement only; no yield of horizontal reinforcement anywhere
    - o **Steep shear mechanism** applicable for heavier shear reinforcement
    - o **Hypotenuse shear mechanism** for lighter vertical shear reinforcement
- **1 'Dogleg bending' mechanism** characterized by yield of the horizontal reinforcement only; no yield of the vertical shear reinforcement
- **2 'mixed' mechanisms** each characterized by yield of both the horizontal reinforcement and the vertical shear reinforcement.
    - o **Steep mixed mechanism** applicable for heavier shear reinforcement
    - o **Hypotenuse mixed mechanism** for lighter vertical shear reinforcement

These provide 3 statically determinate values of collapse load. For the 2 shear mechanisms, the correct mechanism can be identified and the collapse shear found by direct calculation.The mixed mechanisms require iterative solution.

Perhaps this multiplicity of mechanisms has obstructed progress in theoretical analysis but that should be manageable now that desktop computers are ubiquitous.

The present purpose is to reconcile/unify this inventory of 5 mechanisms so as to facilitate software to discriminate between them and calculate the minimum collapse load. Such software seems to be a necessary investigative tool. Other interesting issues such as further details of the kinematics, the strain ratios and the virtual work algebra will have to wait to a later time.

YIELD-LINES AND IN-PLANE STRAINS

Yield-lines are themselves directions of zero direct strain characterized by the relative displacements of the 2 adjacent segments on each side of the yield-line. Yield-lines of 2 types are used in this paper:

- A bending yield-line is characterized by relative rotation about a centre located on the yield-line; there is a principal in-plane strain, tension or compression, perpendicular to the yield-line and the principal in-plane strain parallel to the yield-line is zero. Such yield-lines should be ductile provided that the reinforcement across tension arms is neither too light nor too heavy.
- Translation yield-lines are characterized by principal in-plane strain directions that bisect two directions of zero direct strain (Mohr's circle for strain). The principal strains are of different sense tension/compression and the principal tension is greater, sometimes much greater than the principal compression strain. Ductility is suspect and can be expected to reduce as the ratio of principal strains (tension/compression) increases.

For a bending yield-line, the appropriate concrete compressive strength is the bending compressive strength; a well-established and accepted value which is, in Australia, New Zealand, Canada and USA: $f_b = 0.85 f'_c$ (1)

For translation yield-lines, principal compressive strains (hence strut directions) are offset from yield-lines by the 'attack angle' ε which is half of the acute angle between translation yield-lines and the 'other' direction of zero direct strain:

$$\text{Ratio of principal in-plane strains (tension/compression)} = \frac{1}{\tan^2 \varepsilon} \quad (2)$$

A codified value for the diagonal compression strength across translation yield-lines is, from AS3600 c12.1:

$$f_d = \left(0.80 - \frac{f'_c}{200}\right) f'_c \quad (3)$$

The value (3) is less than (1) as it should be. (3) is discussed by Nielsen 1999 where it seems clear that, for structures that are 'initially cracked' (as from an earlier load cycle), it relies on a minimum closely-spaced rebar content of 0.003 both ways.

The algorithm for the hypotenuse mixed mechanism below will sometimes result in small diagonal forces crossing translation yield-lines at small attack angles hence large strain ratios. It is not yet clear whether this is a real effect or a mathematical oddity. Numeric studies will help. An interim solution for AS 3600 s12 would be to specify a minimum attack angle at, say, 10 degrees corresponding to a strain ratio of 32.

For bending yield-lines, designers already assume a value: $e_{cu} = 0.003$ (4)

for maximum bending compression strain and use that to calculate elasto-plastic strains in tension/compression rebar. A comparable approach might seek to evaluate strains (not just strain-ratios) along translation yield-lines. See Collins 2007, Nielsen 1999.

STEEP SHEAR MECHANISM

Shear mechanisms are here defined as those not involving yield in any horizontal reinforcement. Horizontal direct strains must be zero everywhere and so relative displacements must be vertical. Fig 3 shows the yield-lines for a steep shear mechanism at some initially unknown angle 2θ defining the yield-segment shown in Fig 5. The left wall displaces vertically up and the right wall vertically down as in Fig 4.

- Horizontal direct strains are zero as noted above and (5)
- Direct strains at angle 2θ parallel to the yield-lines are also zero (6)
- Attack angle strut from yield-line $\varepsilon = 2\theta/2 = \theta$ (7)
- Angle of strut from horizontal = θ (8)

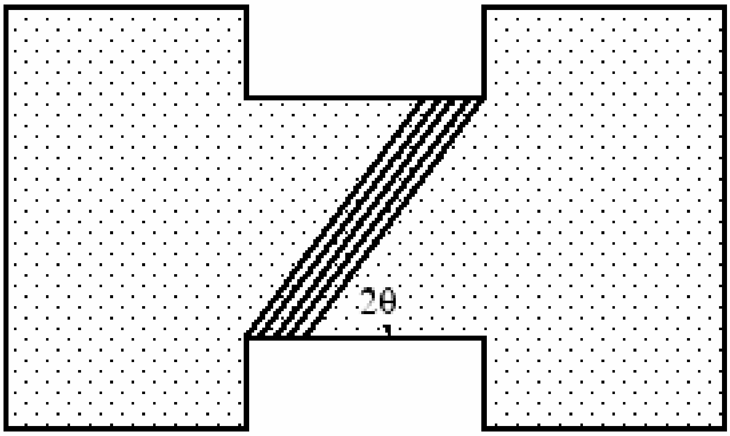

Fig 3: Steep shear mechanism: yield-lines

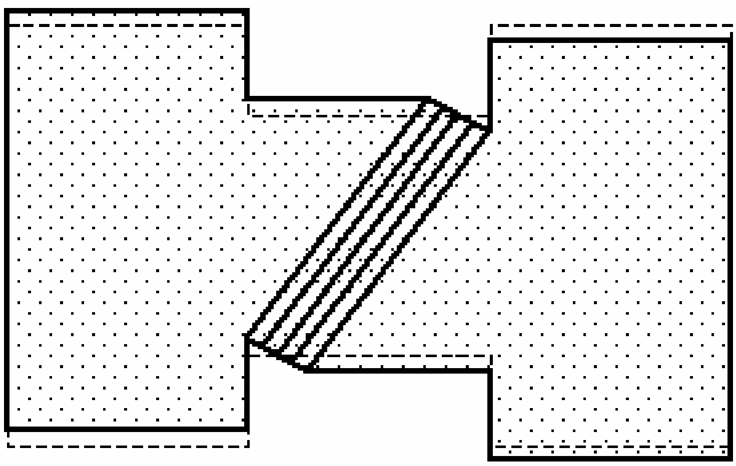

Fig 4: Steep shear mechanism: displaced shape

The angle θ is determined so as to minimize the upper bound estimate of collapse shear. Virtual work analysis leads to: $$\sin^2\theta = \frac{a}{f_d} \qquad (9)$$

Where: Smeared yield strength of vertical shear reinforcement: $$a = \frac{A_v f_{sy}}{b s_v} \qquad (10)$$

(9) is an equation of vertical equilibrium for the web/flange horizontal-shear connection requiring that the vertical component of the diagonal compression-field be balanced by the yield strength of the vertical shear reinforcement. It also ensures that the vertical shear component of the diagonal strut force acting on a vertical cross-section is the same as the total shear acting on either of the inclined yield-lines of Fig 5.

A detailed equilibrium analysis of Fig 5 could follow but instead will be combined with the hypotenuse shear mechanism.

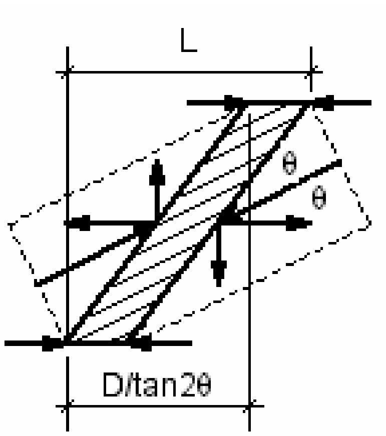

Fig 5: Steep shear mechanism: forces on free-body yield-segment

HYPOTENUSE SHEAR MECHANISM

As the content a of shear-reinforcement reduces so does strut angle θ from (9). Angle 2θ cannot be less than the value 2α for the full-span hypotenuse. If the calculated value $\theta<\alpha$ then (9) is not valid and $\theta=\alpha$. See Fig 6.

Both mechanisms can be calculated from Fig 7:

- For the steep shear mechanism, the full-span hypotenuse at angle 2α is not a yield-line but it is entirely within the yield-segment and it is everywhere crossed by a diagonal compression-field at angle $\theta>\alpha$ from horizontal.
- For the hypotenuse shear mechanism, the full-span hypotenuse at angle 2α is the only yield-line and it is everywhere crossed by a diagonal compression-field at angle $\theta=\alpha$.

ALGORITHM FOR BOTH SHEAR MECHANISMS

- Calculate angle θ from (9) (11)
- If $\theta>\alpha$: Steep shear mode:
  - Horizontal shear-stress at flange: $q=\frac{1}{2}f_d\sin 2\theta=\frac{a}{\tan\theta}$ (12)
- If $\theta<\alpha$ then: Hypotenuse shear mode: $\theta=\alpha$ and:
  - Reduced diagonal compression-field stress at flanges only: $f_{reduced}=\frac{a}{\sin^2\alpha}$ (13)
  - Note that the main field intersecting the hypotenuse is NOT TO BE REDUCED.
  - And horizontal shear-stress at flange: $q=\frac{1}{2}f_{reduced}\sin 2\alpha=\frac{a}{\tan\alpha}$ (14)

Regardless of which shear mode:

- Angle between directions of zero direct shear = 2θ (15)
- Attack angle from yield line to concrete strut: $\varepsilon = \theta$ (16)
- Angle of concrete strut from angle of hypotenuse: $2\alpha - \theta$ (17)
- Width of the concrete-strut crossing the full-span hypotenuse: $h\sin(2\alpha - \theta)$ (18)
- The diagonal force in the strut crossing hypotenuse: $C_{cs} = hbf_d \sin(2\alpha - \theta)$ (19)
- With horizontal component: $C_{csh} = C_{cs}\cos\theta$ (20)
- And vertical component: $C_{csv} = C_{cs}\sin\theta$ (21)
- Shear strength: $V = abL + C_{csv}$ (22)

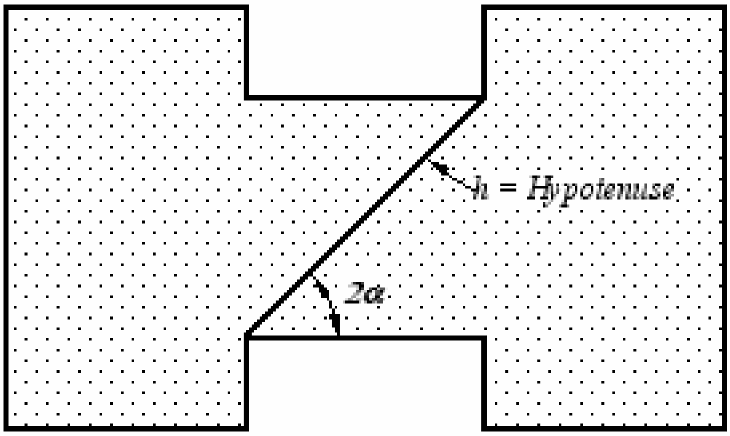

Fig 6 Hypotenuse shear mechanism: yield-line on hypotenuse only

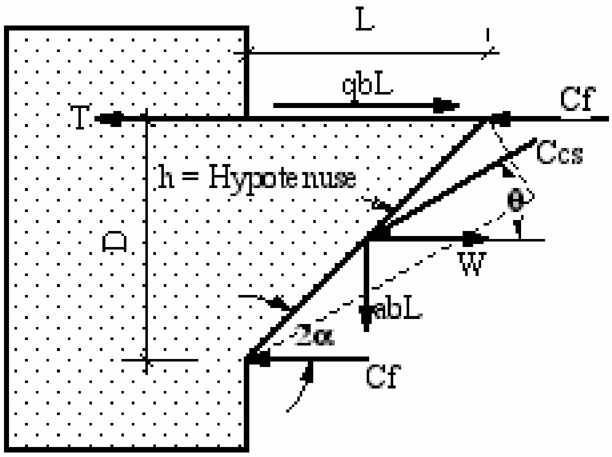

Fig 7 Both shear mechanisms: forces on hypotenuse

- Calculate C_f from the equilibrium of horizontal forces across the hypotenuse :
  $W - 2C_f - C_{csh} = 0$ (23)
- Whence an estimate of flange tension: $T = qbL - C_f$ (24)

(23) implies that, if W (mid-depth horizontal reinforcement) is small or zero then C_f will be tensile. Tensile strains have been noted in experimental reports.

DOGLEG BENDING MECHANISM

Fig 8 shows the yield-lines for the dogleg mechanism which adapts the usual 'hinge at the face of a column' yield-lines so as to provide a load-path for the shear onto the support. The bending yield line now follows a dogleg path with a knee at the rotation centre defined by co-ordinates (x, k). All yield-lines are bending yield-lines.

Fig 9 shows the displaced shape: the yield segment rotates about its own geometric centre and walls each side rotate around the dogleg hinge so as remain vertical. Fig 10 shows the forces acting on the free-body yield segment.

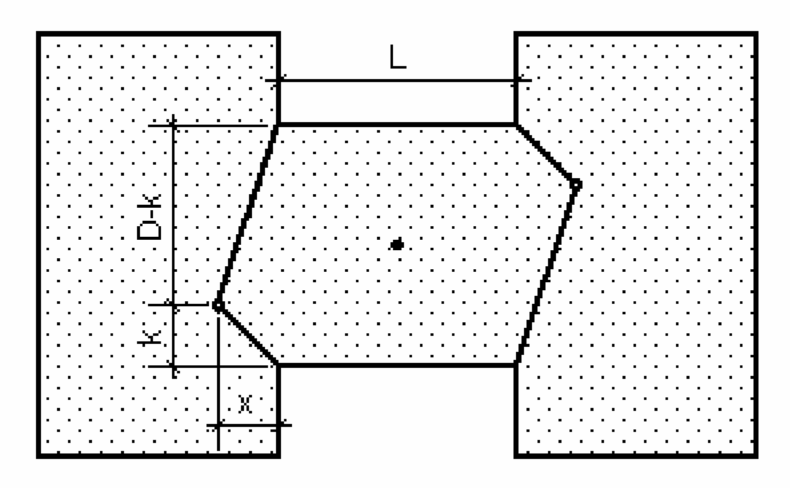

Fig 8: Dogleg bending mechanism: yield lines

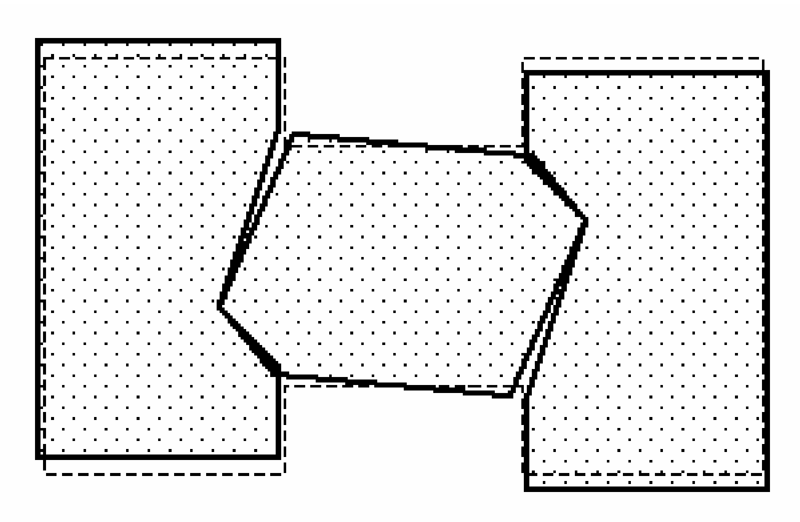

Fig 9: Dogleg bending mechanism: displaced shape

YIELD STRENGTHS OF HORIZONTAL REBARS (Refer Fig 10)

Yield strength of flanges in tension: $T = A_s f_{sy}$ (25)

Yield strength of flanges in compression: $C_f < C_{fy} = A_s(f_{sy} - f_b)$ (26)

Yield strength of horizontal web-reinforcement: $W = A_w f_{sy}$ (27)

The present analysis treats the horizontal web-reinforcement W as lumped at mid-depth. Later it may be appropriate to consider it distributed over most of the web and to use elasto-plastic calculations to check actual strains and stresses.

ALGORITHM FOR DOGLEG BENDING MECHANISM

This algorithm begins with the yield-strengths of horizontal rebars (25 – 27). For the dogleg mechanism (without any internal yield) the compression flange force C_f must be at yield C_{fy} and the algorithm returns a result in a single cycle. For the mixed mechanisms described later, a value of C_f will be supplied from iteration. This algorithm must then check that $C_f < C_{fy}$ and, if necessary, reduce it. Reducing C_f will increase *Cc* so it must also be recalculated.

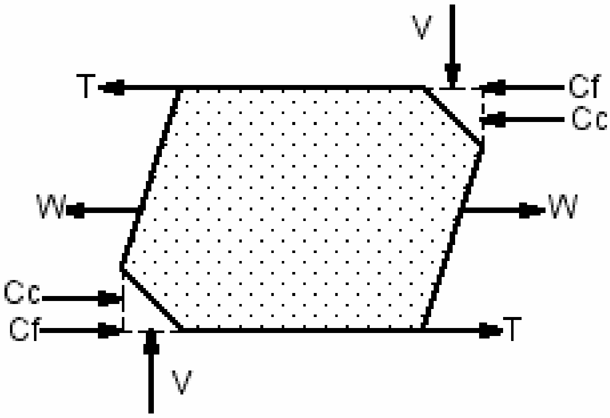

Fig 10: Dogleg bending mechanism: forces on yield segment

- $C_f = C_{fy}$ unless value from iteration: $C_f < C_{fy}$ (28)
- Calculate *Cc* from horizontal equilibrium at ends: $T + H - C_c - C_f = 0$ (29)
- Calculate neutral axis depth k from: $C_c = bkf_b$ (30)
- Calculate end moment strength: $M = TD + \frac{1}{2}HD - \frac{1}{2}C_c k$ (31)
- Calculate *x* from (combination of 33,34): $bf_b x(L + x) = 2M$ (32)
- And V from: $V(L + x) = 2M$ (33)
- Or V from: $V = bxf_b$ (34)

STEEP MIXED MECHANISM

Fig 11 shows the yield-lines with interior yield-lines additional to those of the dogleg mechanism. The small circles indicate 5 distinct rotation centre locations. There are 4 bending yield-lines and no translation yield-lines. The relative rotations at the various hinges can be adjusted so that there is no relative horizontal displacement between the central section and the walls at each 'compression' corner. It follows that the flange compression force C_f is not required to yield.

The displaced shape is shown in Fig 12 and a free-body yield segment in Fig 13.

ALGORITHM FOR STEEP MIXED MECHANISM

- Start with estimate of *V* say from lesser of shear and dogleg mechanisms (36)
- If $V<abL$, steep mixed mechanism, else hypotenuse mixed mechanism (37)
- Calculate segment width *y* from: $V = aby$ (38)

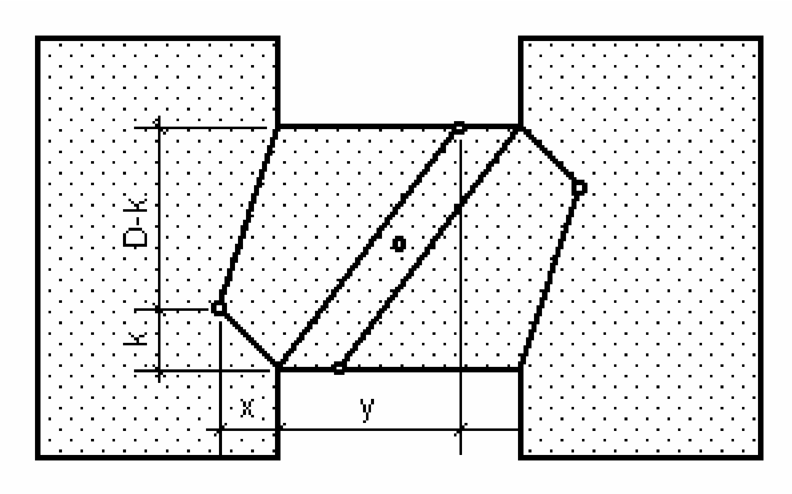

Fig 11: Steep mixed mechanism: yield lines

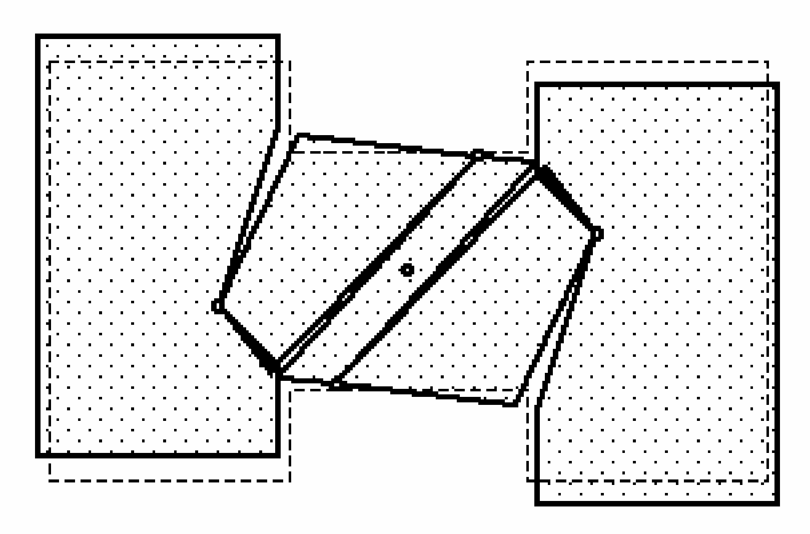

Fig 12: Steep mixed mechanism: displaced shape

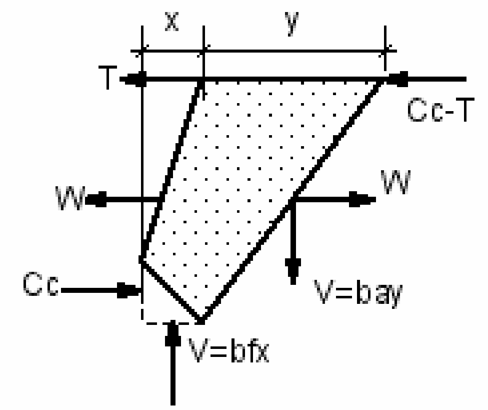

Fig 13: Steep mixed mechanism: forces on a free-body yield segment

- Calculate *Cc* from: $$\frac{1}{2}V(x+y)=C_c(D-\frac{1}{2}k) \qquad (39)$$

- Calculate C_f from: $T + H - C_c - C_f = 0$ (40)
- Calculate C_f, Cc, k, M, x, V from (28-34) (41)
- Repeat from (36) until convergence (42)

HYPOTENUSE MIXED MECHANISM

Fig 14 shows the yield-lines including those of the dogleg mechanism. There are now 4 rotation centres indicated by the 4 small circles. Each half-yield segment rotates through the same small angle about the rotation centre located in THE OTHER HALF YIELD-SEGMENT. These inner rotation centres are joined by a line of rotation centres at some unknown angle 2β to horizontal.

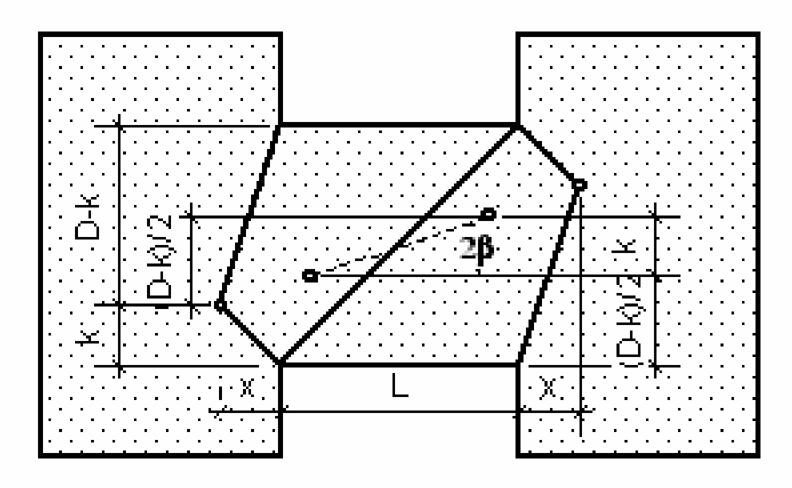

Fig 14: Hypotenuse mixed mechanism: yield-lines

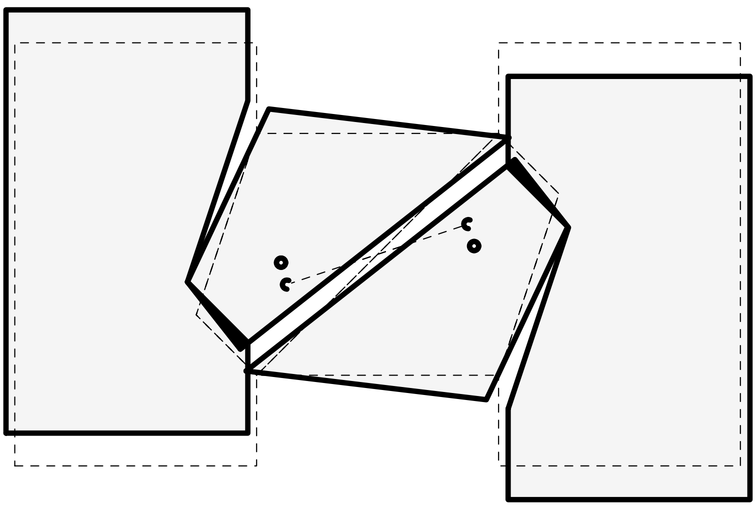

Fig 15: Hypotenuse mixed mechanism: displaced shape

Since both half segments rotate through the same angle in the same sense, there is no relative rotation across the hypotenuse yield-line. There is relative translation which is perpendicular to the line of centres.

There are, again, no relative horizontal displacements at the 'compression' corners and so, as in the steep mixed mechanism, the flange compression force C_f need not reach yield.

For the hypotenuse yield-line:

- The line of rotation centres at angle 2β is a direction of zero direct strain (43)
- The yield-line itself at angle 2α is also a direction of zero direct strain (44)
- Acute angle between directions of zero direct strain $2(\alpha - \beta)$ (45)
- Attack angle: $\varepsilon = \alpha - \beta$ (46)

Fig 15 shows the displaced shape and Fig 16 the forces acting on a yield-segment.

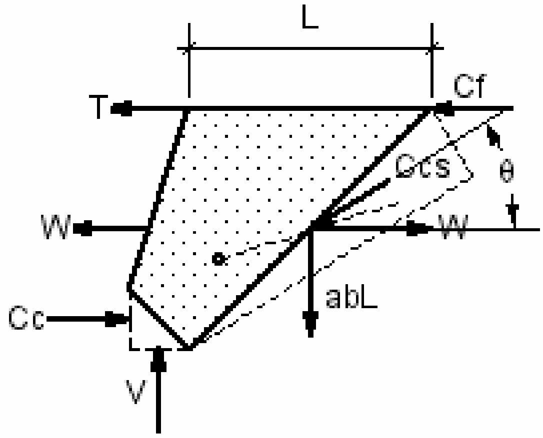

Fig 16: Hypotenuse mixed mechanism: forces on a free body yield-segment

CONCRETE STRUT FORCES

- Angle of principal compression and concrete strut from horizontal: $\theta = \alpha + \beta$ (47)
- In-plane width of the concrete strut across hypotenuse is: $h \sin \varepsilon$ (48)
- And the diagonal strut force: $C_{cs} = f_d bh \sin \varepsilon$ (49)
- Vertical component of strut force:

  $C_{csv} = C_{cs} \sin \theta = f_d bh \sin(\alpha - \beta) \sin(\alpha + \beta) = f_d bh \left(\sin^2 \alpha - \sin^2 \beta\right)$ (50)
- Horizontal component of strut force:

  $C_{csh} = C_{cs} \cos \theta = f_d bh \sin(\alpha - \beta) \cos(\alpha + \beta) = f_d bh \dfrac{\sin 2\alpha - \sin 2\beta}{2}$ (51)

ALGORITHM FOR HPYOTENUSE MIXED MECHANISM

- Start with estimates of V say from shear mechanisms (52)
- If $V > abL$, hypotenuse mixed mechanism, else steep mixed mechanism (53)
- Calculate angle β from: $V = abL + C_{csv} = abL + f_d bh \left(\sin^2 \alpha - \sin^2 \beta\right)$ (54)
- Calculate θ, ε, *Ccs, Ccsh* from (46 - 51) (55)
- Calculate *Cf* from forces across hypotenuse: $W - 2C_f - C_{csh} = 0$ (56)
- Calculate *Cf*, *Cc*, *k*, *M*, *x*, *V* from (28-34) (57)
- Repeat from (52) until convergence. (58)

CONCLUSIONS

It is well-known that the skeletal collapse mechanism (Fig 1) used for the design of moment resisting frames is inappropriate for the design of core coupling-beams.

Five plane-stress (2D two-dimensional in-plane) rigid-plastic collapse mechanisms are proposed. All seem likely to be the correct mechanism for some range of variables. They lend themselves to simple solutions in Microsoft Excel and so they may be of interest to design offices and also to researchers interested in shear in concrete generally.

My next priority will be to write software, probably in Microsoft Excel, that will, for any given structural design in a reasonable range, select the correct mechanism and calculate collapse load and other useful results. I am not certain that this inventory of mechanisms for coupling-beams is complete and the proposed software will serve as an investigative tool to detect other mechanisms, if any.

ACKNOWLEDGEMENTS

Some work on the shear strength of concrete structures dates back the invention of reinforced concrete 1880-1890. Investigations in other areas of structural plasticity in Denmark, England and America date from the 1930s. Focused research on problems of in-plane plasticity, including the shear strength of concrete, began in late1960s/1970s at the Technical University of Denmark (M.P.Nielsen, Mikael Braestrup and others) and at ETH Zurich (Bruno Thurlimann, Peter Marti and Peter Mueller). This lead to the IABSE Colloquium on *Plasticity in Reinforced Concrete* Copenhagen 1979, to Nielsen’s book: 908 pages published 1984 and 1999 and to some Eurocode provisions.

The 2 shear mechanisms mentioned in this paper were described by several of the Copenhagen/Zurich authors in the 1970s. The other 3 mechanisms (dogleg and mixed) described here are, to the best of my knowledge, original although some underlying ideas were mentioned in my 1987 paper.

This paper (and others) was stimulated by an invitation to present a paper at the *Morley Symposium on Concrete Plasticity and Its Application,* University of Cambridge, UK, July 2007. The calibre of the other participants and the quality of the discussion rekindled my interest in structural plasticity. My thanks to Chris Morley, to the organizers from Cambridge University Engineering Department, and to the other participants.

Generally my work has been informed by the observation that differentiated virtual work equations are themselves, necessarily, equilibrium equations (or, rarely, compatibility equations). I think that I read this in the book by Jones & Wood 1967 but I cannot now find the page. It seems obvious that the proposition is as true for the plastic analysis of in-plane problems in concrete as it is for yield-line analysis of slabs. I am only surprised that I have not seen it mentioned since Jones & Wood.

The experimental work on shear walls and coupling-beams at Canterbury University, New Zealand (Tom Paulay, Nigel Priestley and others) started about 1967. While I believe that Canterbury University continues as a world-leader in earthquake engineering, I am not aware of current shear wall research being undertaken there.

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