Teaching Structural Dynamics and Earthquake Engineering - Meeting the Practical Work Challenge

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Abstract

Structural Dynamics and Earthquake Engineering as a teaching subject has progressively been introduced into the Civil/Structural Engineering curricula, principally at the post-graduate level (Masters and PhD) and sometimes offered as an elective unit at final year undergraduate level in more and more technical universities throughout the world.

There are two key reasons why this particular subject is found to be challenging to teacher and student alike. The first is that the technical material this subject encompasses and builds on from earlier years of study is so wide that it can be considered to virtually be a capstone subject. The second key reason is the general lack of easily accessible classroom based hands-on experimental teaching equipment that can be instrumental in facilitating the learning of this material.

The author, an experienced teacher of Structural Dynamics, has long recognized these difficulties and recently taken on the challenge of developing a range of SDOF and MDOF experiment setups for the study of structural dynamics as an extension of his innovative TechnoLab™ series of teaching products in Statics and Mechanics. In addition, he has sourced, and is also developing, suitable sensor/response measurement techniques to interface to these experiment setups and allow analysis and numerical investigation opportunities for students to enhance their learning experience.

This paper describes some of these setups and measurement techniques as currently implemented in the subject CVE80019 Structural Dynamics and Earthquake Engineering, co-delivered bythe author at Swinburne University of Technology.

Keywords: accelerometers, vibration sensors, dynamics, modal analysis

1 INTRODUCTION

"Structural Dynamics and Earthquake Engineering" courses have progressively been introduced into the syllabus of Civil/Structural Engineering curricula of more and more technical universities throughout the world, over the past few decades. Whilst in some of these universities it forms part of their core program at the post-graduate level (Masters and PhD), in several others it is also being offered as an elective unit at final year undergraduate level.

Structural Dynamics and Earthquake Engineering can be a very challenging subject both for the staff who are tasked to teach the material and for the students who have chosen to do the subject. There are two key reasons why this particular subject is so challenging to teacher and student alike. The first is that the technical material this subject encompasses, builds on quite a wide range of subjects delivered in earlier years of study. Structural Mechanics subjects (modelling for stiffness/deflections due to loading and the Finite Element Method, in particular) in combination with subjects dealing with material behaviour (elasticity, plasticity, hysteresis in load/deflection behaviour, plastic hinge formation, collapse mechanisms) need be mastered and built on as well as those subjects that dealt with statistics,experiment design, mathematical and numerical modelling (second order differential equations, matrix methods/linear algebra, numerical integration, time-domain/frequency domain analysis, stochastic processes, extreme-value analysis etc), amongst others. To many engineering academics, Structural Dynamics and Earthquake Engineering is an ideal "capstone" subject, because it brings together such a wide range of topics taught in earlier years.

The second key reason as to why Structural Dynamics and Earthquake Engineering as a teaching subject can be very challenging is the general lack of in-class hands-on experimental teaching aids that may facilitate the learning of this material. Many universities rely on dealing with case studies drawn from the lecturer's experience to bring some form of "realism" to an otherwise "abstract" treatment of the subject. Often lecturers in the subject resort to numerical modelling and simulation techniques both for illustrating the material concerned and also in the setting of assignment work for students to perform and learn from which at the same time may form part of their assessment. A few universities may provide limited access (sometimes in the form of a live demonstration to the class, but more often, a video capture) of dynamic testing being performed on a research-scale shaking table typically on a simple scale model building structure. In some cases, not only do the students get to observe the experiment, but data collected of the response from sensors is made available to them to explore some of the modelling and analysis techniques treated in their course.

As a long time teacher of Structural Dynamics, the author has recognized there to be a lack of affordable hands-on equipment, (both in the form of experimental apparatus and for sensors/instrumentation needed for measuring dynamic response), that is suitable for the teaching of structural dynamics relevant material.

Over the past few years, he has therefore taken up the challenge of developing a range of SDOF and MDOF experiment setups for the study of structural dynamics and sourced suitable sensor/response measurement techniques to interface to them. These experiments can be assembled onto a desktop mounted TechnoLab™ Pixi Frame (see https://www.technolab.net.au/) or onto a battery-operated actuator based Ground Motion Simulator (GMS). Live experiments can then be performed by students in the classroom and their investigation and reporting of these can be integrated as part of their assessment in their Structural Dynamics and Earthquake Engineering subject.

Some of these setups and measurement techniques from TechnoLab™, as introduced in the subject CVE80019 Structural Dynamics and Earthquake Engineering, co-delivered by the author at Swinburne University of Technology, are detailed below.

2 TECHNOLAB™ DYNAMICS EXPERIMENT KITS

The TechnoLab™ learning platform offers a number of purpose-specific experiment kits for investigation of basic dynamic properties of single and multi-degree-of-freedom systems, (SDOF and MDOF systems). The experiments are supported with "Experiment Performance Instruction Sheets" and "Experiment Report Sheets" to guide students to perform and report on the experiments concerned. Additional support material in the form of EXCEL spread-sheets and/or other software, directly useful to the experiment exercises, are also provided by TechnoLab™ as appropriate.

2.1 Mass Spring SDOF combinations

Stainless steel (SS) close-coiled helical springs of the same mean coil diameter, pitch and length but of three different wire diameters to produce three different base spring stiffnesses are available for performing basic dynamics investigations of the vibration frequency (and damping) of simple SDOF dynamic systems from TechnoLab™. A thin Aluminium threaded disc allows spring assemblies of multiple springs to be formed in series by screwing their two closely spaced end coils onto a disc from above and below. The top end disc in the spring assembly is bolted onto the top of the Pixi Frame™ in portrait orientation and the disc at the free end has a threaded rod from which a number of 5 mm thick threaded stainless steel discs chosen from a range of diameters can be attached to produce a suspended point mass thereat.

Figure 1 depicts setups where five of the SS springs of 1.0 mm wire diameter have been assembled in series and from one to six 44 mm diameter 6 mm thick SS disks are attached ready to perform a simple "Pluck test" and observing the resultant dynamic response. The response is videoed using any suitable digital video recorder with a reasonable resolution, that would include smart phones which typically record at 25 or 30 frames per second (fps) with High Definition (HD of 1024 x 768 pixels). More recent smart phones, such as the iPhone6, can also record HD video at 60 fps which doubles their effective dynamic range for use in studies of dynamics experiments.

Tracker (https://physlets.org/tracker/) is a free video analysis and modelling package that can track a user-defined feature on the video capture of a dynamic experiment to yield its (x, y) co-ordinates over the range of video frames chosen to be investigated.

Figures 2(a) and 2(b) depict portions of the pluck test time traces from Tracker for the (x, y) displacement of the end disc of the spring assembly for the 5 SS and 6 SS end mass conditions, respectively, for the 5 springs in series model condition of Fig. 1. The HD video capture was at 25 fps. The insets on the 15 second long traces show the full forms of these traces captured over a 50 second long duration from the video.

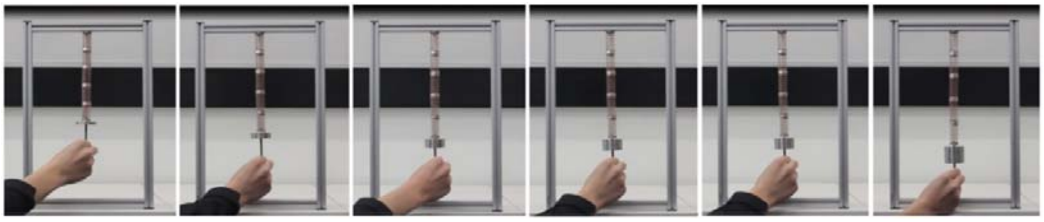


Figure 1: Five springs of 32mm length connected in series with one to six disc masses

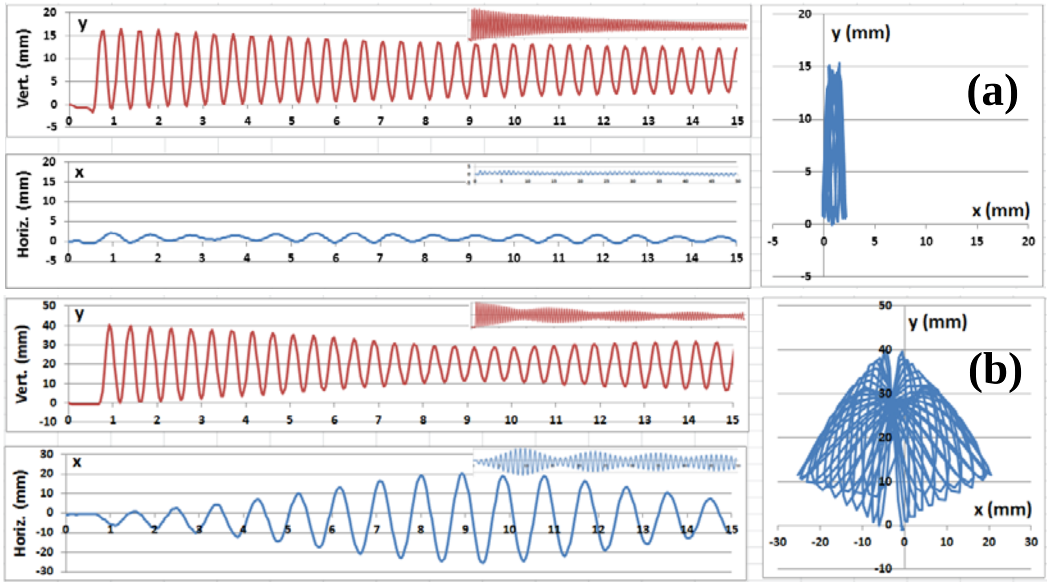


Figure 2: 5 Springs in Series and (a) 5 SS Disc Masses (b) 6 SS Disc Masses

Clearly the behaviour of the two vertical spring-mass systems is quite different. Case (a) vertical motion closely resembles a low damped cosine of a Pluck test trace for a SDOF system. Horizontal (pendulum) motion is small compared with the vertical motion as the pluck was manually introduced to be as close as possible to vertical. For both the vertical and horizontal time traces, cycle counting or optimal exponential decay cosine fitting using the TechnoLab™-supplied Excel spreadsheet can be used to evaluate the frequency of vibration, and in the case of the latter, also the damping.

For Fig. 2 Case (a) above, an Excel Least Squares (LS) fit using the "Solver" Add-in over 1 to 15 seconds produced a response frequency of 2.384 Hz and a quite low damping value of 0.30% for the vertical response. Figure 3 shows the fitted damped cosine to the Tracker data from which these results were obtained. Cycle counting for the rather small horizontal oscillatory response produced a frequency of 1.099 Hz – a little less than half of the frequency in the vertical direction. The "wavy" response in the horizontal direction is ill-conditioned for fitting a damped cosine model so this wasn't attempted. The frequencies for the SDOF modelling (spring *k* and mass *m*) of the vertical motion and for the pendulum sway modelling, (length to COG of *L*), correspond closely to the theoretical $\sqrt{k/m}/2\pi$ and $\sqrt{g/L}/2\pi$, respectively, where g is acceleration due to gravity.

For Fig. 2 Case (b) above, there appears to be a waxing/waning style of interaction between the oscillatory motions in the vertical and horizontal directions. This interaction can be visually identified in the x-y trajectory patterns depicted alongside the horizontal x(t) and vertical y(t) time trace segments to which these apply.

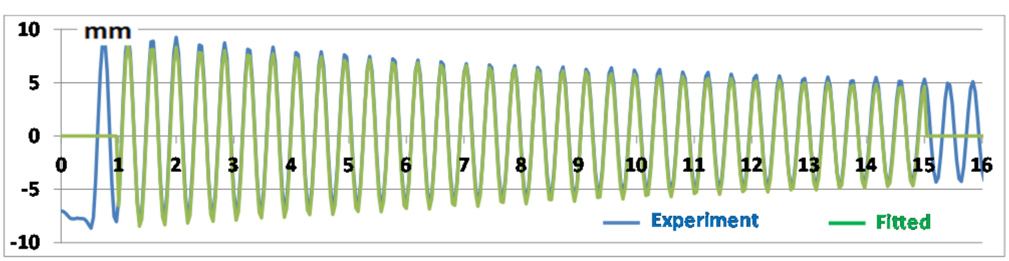


Figure 3: Least Squares Exponential Decay Cosine Fit to Fig 2 Case (a) data

The response frequencies were determined as 2.20Hz and 1.07 Hz for the y and x directions, respectively. On this occasion, the y response frequency is very close to twice that of the x direction which can explain the rather rhythmic waxing/waning interaction behaviour. Again, these values correspond closely to theory for adjusted values of *m* (now six SS disc masses) and increased length to COG, *L*, for this case.

The Case(a) style observations are observed in most of the spring-mass combinations in the TechnoLab™ kit for this experiment, with the occasional Case(b) style "cropping up" when the vertical spring-mass assembly frequency is very close to a 2 (or 3) times integer factor of the horizontal (pendulum) frequency for that assembly.

2.2 Two-storey Sway Frame Pluck tests

The investigation of a two-storey sway frame model for its modal properties using simple Pluck tests is a recent addition to the TechnoLab™ platform Dynamics experiment series, (see Fig. 4). For the past two years, students enrolled in the subject CVE80019 Structural Dynamics and Earthquake Engineering at Swinburne University of Technology, have used this setup with an iPhone attached to the top storey rigid beam as its principal mass to capture the frame's acceleration response to Pluck tests with different initial displacements using the "Vibration" App from Diffraction Ltd Design (https://itunes.apple.com/au/app/vibration/id301097580?mt=8) for capturing *.csv* data files of this response. These files are "worked" on by students to evaluate the experimentally obtained modal properties of the sway frame. Results are then compared with their "hand" solutions for the model frame modal properties. Video capture of the individual Pluck tests using smart phones on small desktop tripods is also performed for subsequent Tracker analysis to provide an alternative experimental determination of the 2-storey sway frame modal properties. Individual student reporting of this investigation forms part of their continuous assessment.

2.2.1 Modal properties of two-storey sway frame from top storey accelerations

At "first look" it would seem that it would be impossible to determine the mode shapes of a MDOF sway frame structure solely from measurements at its Top storey of the acceleration response to pluck tests. Yes, modal frequencies would not be a problem, but mode shapes from a single point measurement – *big question mark.*

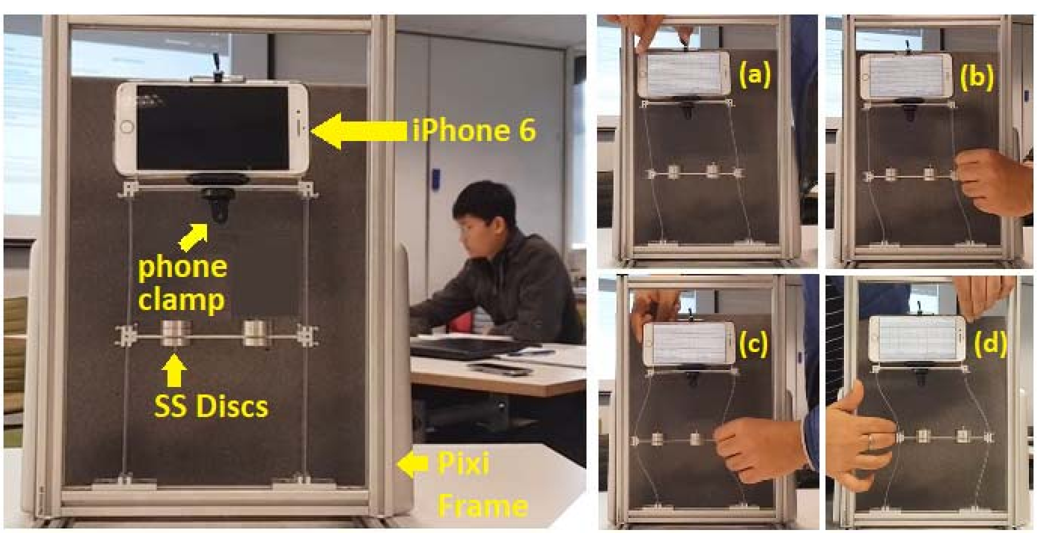


Figure 4: Two-storey Sway Frame Investigation of Modal Response Characteristics

The first (1st) storey of the model sway frame has stainless steel discs attached to its rigid beam to produce a total mass of 180 gram. Which model iPhone (5 or 6, 6s or other), and whether it has additional memory and/or a cover produces a total phone mass in the range 112 to 150 gram. The top storey beam and attachment for securing the iPhone total 60 gram, so that a standard iPhone 5 with 32 Gigs of memory would closely "match" the first storey mass. With all rigidly connected columns in this two-storey sway frame of the same Polycarbonate material, cross-section and length, this leads to stiffness $[K]$, and mass $[M]$, matrices of:

$$[K] = \begin{bmatrix} k & -k \\ -k & 2k \end{bmatrix}; \quad [M] = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \tag{1}$$

where *k* is the storey sway stiffness and *m* the storey mass and the DOF numbering is taken from Top-storey downwards. Considering free undamped vibration and a storey sway displacement response vector of $\{X\}$ then:

$$[M]\{\ddot{X}\} + [K]\{X\} = \{0\} \text{ which for } \{X\} = \{U\}e^{-i\omega t} \text{ results in } |[K] - \omega^2[M]| = 0 \tag{2}$$

The solution to the eigenvalues, ω^2, and eigenvectors, $\{U\}$, for conditions of Equation (2) above, after adopting a unit value for the top-storey sway (DOF "1"), results in:

$$\omega_1^2 = 0.382p;\ \omega_2^2 = 2.62p \text{ where } p = \frac{k}{m} \text{ and } \{U_1\} = \begin{Bmatrix} 1 \\ 0.618 \end{Bmatrix} \text{ and } \{U_2\} = \begin{Bmatrix} 1 \\ -1.618 \end{Bmatrix} \tag{3}$$

and where ω_1 and ω_2 are the first (lowest) and second mode natural circular frequencies respectively of the two storey sway frame.

Figures 4 (a) to (d), depict the initial Pluck storey displacement configurations as used by one group of four students in CVE80019 to produce acceleration response records of the ensuing free vibrations using the Vibration App on their iPhone6.

Now, should the initial displacement configuration of one of the cases in Fig. 4 correspond identically to a scaled version of one of the two sway mode shapes, then the recorded acceleration response trace on the Top storey using the Vibration App would not show any contribution from the other mode shape.

This suggests a "trial and error" procedure for determining each mode shape – *introduce a displacement configuration for the Pluck test that would show response only at the one frequency corresponding to that identified mode*. This procedure for identifying the mode shapes can be rather time consuming. More expedient methods would be desirable that not only reduce the amount of experimental effort but retain a reasonable level of accuracy.

Consider the dynamic modes of the model 2-storey sway frame under test to be lightly damped. Consider also that the acceleration response records are available from two separate Pluck tests with significantly different initial displaced shapes $\begin{Bmatrix} H_{1,j} \\ H_{2,j} \end{Bmatrix}$ for the j^{th} configuration, (here j = 1, 2). These shapes are declared to be a scaled super-position $\begin{Bmatrix} A_j \\ B_j \end{Bmatrix}$ of the two mode shapes, $\{U_1\} = \begin{Bmatrix} 1 \\ u_1 \end{Bmatrix}$; $\{U_2\} = \begin{Bmatrix} 1 \\ u_2 \end{Bmatrix}$, as follows:

$$\begin{Bmatrix} 1 & 1 \\ u_1 & u_2 \end{Bmatrix}\begin{Bmatrix} A_1 \\ B_1 \end{Bmatrix} = \begin{Bmatrix} H_{1,1} \\ H_{2,1} \end{Bmatrix} \text{ for the } 1^{st} \text{ configuration and } \begin{Bmatrix} 1 & 1 \\ u_1 & u_2 \end{Bmatrix}\begin{Bmatrix} A_2 \\ B_2 \end{Bmatrix} = \begin{Bmatrix} H_{1,2} \\ H_{2,2} \end{Bmatrix} \text{ for the } 2^{nd} \tag{4}$$

The acceleration response time trace can be easily transformed to displacement via band-pass filtering using FFT and IFFT transforms in EXCEL on the *.csv* data file from the Vibration App for these to obtain the two separate modal contributions A_j and B_j for the j^{th} Pluck test (here j = 1, 2).

The procedure for this is outlined in Fig. 5 in which (a_n, ib_n), represent the Complex Fourier coefficients for the n^{th} harmonic of the recorded acceleration response trace on the Top storey of the frame under test. The notch-filter start and end (finish) frequencies for the first and second mode for the decoupling procedure are given by (f_{s1}, $f_{f,1}$) and ($f_{s,2}$, $f_{f,2}$), respectively. These two ranges "straddle" the first mode and second mode frequencies, f_1 and f_2, and are chosen such that $f_{f,1} \leq f_{s,2}$. The Fourier coefficients outside this range of frequencies are set to zero so their contributions to the response are completely removed. This is especially important to the low end frequencies as the transformation from acceleration to displacement involves division of acceleration Fourier coefficients by $-\omega_n^2$, where $\omega_n = 2\pi f_n$ which is very low valued at low frequencies and hence magnifies any low frequency errors in the experimentally obtained data.

The traces for $x_{j,1}(t)$ and $x_{j,2}(t)$ are examined as close to the initiation time of the Pluck test to obtain an estimate of the A_j and B_j amplitude contributions to the top storey initial displacement. The sum of these two estimates should be close to $H_{1,j}$, for low level damping.

From Equations (4), and the now known values of A_j and B_j, (j = 1,2), we obtain:

$$\begin{Bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{Bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} H_{2,1} \\ H_{2,2} \end{Bmatrix} \rightarrow \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \frac{1}{A_1 B_2 - A_2 B_1} \begin{Bmatrix} B_2 & -B_1 \\ -A_2 & A_1 \end{Bmatrix} \begin{Bmatrix} H_{2,1} \\ H_{2,2} \end{Bmatrix} \qquad (5)$$

Equation (5) provides a direct solution to u_1 and u_2, the modal amplitudes at the 1st storey for the first and second modes of vibration, respectively.

Because the inter-storey stiffness and the storey masses (with iPhone) are not exactly equal in the experiment setups, the mass [M] and stiffness [K] matrices do not exactly conform to those depicted in Equation(1). Hence u_1 and u_2 would not be equal to 0.62 and -1.62, respectively. However, simple experiments are conducted by students to determine [M] and [K] more precisely for the particular setup under test, then an Eigen-analysis is performed to determine the applicable mode shapes and frequencies.

To avoid the complication of attempting to manually invoke an initial displaced configuration of the 2-storey test sway frames using both hands when performing a Pluck test, the setup conditions of Fig. 4(a) and 4(b) involve only the use of the one hand and so are easier to perform as it is difficult to "release" the displaced structure from "at rest" conditions simultaneously using both hands.

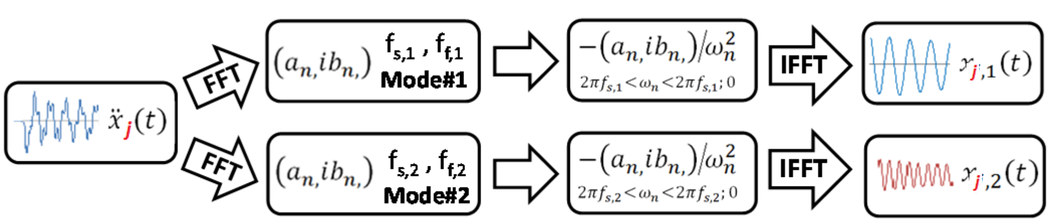


Figure 5: Decoupling the Top Storey Modal Displacements for 2-storey Test Frame

Figure 6 depicts an extract of the initial 5 seconds of decoupled displacement response for Mode#1 (f_1 = 1.11 Hz) and Mode#2 (f_2 = 3.78Hz) as derived from the Vibration App using the technique described above. It is clearly evident that damping (associated with the rate of decay in successive cycles) is quite small.

Coefficients (A_1, B_1), and (A_2, B_2), are evaluated for Case 2(a) and 2(b), respectively, in Fig. 5 and depicted therein. Values for $H_{2,1}$ and $H_{2,2}$ at the 1st storey are determined to be approx. -12mm and -18mm, respectively. Solution of Equation (5) for these values yields u_1 = 0.67 and u_2 = -1.81.

The second mode contribution to the vibration of the top-storey in Case 2(a) is quite small (dominated by the first mode) implying that the initial displaced shape is close to that of the first mode. For this test frame the initial pluck shape, normalised to a top storey of “1” is very close to 0.61, implying that the top storey columns are stiffer than those of the first storey,as a value of 0.5 would result for equal such stiffnesses.

The 30 fps video recording of the model sway frame Pluck test for Case 2(b) was analysed by Tracker for Top and first storey motion $x_1$and x_2. Notch filters of 0.4 to 2.7 Hz and 2.7 to 15 Hz, respectively, were applied to the resultant time traces x_1 and x_2, to decouple the first and second mode contributions to the responses at these storeys. The initial 5 seconds of these contributions are depicted in Figs. 7 and 8.

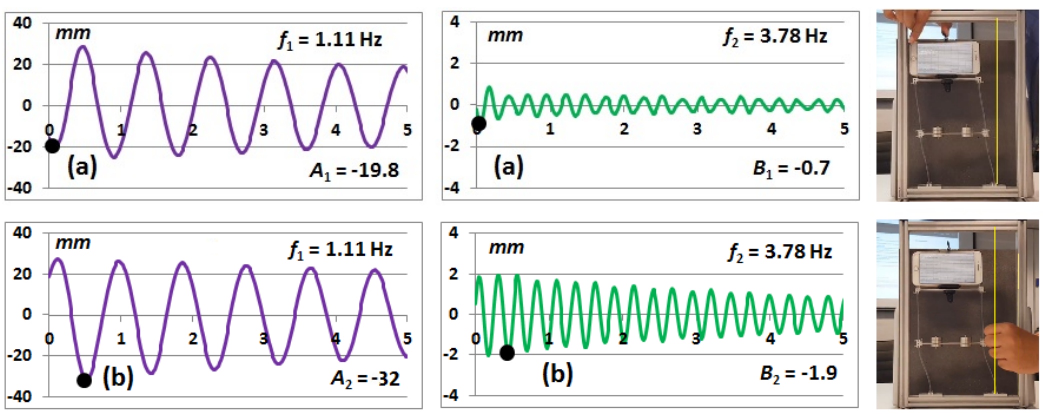


Figure 6: Top Storey Decoupled Modal Displacements for 2-storey Test Frame

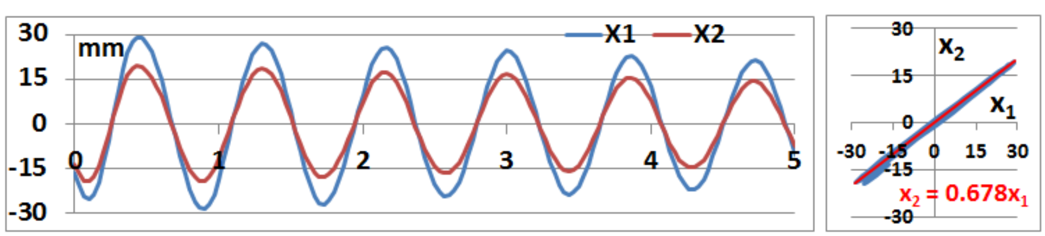


Figure 7: Top (x_1) and First (x_2) Storey Displacements for Mode#1 for Case 2(b)

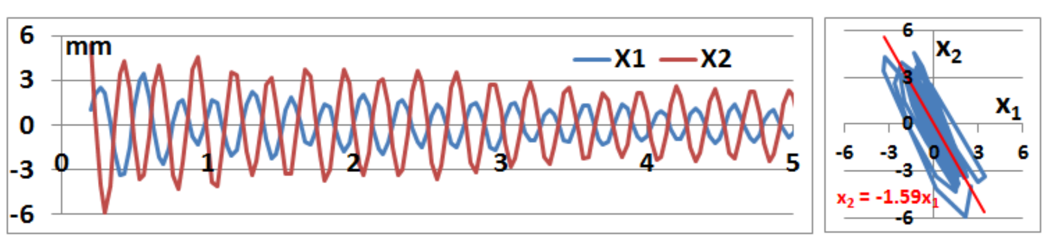


Figure 8: Top (x_1) and First (x_2) Storey Displacements for Mode#2 for Case 2(b)

The resultant u_1 and u_2 values of 0.68 and -1.59 respectively can be obtained from the plots of x_2 vs x_1 also depicted in Figs. 7 and 8. These values compare favourably with those obtained from the acceleration response records at the top-storey using the Vibration App noted earlier herein of u_1 = 0.67 and u_2 = -1.81.

An alternative to the Fourier notch-filter decoupling procedure described above for determining mode shapes of the two-storey model sway frame from only the acceleration response records taken at the Top storey from the two separate Pluck tests (j = 1, 2) is to obtain the Least Squares (LS) fit to parameters A_r, ω_r, ζ_r and t_j, in the following assumed forms, for these traces:

$$\ddot{x}_j(t) = \ddot{x}_{1,j}(t) + \ddot{x}_{2,j}(t) \quad \text{where} \quad \ddot{x}_{r,j}(t) = a_{r,j} e^{-\zeta_r \omega_r t} \cos(2\pi f_r t - t_j) \tag{6}$$

In Equation (6) above, $a_{r,j}$, ω_r (= $2\pi f_r$) and ζ_r, represent the initial acceleration response amplitude, the natural circular frequency and critical damping ratio of Mode#r. Time "lag" t_j is required in order to "align" the digital time domain record capture of the top storey acceleration response to the "start" time of the Pluck. This requirement necessitates, in turn, that the digital time domain record capture of the top storey acceleration response needs commence before the start of the actual Pluck tests.

The student records analysed earlier were commenced close to but not before the actual start of the Pluck test, so a repeat test using one of the replicate TechnoLab™ 2-storey sway frame test rigs as used in subject CVE80019 was performed by the author to obtain conforming data for exercising the LS fitting of Equation (6) using the Solver function in EXCEL.

On this occasion, the acceleration response at the Top storey was obtained using a MetaWear C BLE4.0 Motion Sensor from mbientLab sampling at 100Hz mounted thereon, (see https://mbientlab.com/product/adhesive-sensor-research-kit/). The free-of-charge MetaBase App from the manufacturers allows streaming data at up to a 200Hz sampling rate from the on-board tri-axial accelerometer sensor via Bluetooth connection to a range of smart phones. The .csv files can then be forwarded by email to users for subsequent analysis using EXCEL programs supplied by TechnoLab™, which includes ones for the LS fitting of Equation (6).

Figure 8 depicts the results of the fitting procedure to the first few seconds of the captured Top storey acceleration response traces for a nominal 50mm Pluck displacement at the top storey (so approx. half of this at the 1st storey) and an approx. 50 mm Pluck displacement at the 1st storey (so approx. the same value at the Top storey). The modal frequencies for the fitting procedure were set at the values obtained from the response frequencies at peak energy of the acceleration response spectrum, viz: f_1 = 1.02 Hz and f_2 = 3.42 Hz.

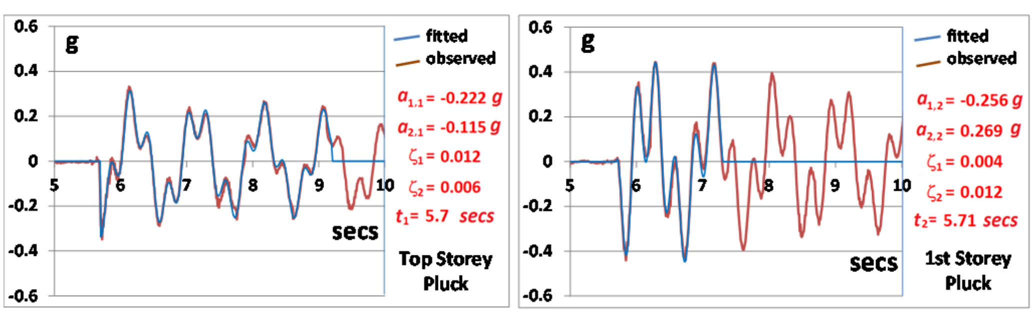


Figure 8: Least Squares fitted Modal Contributions to Top Storey Pluck Test Accelerations

It is clear from the fitted values that the damping is quite small for both modes but not reliably estimated using this method as values are inconsistent between the results in the two separate Pluck tests. However, the primary purpose of this method is to obtain estimates of A_j *and* B_j, (j = 1, 2) - the initial displacement amplitude contributions by the first and second modes to the Top storey Pluck displacement, separately, for the two Pluck tests. These values, expressed in mm, can be determined as follows:

$$A_j = -9810\left(\frac{a_{1,j}}{\omega_1^2}\right); \quad B_j = -9810\left(\frac{a_{2,j}}{\omega_2^2}\right) \tag{7}$$

Results obtained from Equation(7) in mm for values depicted in Fig. 8 become A_1 = 53.1, B_1 = 2.4 and A_2 = 61.1 , B_2 = -5.7. (These pairs add to 55.5 and 55.4 mm respectively for the two pluck tests which were nominally displaced by 50mm at the Top storey in both Pluck tests, implying errors of approx. 10% associated with this approach). Adopting these values and substituting into Equation(5) we obtain the solution to mode shape amplitudes at 1st storey level for Mode#1 (u_1) of 0.65 and for Mode#2 (u_2) of -1.88. Table 1 below summarises results for comparison purposes.

Table 1: Comparison of Experimental Results for Mode Shapes cf Nominal

	Theoretical		Decoupled Displacement		Exponential Decay Fit	
	Mode#1	Mode#2	Mode#1	Mode#2	Mode#1	Mode#2
Top Storey	1	1	1	1	1	1
1st Storey	0.62	-1.62	0.67	-1.81	0.65	-1.88

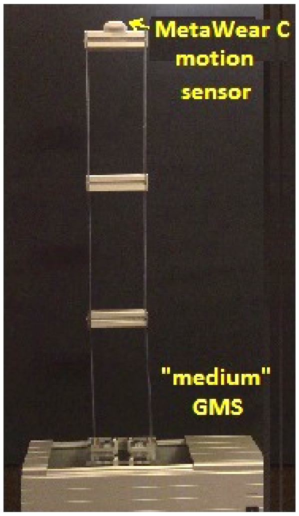
2.3 Ground Motion Simulator - Swept Sine Wave Input

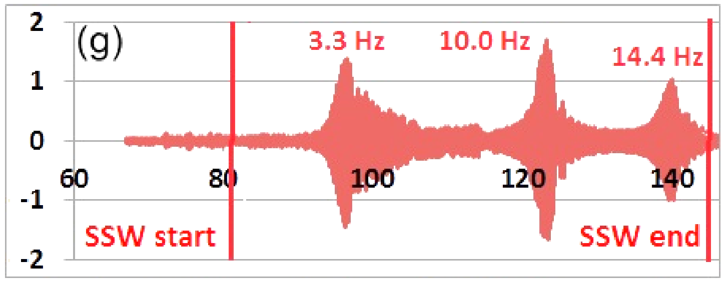
TechnoLab™ has developed a small range of desk-top Ground Motion Simulators (GMSs) that allow a number of choices of base motions as input to model test frames mounted to their base plates. These choices include: Swept Sine Waves(SSWs) with nominated start and end frequencies, f_s and f_e, and duration of shake, T_d, and scaled horizontal components of well-known recorded earthquakes such as El Centro, Northridge, Kobe, Ache, etc, as well as user-designed simulated inputs.

The “standard” SSW adopted by TechnoLab™ in its “medium” GMS model has T_d = 64 s, f_s = 0.5 Hz and f_e = 16.5 Hz. This provided a base sine-wave input frequency that linearly varies between f_s and f_e at a rate of (16.5 – 0.5)/64 = 0.25 Hz per second. Since frequency varies linearly with time, the frequency associated with the observed resonance condition of a model structure mounted on the GMS base plate can be immediately identified by the associated time value of the shake at that point.

Figure 9 depicts a 3-storey model sway frame tested with TechnoLab™’s “standard” SSW on its “medium” GMS model version. Also shown are results obtained using a MetaWear C BLE4.0 Acceleration Motion Sensor from mbientLab mounted on the top storey of this frame.

The model sway frame under test in Fig. 9 nominally has the same inter-storey sway stiffness from its vertical columns rigidly connected to the GMS base plate and the Aluminium sections (same mass) at each storey. Results obtained for the three modal frequencies from this experiment compare favourably with theoretical values.





Experimental (Observed) vs Theoretical Modal Frequencies
Three-storey Sway Frame

Observed		Ratio	Ratio	Theory	
1st Mode	3.32 Hz	0.23	0.25	0.0708	$\sqrt{\frac{k}{m}}$
2nd Mode	10.23 Hz	0.71	0.69	0.198	
3rd Mode	14.41 Hz	1.00	1.00	0.287	

Figure 9: Experiment on a 3-storey Model Sway Frame using TechnoLab™'s GMS

3 TECHNOLAB™ STATICS EXPERIMENTS EXTENDED TO DYNAMICS

The majority of the experiment kits developed for the Basic Statics and Flexure modules of the TechnoLab™ learning system can be utilised with little further modification or intervention to enable investigations of their dynamic properties.

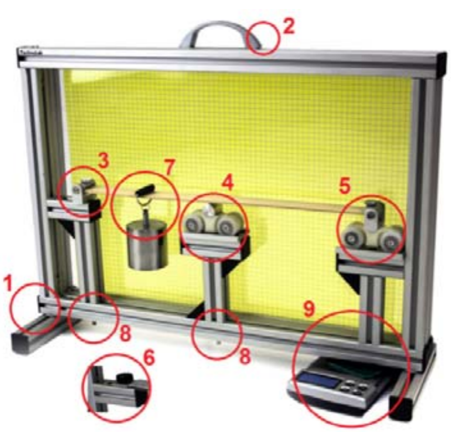
Figure 10 depicts the General Flexure series experiment kit from TechnoLab™ and the versatility it offers for a number of variations in beam support and loading conditions. The beams for this series are "ruler-like" and rectangular in cross-section, or circular solids or tubes. Several different materials and width/thickness combinations for the rectangular and circular cross-sections are available as an option.

Investigation of the accuracy of Rayleigh's Method for the natural circular frequency ω_o of a flexural system based upon a "guessed" form of its mode-shape, Y(x), viz:

$$\omega_o = \sqrt{\frac{EI}{\mu}} \sqrt{\frac{\int_0^L \left(\frac{d^2Y(x)}{dx^2}\right)^2 dx + \sum_j^{NS} k_j Y_j^2(x_j)}{\int_0^L Y^2(x)dx + \sum_i^{NM} M_i Y_i^2(x_i)}} \tag{8}$$

EI — *cross section flexural rigidity*
μ — *mass per unit length*
k_j — *point spring stiffness (NS springs)*
M_i — *point mass (NM masses)*

can be readily performed using Beam/Cantilever configurations set up using this kit and the application of distributed and/or point mass combinations.



General Flexure Series Experiments

1 Pixi Frame™ extruded T-slot ~A3 in size
2 Window Frame with handle and graticule
3 Pinned swivel joint
4 Knife-edge roller joint
5 Roller swivel joint
6 Clamp end-joint – substitute for 3 and/or 5
7 Point Load (variable location)
8 Support stand with internal reaction rod
9 Digital scales to measure reaction via rod

Remove 4 & 5 and replace 3 with 6 → cantilever
Remove 4 & replace 3 with 6 → propped cantilever
Remove 4→ single span simply supported beam
Remove 5 → overhung simply supported beam, etc

Figure 10: General Flexure Series Experiment Kit from TechnoLab™

3.1 Cantilever with Uniformly Distributed (Mass) Loading

Figure 11 (a) depicts the Tracker analysis on a short video clip taken by a hand-held iPhone camera at 30 fps of the (x, y) co-ordinates of the 4th SS ball in from the root of a simple cantilever beam with a Uniformly Distributed Load of regularly spaced SS balls over its span, of the response to a simple Pluck test. Results are depicted in Fig. 11 (c). Figure 11 (b) depicts the Tracker analysis of this same clip of the (x, y) co-ordinates of the edge of the left support leg of the Pixi Frame™. Results are depicted in Fig. 11 (d). The (x, y) response of this SS ball, corrected for the base support leg "motion" from the hand-held video is depicted in Fig. 11 (e). Despite making this correction, there still appears to be some residual low frequency motion in the vertical vibration movement of the target SS ball, which ideally should be "filtered out".

Figure 12 depicts the y-displacement trace "re-zeroed" in time to the beginning of the Pluck after "filtering out" the low frequency content below approx. 0.5 Hz. Also shown superposed to this plot is the least squares fit of the theoretical exponential decay cosine for a Pluck test of a SDOF model using Solver in Excel. The fit is excellent for this very lightly damped condition, whilst the observed frequency is also found to be close to the theoretical prediction.

4 CONCLUDING REMARKS

TechnoLab™ has developed a number of simple experiment models suitable for performing SDOF and MDOF investigations of their dynamic properties and response to base excitation. These classroom-based experiments can greatly assist students in consolidating their understanding of structural dynamics and earthquake engineering.

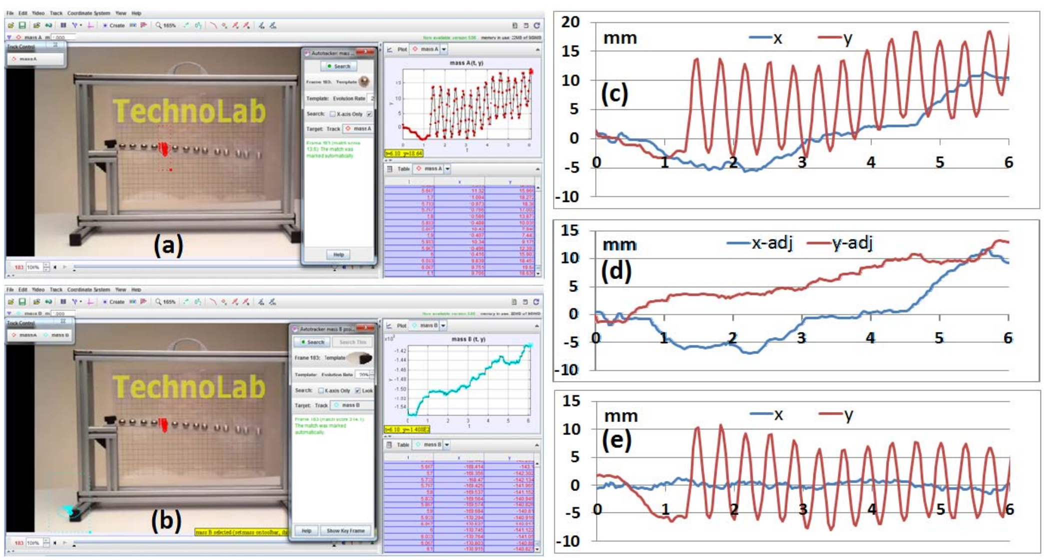


Figure 11: Tracker Investigation of a Pluck Test on a Cantilever with UDL

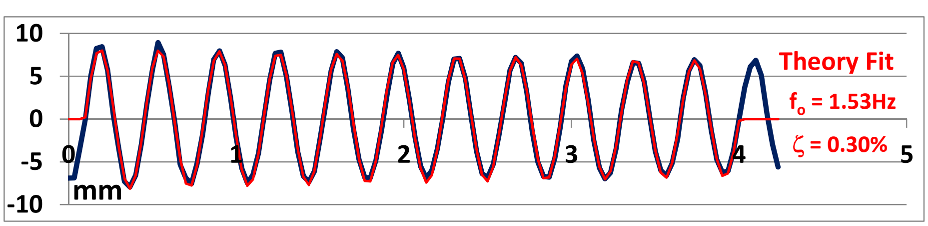


Figure 12: Exponential Decay Fit using Solver in Excel of Low Pass Filtered y-trace