Development of Simple and Transparent Non-Linear Analysis Methods for RC Walls

Scott J. Menegon[1], John L. Wilson[2], Nelson T. K. Lam[3] and Emad F. Gad[4]

1. Research Fellow, Department of Civil and Construction Engineering, Swinburne University of Technology, Melbourne, VIC, Australia; and
   Structural Engineer, Wallbridge Gilbert Aztec, Melbourne, VIC, Australia
   Email: smenegon@swin.edu.au (Corresponding Author)

2. Deputy Vice-Chancellor and Chief Executive Officer of Swinburne Sarawak, Swinburne University of Technology, Melbourne, VIC, Australia
   Email: jwilson@swin.edu.au

3. Professor of Structural Engineering, Department of Infrastructure Engineering, University of Melbourne, Melbourne, VIC, Australia
   Email. ntkl@unimelb.edu.au

4. Dean of School of Engineering, Faculty of Science, Engineering and Technology, Swinburne University of Technology, Melbourne, VIC 3122, Australia
   Email: egad@swin.edu.au

Abstract

Displacement-based seismic design procedures require knowledge of the force-displacement behaviour of both the overall building and individual lateral load resisting elements, i.e. walls or building cores. This paper will introduce the development of a user-friendly and transparent analysis program for predicting the back-bone force-displacement behaviour of slender (i.e. flexure controlled) RC walls and building cores. The program has been validated and benchmarked theoretically against commonly and widely used analysis packages and experimental test data. The program, which is called WHAM, is written using Microsoft Excel spreadsheets and the intent is to release the program free-of-charge as a design tool to assist structural engineers or be used as an educational tool for students or researchers. The paper is concluded with the results of a parametric study using WHAM, which has allowed the development of a simple empirical model for estimating the non-linear moment-curvature response of both limited ductile RC rectangular walls and building cores.

Keywords: reinforced concrete walls, non-linear analysis of RC walls, non-linear moment-curvature analysis of RC walls.

1 Introduction

A simple-to-use and transparent analysis program was developed using Microsoft Excel spreadsheets for predicting the force-displacement behaviour of RC walls and building cores. The program is called WHAM and was developed using Excel spreadsheets as they offer complete transparency, such that the user can easily examine and understand how the program works, while also being able to easily further develop or expand the capabilities of the program to suit their respective needs.

One of the primary objectives while developing WHAM was to ensure it was simple-to-use and had a user-friendly interface so designers or students, which whom have had little or no exposure and experience using non-linear analysis packages, could easily understand, adopt and use the program.

2 Program Development and Interface

WHAM is a fibre-element analysis program, which determines the non-linear moment-curvature response of the section based on the concrete and reinforcement non-linear stress-strain material models selected and the axial load applied to the wall. The fibre-element analysis procedure accounts for tension stiffening of the section using the model proposed by Menegon (2018). The force-displacement response of the wall is calculated using Equations 1 to 6, which assumes an equivalent plastic hinge at the base of the wall and a linear curvature profile up the height of the wall. The plastic hinge model adopted in WHAM is the Priestley, Calvi and Kowalsky (2007) model. An idealised force-displacement response is presented in Figure 1.

Cracking:

$$\Delta_{cr} = \frac{\phi_{cr} H_{eff}^2}{3} \quad \text{. . . 1}$$

$$F_{cr} = \frac{M_{cr}}{H_{eff}} \quad \text{. . . 2}$$

Point of first yield:

$$\Delta'_y = \frac{\phi'_y H_{eff}^2}{3} \quad \text{. . . 3}$$

$$F_y = \frac{M_y}{H_{eff}} \quad \text{. . . 4}$$

The i-th point after first yield:

$$\Delta_i = \Delta'_y \left(\frac{M_i}{M_y}\right) + \left[\phi_i - \phi'_y \left(\frac{M_i}{M_y}\right)\right] L_p \left[H_{eff} - \left(\frac{L_p}{2} - L_{sp}\right)\right] \quad \text{. . . 5}$$

$$F_i = \frac{M_i}{H_{eff}} \quad \text{. . . 6}$$

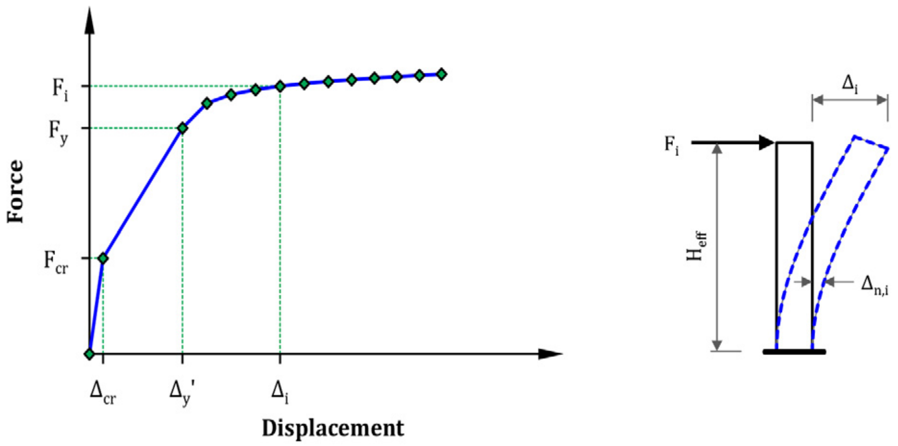


Figure 1: Idealised force-displacement response (Menegon 2018).

The program is split across several worksheets in a macro enabled Microsoft Excel file. The user enters the cross section of the wall or building core by entering the x and y nodal coordinates of the section, as shown in Figure 2. The program can handle various wall cross sections, e.g. Figures 3(a) to 3(e). The reinforcement in the wall can be generated automatically by entering a maximum reinforcement centre-to-centre spacing or desired reinforcement ratio (i.e. $p_v = A_{sv}/A_g$). The automatic function can also be disabled and the user can input each individual reinforcing bar using the x and y coordinates of each respective bar. Further, the automatic and manual functions can also be used together. The user can also enter confined regions of concrete within the cross section of the wall.

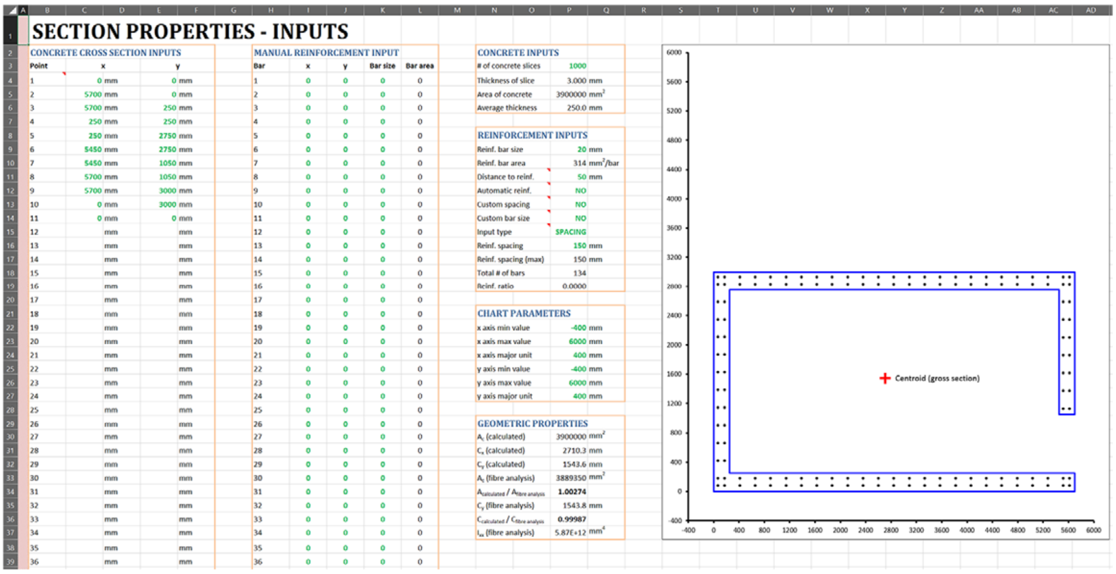
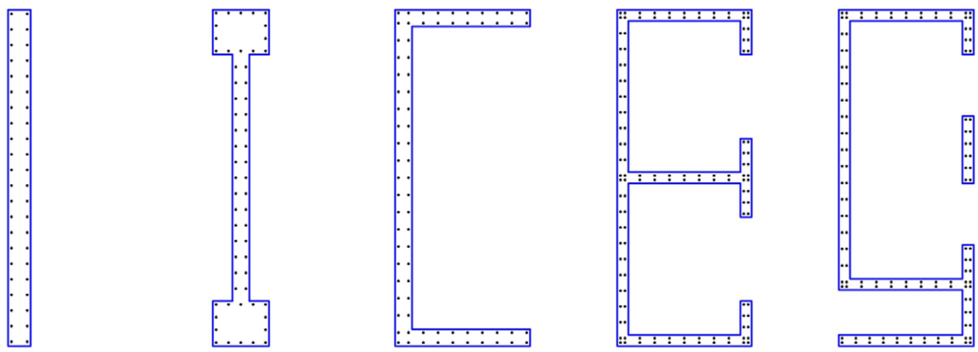


Figure 2: WHAM section input page (Menegon 2018).



(a) rectangular **(b) flanged wall** **(c) 'C' section** **(d) Bundled box** **(e) Geometric**

Figure 3: Example wall and building core cross sections that can be analysed.

The overall framework of program was initially based on the methodology presented by Lam, Wilson and Lumantarna (2011). The approach used to divide the complex non-rectangular cross sections into finite constant thickness fibres and to automatically generate reinforcement was adopted from Lam et al. (2011), however a different approach was developed for performing the required iterations to solve each respective point on the non-linear moment-curvature response curve, which uses a Visual Basic Applications (VBA) macro.

The program allows the user to choose from a selection of different non-linear material models, in addition to manually entering their own model using tabulate stress-strain data. However, the Mander, Priestley and Park (1988) concrete model and a bilinear reinforcement model is recommended for limited ductile RC walls in Australia. The Mander et al. (1988) model was used for the experimental validation below. Further details regarding the development of the program are provided in Menegon (2018).

3 Program Validation

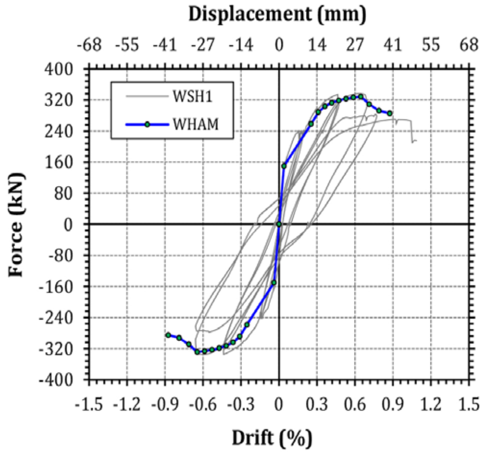
Two approaches were adopted for validating the program. The first approach was a theoretical validation, which was being used to confirm that both the general coding and fibre element analysis engine worked and was written correctly. The theoretical approach was performed by comparing the moment-curvature results obtained from WHAM against the results from two independent software packages for two different wall cross sections. The first was the commercial software package RAPT (Prestressed Concrete Design Consultants Pty Ltd 2007), which is a widely used structural analysis package for analysing conventional reinforced, prestressed and post-tensioned elements. The second was a analysis package called Response-2000 (Bentz 2000a), which is a sectional analysis program developed by researchers at the University of Toronto and is available to download free-of-charge online.

Two wall cross sections were analysed in WHAM and the two independent software packages for incrementally increasing axial load ratios. Very good correlation between WHAM and RAPT was observed. However, slightly different moment-curvature responses were observed between WHAM and Response-2000. This was due to the tension stiffening approach adopted by each respective program. Response-2000 adopts the tension stiffening approach proposed by Bentz (2000b), whereas WHAM uses the procedure proposed by Menegon (2018). When tension stiffening is turned off in each respective program the moment-curvature results correlate very well. This methodology provided good validation that the fibre element analysis engine written for WHAM works as intended. Detailed discussions and visual comparisons of this first theoretical validation are presented in Menegon (2018).

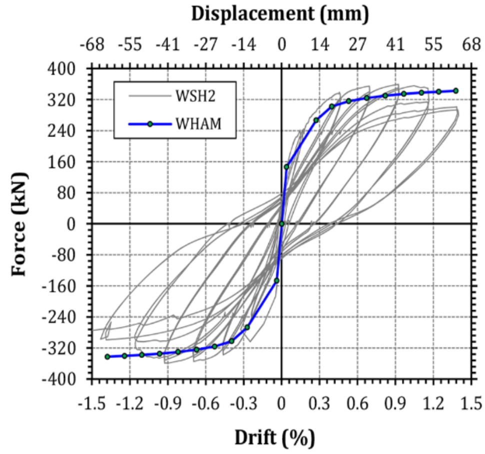
The second approach was an experimental validation, which was used to confirm that the overall process resulted in back-bone force-displacement curves that correlated well with experimentally tested laboratory specimens of RC walls. The experimental approach was performed using the results of 16 test specimens, which included the two cast in-situ wall specimens tested by the authors in Menegon et al. (2017) and another 14 test specimens from literature (Dazio, Beyer and Bachmann 2009; Lu et al. 2017; Thomsen and Wallace 2004; Tran and Wallace 2015).

The 16 test specimens had a wide range of parameters, which included shear-span ratios that varied from 2.0 to 6.5, axial load ratios varying from 0.035 to 0.128 and vertical reinforcement ratios varying from 0.005 to 0.071. The details of all 16 test specimens and comprehensive discussions and comparisons of the calculated WHAM response and the observed experimental results are presented in Menegon (2018). The contents in this paper however, are limited to the comparisons between WHAM and the walls tested by Dazio et al. (2009), which are shown in Figure 4.

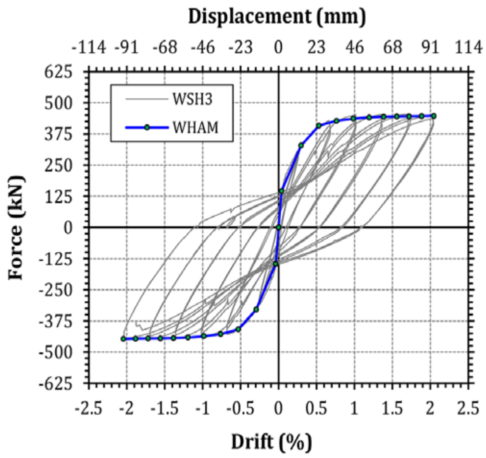
Very good correlation was generally observed between the test specimens tested by Dazio et al. (2009) (i.e. WSH1 to WSH6) and the program. Particularly good correlation was observed in specimens WSH1, WSH3, WSH4 and WSH4. The strength was slightly underpredicted in specimens WSH2 and WSH5.



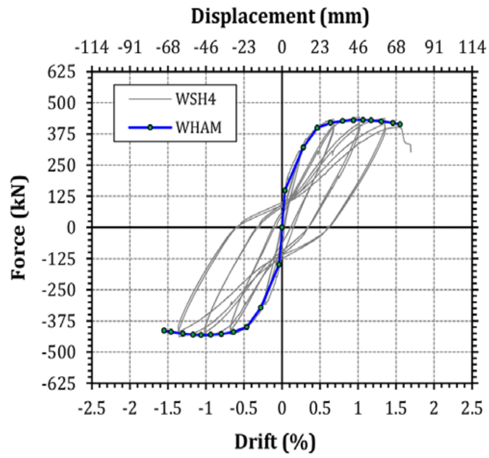
(a) test specimen WSH1



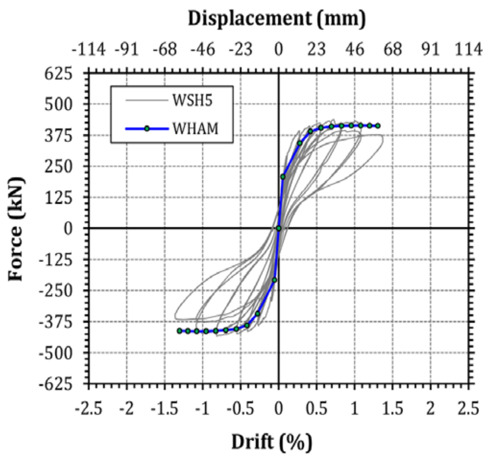
(b) test specimen WSH2



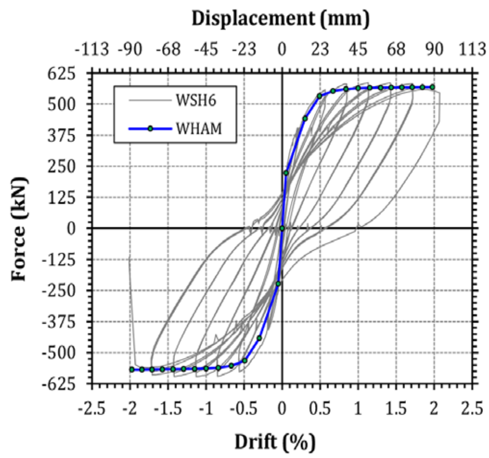
(c) test specimen WSH3



(d) test specimen WSH4



(e) test specimen WSH5



(f) test specimen WSH6

Figure 4: Experimental validation of WHAM (Menegon 2018).

Overall, very good correlation was observed between the experimental test data of the rectangular wall specimens and the theoretical predictions of the test program. This included test specimens with a wide range of shear-span ratios, axial load ratios and vertical reinforcements ratios. This shows the program can quite confidently predict the back-bone force-displacement behaviour of rectangular walls and particularly, limited ductile rectangular walls, which are of particular importance to seismic design in Australia.

A limited number of specimens were used to validate the programs ability to predict the back-bone force-displacement behaviour of non-rectangular walls. For the most part, the correlation was not as strong as the rectangular walls. Further research and development is recommended for non-rectangular walls, which may require the development and implementation of a different plastic hinge model specifically for non-rectangular walls of various cross sections. In the interim, with respect to non-rectangular wall sections, the program should be limited to performing only non-linear moment-curvature analyses.

4 Parametric Study and Empirical Moment-Curvature Relationships

WHAM was used to perform a parametric study to investigate the non-linear behaviour of limited ductile RC walls and building cores. The study was then used to develop simplified empirical models for determining the bilinear moment curvature response of a rectangular wall or building core cross section, which can then simply be converted to a bilinear force-displacement response using Equations 1 to 6.

The parametric study was undertaken for typical rectangular wall and box-shaped building core cross sections, as shown in Figure 5. Wall lengths of 2000, 3500 and 5000 mm were considered for the rectangular wall section. Similarly, for the building core section, three geometric combinations were used, which were: 2500 mm long and 2500 mm wide; 3500 mm long and 2500 wide; and 2500 mm long and 3500 mm wide. The other parameters that varied for each section was: wall thickness, which varied from 200 mm to 300 mm; concrete grade, which varied from 40 to 65 MPa characteristic compressive strengths; axial load ratio, i.e. $N^*/(A_g f_{cmi})$, which varied from 0.0 to 0.15; bar diameter, which varied from 16 to 24 mm; and reinforcement ratio, which varied from 0.7% to 3.0%. Mean material properties for concrete and reinforcement, as proposed by Foster et al. (2016) and (Menegon et al. 2015) respectively, were adopted for the analysis. The Mander et al. (1988) concrete model and a bilinear reinforcement model were used for the analysis.

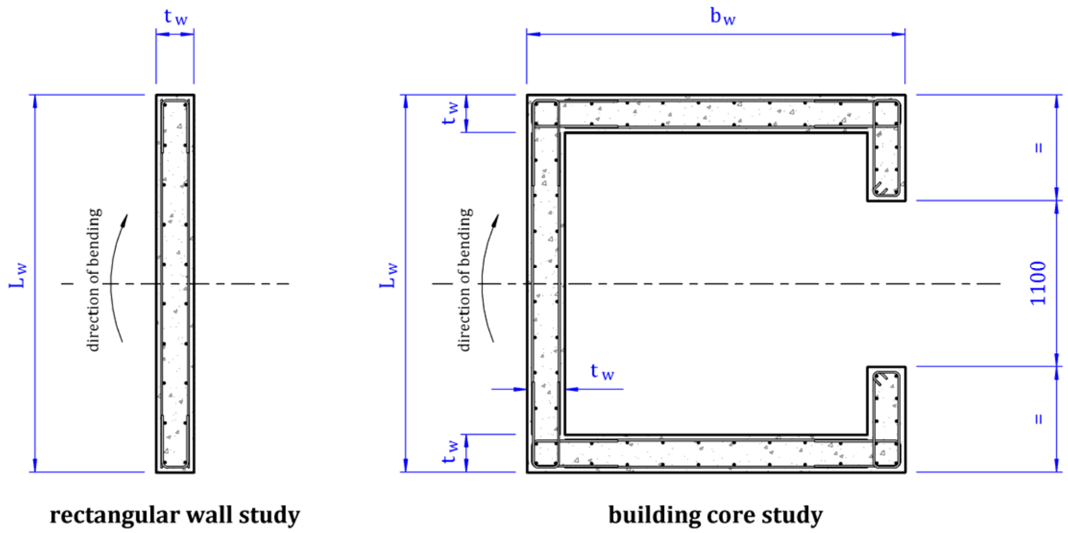


Figure 5: Parametric study cross sections (Menegon 2018).

The failure criteria adopted in the parametric study was determined with reference to the recommendations provided by Menegon (2018), where it is proposed for limited ductile RC walls or building cores that a maximum concrete compressive strain and reinforcement tensile strain of 0.6% and 5% respectively be adopted for the no collapse (i.e. ULS) performance of the structure.

As such, failure for the walls in the parametric study was taken to be when either a compressive strain of 0.6% in the concrete or tensile strain of 5% in the reinforcement is reached, whichever occurs first.

A bilinear approximation of the non-linear moment-curvature response of each wall analysis was constructed around the yield moment (i.e. M_y), the maximum moment (i.e. M_u), the notional yield curvature (i.e. ϕ'_y), the yield curvature (i.e. ϕ_y) and the ultimate curvature (i.e. ϕ_u). The yield moment is taken as the point of first yield, i.e. the point when the extreme tensile reinforcing bar yields or a compressive strain of 0.2% is reached. The overstrength (i.e. Ω) is then taken as the maximum moment divided by the yield moment and similarly, the curvature ductility (i.e. μ_ϕ) is taken as the ultimate curvature divided by the yield curvature. The effective moment of inertia of the cross section (i.e. I_{eff}) is taken as the slope of line from origin through the point of first yield (i.e. the notional yield curvature) divided by the elastic modulus of the concrete, i.e. $I_{eff} = \left(M_y/\phi'_y\right)/E_c$. Each of these parameters are further illustrated in Figure 6.

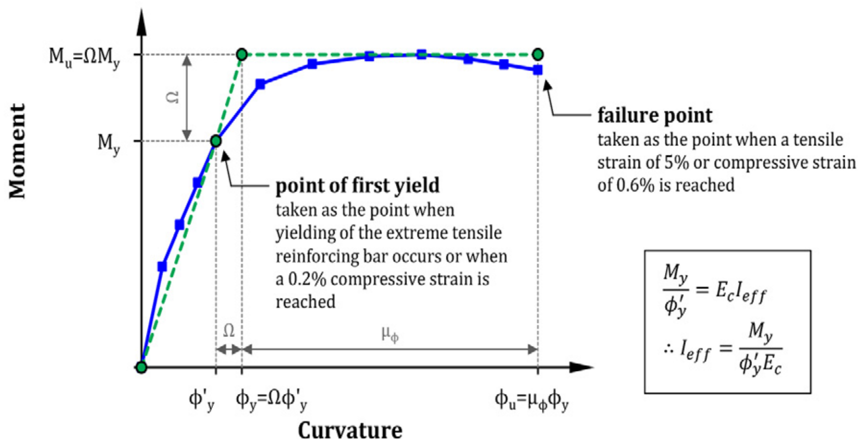


Figure 6: Bilinear approximation of moment-curvature response.

The results of the parametric study were used to create empirical models for determining a bilinear moment-curvature response of both limited ductile rectangular walls and building cores. The moment capacity of the section in each model is calculated using the elastic modulus of the concrete, the effective moment of inertia of the cross section and the yield curvature. The moment capacity of an RC wall section is usually quite difficult to calculate using 'simple hand calculations' since walls are commonly detailed to have multiple vertical bars at varying locations across the depth of the section (i.e. wall length). This means that computer programs generally have to be heavily relied on to calculate their moment capacities. The proposed method however, allows a designer to approximately calculate the moment capacity with relative ease and without the need to rely on commercial software packages or other computer-based design aids.

The empirical model for limited ductile rectangular walls is presented by Equations 7 to 13 and similarly, the empirical model for limited ductile building cores is presented by Equations 14 to 20. The proposed empirical models for rectangular walls and building cores provides reasonably accurate results, as indicated by Table 1, which show the percentage of predicted values using the respective model that are within ±15% of the actual value determined in the parametric study using WHAM. The rectangular wall model generally provided more accurate predictions than the building core model. Further details are presented in Menegon (2018).

Rectangular walls:

$$M_y = (E_c I_{eff})\phi'_y \quad \ldots 7$$

$$M_u = (E_c I_{eff})\phi_y \quad \ldots 8$$

$$\phi'_y = \frac{\phi_y}{\Omega} \quad \ldots 9$$

$$\phi_y = \frac{0.15p_v - 2p_v^2 + 0.0031}{L_w} \quad \text{where: } 0.005 \le p_v \le 0.035 \quad \ldots 10$$

$$\phi_u = \frac{a_1 p_v^3 + a_2 p_v^2 + a_3 p_v + a_4}{L_w} \quad \text{where: } 0.005 \le p_v \le 0.035 \quad \ldots 11$$

$$\frac{I_{eff}}{I_g} = p_v(10 - 30n) + 0.03 f_{cmi} n + 0.1 \quad \ldots 12$$

$$\Omega = 9.1n^2 - 3.6n + 1.6 \quad \text{where: } 0.0 \le n \le 0.2 \quad \ldots 13$$

Where: E_c = elastic modulus in accordance with AS 3600
= $(2400^{1.5}) \times (0.043\sqrt{f_{cmi}})$ where: $f_{cmi} \le 40$ MPa
= $(2400^{1.5}) \times (0.024\sqrt{f_{cmi}} + 0.12)$ where: $f_{cmi} > 40$ MPa
n = axial load ratio, i.e. $N^*/(f_{cmi}A_g)$
p_v = vertical reinforcement ratio, i.e. A_{sv}/A_g
f_{cmi} = mean in-situ concrete compressive strength
a_1 to a_4 = curve fitting constants for curvature ductility factor
a_1 = $3500 - 22000n$
a_2 = $1700n - 270$
a_3 = $5.8 - 37n$
a_4 = $0.135n - 0.004$

Building cores:

$$M_y = (E_c I_{eff})\phi'_y \quad \ldots 14$$

$$M_u = (E_c I_{eff})\phi_y \quad \ldots 15$$

$$\phi'_y = \frac{\phi_y}{\Omega} \quad \ldots 16$$

$$\phi_y = \frac{0.13p_v - 1.9p_v^2 + 0.0024}{L_w} \quad \text{where: } 0.005 \le p_v \le 0.035 \quad \ldots 17$$

$$\phi_u = \frac{2.2p_v - 31p_v^2 + 0.0097}{L_w} \quad \text{where: } 0.005 \le p_v \le 0.035 \quad \ldots 18$$

$$\frac{I_{eff}}{I_g} = p_v(10 - 25n) + 0.025 f_{cmi} n + 0.05 \quad \ldots 19$$

$$\Omega = 3.8n^2 - 1.5n + 1.33 \quad \text{where: } 0.0 \le n \le 0.2 \quad \ldots 20$$

Where: E_c = elastic modulus in accordance with AS 3600
= $(2400^{1.5}) \times (0.043\sqrt{f_{cmi}})$ where: $f_{cmi} \le 40$ MPa
= $(2400^{1.5}) \times (0.024\sqrt{f_{cmi}} + 0.12)$ where: $f_{cmi} > 40$ MPa
n = axial load ratio, i.e. $N^*/(f_{cmi}A_g)$
p_v = vertical reinforcement ratio, i.e. A_{sv}/A_g
f_{cmi} = mean in-situ concrete compressive strength

Table 1: Percentage of predicted values within ±15% of the actual value.

Quantity	Symbol	Rectangular wall model	Building core model
Yield moment	M_y	99.7%	73.1%
Maximum moment	M_u	98.1%	69.8%
Notional yield curvature	ϕ_y'	100%	100%
Yield curvature	ϕ_y	100%	99.4%
Ultimate curvature	ϕ_u	91.0%	87.0%
Overstrength	Ω	100%	100%
Moment of inertia ratio	I_{eff}/I_g	100%	98.9%

5 Conclusions

This paper has outlined the development of a simple, user-friendly and transparent analysis program for predicting the back-bone force-displacement behaviour of slender (i.e. flexure-controlled) RC walls and building cores. The program is called WHAM and is written using Microsoft Excel spreadsheets and will be released as a free-of-charge design tool for practicing structural engineers or as an educational tool for undergraduate students or researchers. The program was validated using both theoretical and experimental approaches.

The program was used to perform a parametric study into limited ductile rectangular walls and building cores. The results of the parametric study were used to develop empirical models for both rectangular walls and building cores, which can be used to quickly determine a bilinear moment-curvature response without performing any complex calculations or needing to use computer-based design aids. The empirical models provided predictions that were reasonable accurate and consistently within ±15% of the actual value calculated using the program.

6 Acknowledgements

The authors would like to thank the Brown family for their generous donation in establishing the Dr William Piper Brown AM Scholarship, of which the lead author is the recipient. Financial support from the Australian Research Council (ARC) Discovery Project DP140103350 entitled *Collapse Assessment of Reinforced Concrete Buildings in Regions of Lower Seismicity* is gratefully acknowledged.

7 References

Bentz, E., 2000a, *Response-2000*, v1.0.5 (computer program).

Bentz, E.C., 2000b. *Sectional Analysis of Reinforced Concrete*, Doctor of Philosophy thesis, Department of Civil Engineering, University of Toronto.

Dazio, A., Beyer, K. and Bachmann, H., 2009. Quasi-static cyclic tests and plastic hinge analysis of RC structural walls, *Engineering Structures*, Vol. 31(7), pp. 1556-1571.

Foster, S.J., Stewart, M.G., Loo, M., Ahammed, M. and Sirivivatnanon, V., 2016. Calibration of Australian Standard AS 3600 Concrete Structures: part I statistical analysis of material properties and model error, *Australian Journal of Structural Engineering*, Vol. 17(4), pp. 242-253.

Lam, N., Wilson, J. and Lumantarna, E., 2011. Force-Deformation Behaviour Modelling of Cracking Reinforced Concrete by EXCEL spreadsheets, *Computers and Concrete*, Vol. 8(1), pp. 43-57.

Lu, Y., Henry, R.S., Gultom, R. and Ma, Q.T., 2017. Cyclic Testing of Reinforced Concrete Walls with Distributed Minimum Vertical Reinforcement, *Journal of Structural Engineering*, Vol. 143(5).

Mander, J.B., Priestley, M.J.N. and Park, R., 1988. Theoretical stress-strain model for confined concrete, *ASCE Journal of Structural Engineering*, Vol. 114(8), pp. 1827-1849.

Menegon, S.J., 2018. *Displacement Behaviour of Reinforced Concrete Walls in Regions of Lower Seismicity*, Doctor of Philosophy thesis, Department of Civil and Construction Engineering, Swinburne University of Technology.

Menegon, S.J., Tsang, H.H., Wilson, J.L. and Lam, N.T.K., 2015. Overstrength and ductility of limited ductile RC walls: from the design engineers perspective, *Proceedings of the Tenth Pacific Conference on Earthquake Engineering, Building an Earthquake-Resilient Pacific, 6-8 November 2015*, Sydney, Australia.

Menegon, S.J., Wilson, J.L., Lam, N.T.K. and Gad, E.F., 2017. Comprehensive Experimental Testing Program of Limited Ductile Reinforced Concrete Walls, *Proceedings of the Australian Earthquake Engineering Society 2017 Conference, November 24-26*, Canberra, ACT.

Prestressed Concrete Design Consultants Pty Ltd, 2007, *RAPT*, v6.2.2.2 (computer program).

Priestley, M.J.N., Calvi, G.M. and Kowalsky, M.J., 2007. *Displacement-Based Seismic Design of Structures*, IUSS Press, Pavia, Italy.

Thomsen, J.H. and Wallace, J.W., 2004. Displacement-Based Design of Slender Reinforced Concrete Structural Walls - Experimental Verification, *Journal of Structural Engineering*, Vol. 130(4), pp. 618-630.

Tran, T.A. and Wallace, J.W., 2015. Cyclic Testing of Moderate-Aspect-Ratio Reinforced Concrete Structural Walls, *ACI Structural Journal*, Vol. 112(6), pp. 653-665.