Methods of Structural Analysis of Buildings Incorporating Higher Modes Effects

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Abstract

Dynamic analysis method has been suggested for the analysis of irregular buildings in the seismic standards. However, dynamic analyses can be computationally expensive and there are uncertainties associated with the selection of ground motion records for the regions of low to moderate seismicity. In a relevant research by the authors, it has been shown that linear response of irregular buildings featuring transfer beams resembles the response of regular buildings in the regions of low to moderate seismicity such as Australia. Hence, *Generalised Lateral Force Method* (GLFM) of Analysis is introduced and developed to incorporate the effects of higher modes based on generic modal displacement values. This method can be used for shear wall dominant RC buildings with or without transfer beam irregularity features. Shortcomings of the conventional *Equivalent Static Analysis Method* are resolved and the robustness of the method in estimating the seismic demands within the elastic limit is demonstrated by comparison with dynamic analyses.

Keywords: multi-storey buildings, higher mode effects, static analysis method, dynamic analysis method

1 INTRODUCTION

Dynamic analysis method has been suggested for the analysis of irregular buildings in the seismic standards e.g. *Eurocode 8* (British Standard, 2005). However, dynamic analyses can be computationally expensive and there are uncertainties associated with the selection of ground motion records for the regions of low to moderate seismicity. There are also currently no consistent guidelines on the selection of ground motions for the analyses. For instance, seismic design codes and assessment guidelines such as *Eurocode 8* and *FEMA 356* (FEMA, 2000) require the use of a minimum of three sets of ground motions for the time history analyses. The National Institute of Standards and Technology (*NIST*) (Venture, 2011) recommends the use of seven sets of ground motions if the average maximum responses are to be obtained whilst no less than 30 sets of motions are required for construction of fragility curves. Furthermore, there are uncertainties associated with the selection of ground motion records for the analyses in regions of low to moderate seismicity such as Australia.

On the other hand, the equivalent static analysis method generally assumes that stiffness and mass quantities are gradually decreasing along the height of buildings and the total mass of building is contributing wholly to the first mode. Based on these assumptions, a distribution function to compute inertia forces at floor levels is being recommended. It is well-known that for the cases which contribution of higher modes is not negligible this method leads to overestimation of displacements and inaccurate estimation of storey shear forces.

Parametric studies based on the dynamic analyses have been conducted by the authors to investigate the effects of discontinuities in the gravity load carrying elements of multi-storey buildings (Mehdipanah et al., 2016a). It has been shown that the linear response of irregular buildings featuring transfer beams resembles the response of regular buildings in the regions of low to moderate seismicity such as Australia. There is a scope to develop a method based on generalised mode of response.

Generalised Lateral Force Method (GLFM) of Analysis has been developed to incorporate the effects of higher modes on the response of buildings using a simple analysis approach. This method can be used for shear wall dominant RC buildings with or without transfer beam irregularity features. Shortcomings of the conventional *Equivalent Static Analysis Method* are resolved and the robustness of the method in estimating the seismic demands within the elastic limit is demonstrated by comparison with the results from dynamic analyses.

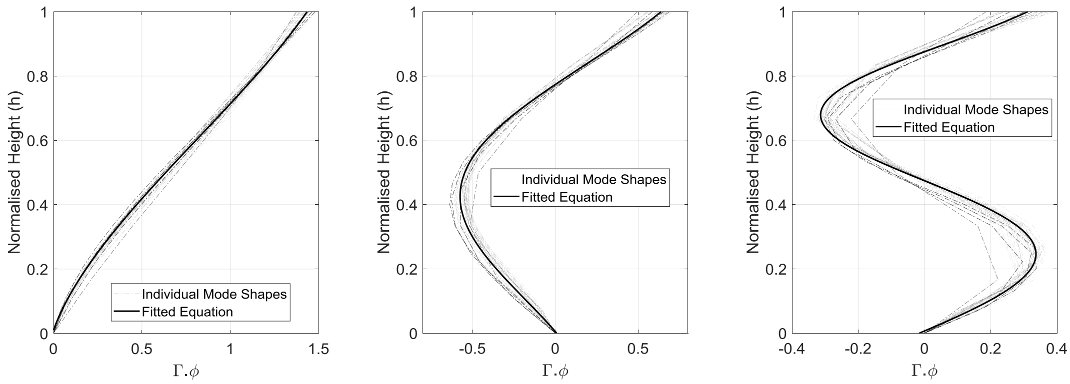
2 GENERALISED MODE SHAPES

Parametric studies had been carried out by the authors to investigate the seismic performance of reinforced concrete buildings, supported by a combination of shear walls or cores and frames featuring vertical irregularities. Effects of interruptions in the load path by discontinuities of columns (irregularity class associated with buildings featuring transfer beam) had been studied. 75 case study buildings were selected to feature different extent and locations of vertical irregularities. The height of the studied buildings varies between 19.8 and 50.0 m. The case study buildings were designed in accordance with the Australian Standards (*AS/NZS 1170.0:2002, AS/NZS 1170.2:2011*

and *AS/NZS 1170.4:2007* for the gravity, earthquake and wind loads, respectively. Description of the buildings, modelling approach and the analysis are presented in Mehdipanah et al. (2016a and 2016b).

It was found that linear response of irregular buildings featuring transfer beams resembles the response of regular buildings in the regions of low to moderate seismicity such as Australia and the modal displacement values are not significantly affected by the building parameters Mehdipanah et al. (2016a).

Based on the study, generalised mode shapes of the first, second and third modes of vibration for the multi-storey wall-frame buildings are provided in this section. Results from the studies presented in the form of generalised modal displacement versus normalised height of the buildings (Figure 1). The modal displacement values are defined as the product of the modal deflection shapes and participation factors ($\Gamma.\phi$). The elevation of each level of the buildings was normalised to the total height of the buildings. Hence, the normalised height (h) for the ground level floor is 0.0 and this value for the roof level floor is 1.0.The first three modes of vibration of the entire building models are plotted in Figure 1. Mean and median values of the modal displacements have been computed and were not found to be significantly different.



(a) Fundamental mode (b) Second mode (c) Third mode

Figure 1 Generalised mode shapes for the first 3 modes of vibration

Equations (1-3) were determined by curve fitting to the mean of the modal displacements for the first, second and third mode of vibration:

a) For mode 1: (1)

$$\Gamma.\phi = p_1 h^3 + p_2 h^2 + p_3 h + p_4$$

where,

$p_1 = -0.9849, p_2 = 1.795, p_3 = 0.6316, p_4 = -0.007134$

(Goodness of fit: R2: 0.9998)

b) For mode 2:

$$\Gamma.\phi = a_1 \sin(b_1.h + c_1) + a_2 \sin(b_2.h + c_2) \quad (2)$$

where,

$a_1 = 0.5793, b_1 = 4.283, c_1 = 2.53, a_2 = 0.357, b_2 = 2.462, c_2 = -1.142$

(Goodness of fit: R^2: 0.9988)

c) And for mode 3:

$$\Gamma.\phi = a_1 \sin(b_1.h + c_1) + a_2 \sin(b_2.h + c_2) + +a_3 \sin(b_3.h + c_3) \quad (3)$$

where,

$a_1 = 0.3549\ , b_1 = 6.748, c_1 = 0.07549, a_2 = 0.2757\ , b_2 = 0.5729, c_2 = -0.1618\ , a_3 = 0.02189\ , b_3 = 11.61, c_3 = 3.059$

(Goodness of fit: R^2: 0.998)

From the study conducted by the authors (Mehdipanah et al., 2016a), the ratio of the second mode period (T_2) to the fundamental period of vibration (T_1) was found to be approximately constant and equal to 0.25. The value for the ratio of third mode (T_3) to first mode (T_1) was found to be approximately equal to 0.1.

3 GENERALISED LATERAL FORCE METHOD (GLFM) OF ANALYSIS

A simple, rapid and robust method to provide estimates of seismic demands of multi-storey buildings is proposed. First, the application of the method will be described for buildings that respond primarily in the first mode (section 3.1) and then its application will be extended to incorporate higher mode effects (section 3.2).

3.1 GENERALISED LATERAL FORCE METHOD (GLFM) OF ANALYSIS CONSIDERING THE FUNDAMENTAL MODE OF VIBRATION

The *Generalised Lateral Force Method (GLFM) of Analysis* considering the fundamental mode of vibration involves a demand curve in the form of acceleration-displacement response spectrum and a capacity curve based on the linear elastic response assumption of the building. The capacity curve can be constructed based on displacement values obtained from the equivalent static analyses in accordance with seismic design procedures. First, the equivalent static analysis is performed in accordance with the seismic design standards (e.g., AS 1170.4:2007). The deflection values obtained by the equivalent static analysis method are shown schematically in Figure 2.

The multi-storey building response can be idealised into a single-degree-of-freedom (SDOF) response by calculating the effective displacement of the buildings:

$$\delta_{eff} = \frac{\sum m_j \delta_j^2}{\sum m_j \delta_j} \tag{4}$$

and the effective mass of the buildings is:

$$m_{eff} = \frac{\left(\sum m_j \delta_j\right)^2}{\sum m_j \delta_j^2} \tag{5}$$

where, δ_j and m_j are the displacement and mass of storey *j*.

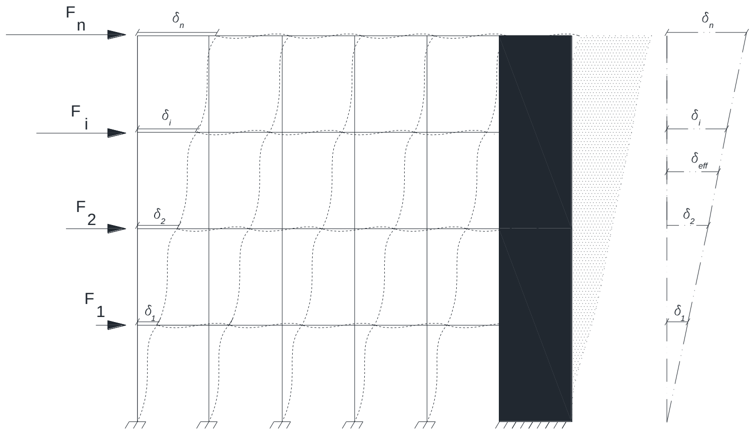


Figure 2 Deflected shape of the building

RSD (mm) Step 2-?*eff by GLFM Step 1-?eff by ESA

The effective stiffness (k_{eff}) and effective natural period (T_{eff}) of the buildings can be calculated by Equations (6-7).:

$$k_{eff} = \frac{V}{\delta_{eff}} \tag{6}$$

where, V is the horizontal equivalent static shear force.

$$T_{eff} = 2\pi\sqrt{\frac{m_{eff}}{k_{eff}}} \tag{7}$$

The capacity diagram can be plotted in the acceleration vs displacement format (Figure 3), where the effective acceleration (a_{eff}) is:

$$a_{eff} = \frac{V}{m_{eff}} \tag{8}$$

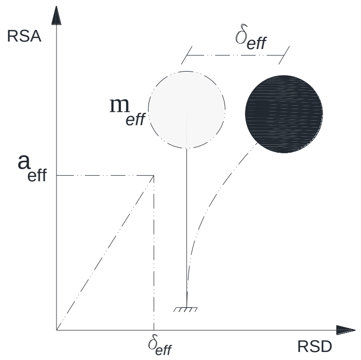


Figure 3 Capacity diagram in the acceleration vs displacement format

The capacity diagram is superposed onto the demand diagram in the acceleration-displacement response spectrum format (the ADRS diagram) as shown schematically in Figure 3 to identify the performance point representing the displacement demand on the buildings (δ^*_{eff}). The construction of the ADRS diagram has been presented in Lam et al. (2016) and Mehdipanah et al. (2016b). The displacement demand (δ^*_{eff}) is then used to obtain the displacement demand values at each floor level of the buildings (Figure 5).

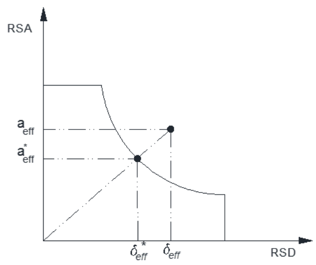


Figure 4 Superposition of the capacity and demand diagrams

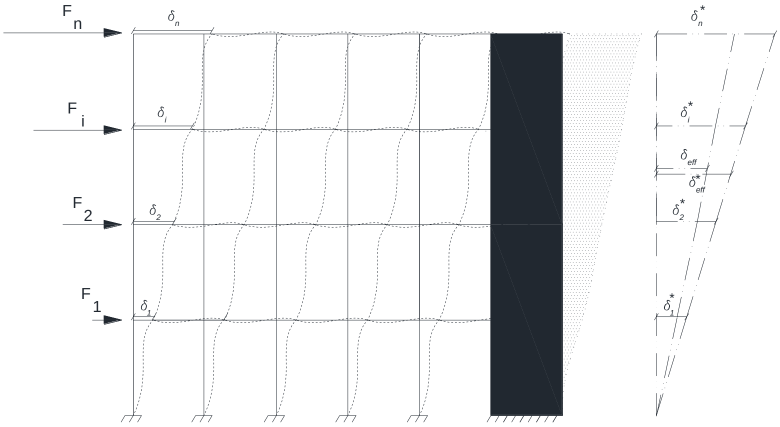


Figure 5 The displacement demand values of the buildings

3.2 GENERALISED LATERAL FORCE METHOD (GLFM) OF ANALYSIS CONSIDERING HIGHER MODES

For taller buildings, usually contributions from higher modes is not negligible. The effects of higher modes can become significant when the building height is larger than 30 m. For most buildings, at least 90% of the total mass is participating in the first three modes of vibration. Hence, proposed method has been extended to cover the contribution from the second and third modes of response to the overall lateral response.

3.2.1 Estimating the displacement profile

The mean values of the modal displacements are presented in Section 2 for the first, second and third modes of response. Based on the means of T_2/T_1 and T_3/T_1 ratios and the generalised modal displacements presented in Section 2, the contribution from the second and third modes to the overall displacement response ($\delta^*_{i,2}$and $\delta^*_{i,3}$) can be defined by:

$$\delta^*_{i,2} = [\Gamma.\phi]_{i,2}.[RSD(T_2 = 0.25T_1)] \tag{9}$$

$$\delta^*_{i,3} = [\Gamma.\phi]_{i,3}.[RSD(T_3 = 0.1T_1)] \tag{10}$$

where, $[\Gamma.\phi]_{i,j}$ is the modal displacement value at the *i^th^* floor of the building for the mode *j*, $RSD(T_j)$ is the spectral displacement at the *j^th^* mode period of vibration (taken as *0.25* of T_1 for the second mode and *0.1* of T_1 for the third mode of vibration) of the building.

Contributions from the higher modes defined by Equations (9 and 10) can be combined with the GLFM method introduced in Section 3.1 based on the *square-root-of-the-sum-of-the-square* (SRSS) combination rule:

$$\delta^*_{i,higher\ modes} = \sqrt{(\delta^*_i)^2 + ([\Gamma.\phi]_{i,2}.[RSD(T_2 = 0.25T_1)])^2 + ([\Gamma.\phi]_{i,3}.[RSD(T_3 = 0.1T_1)])^2} \tag{11}$$

where, $\delta^*_{i,higher\ modes}$ is the displacement value at the i^{th} floor of the building taking into account higher modes effects and δ^*_i is the displacement value at the i^{th} floor level of the building based on the GLFM presented in Section 3.1. T_1 is the fundamental period of vibration of the building defined by Equation (7).

3.2.2 Estimating the storey shear force profile

By using the modal displacements presented in Section 2, inertia forces of the floor levels can be estimated. The modal inertia force for the second and third modes ($F^*_{i,2}$) and ($F^*_{i,3}$) can be defined by:

$$F^*_{i,2} = [\Gamma.\phi]_{i,2}.[RSA(T_2 = 0.25T_1)].m_i \quad (12)$$

$$F^*_{i,3} = [\Gamma.\phi]_{i,3}.[RSA(T_3 = 0.1T_1)].m_i \quad (13)$$

where, $[\Gamma.\phi]_{i,j}$ is the modal displacement value at the i^{th} floor of the building for the mode j, $RSA(T_j)$ is the spectral acceleration at the j^{th} mode period of the building in terms of g, g is the gravity acceleration and m_i is the mass of the i^{th} floor.
For the mode j, the storey shear can be computed using Equation (14).

$$V^*_{i,j} = \sum_{k=i}^{n} F^*_{k,j} \quad (14)$$

The contribution from the higher modes defined by Equation (14) can be combined by the storey shear from the GLFM method ($V^*_{i,1}$) introduced in Section 3.1 based on the *square-root-of-the-sum-of-the-square* (SRSS) combination rule:

$$V^*_{i,higher\ modes} = \sqrt{{V^*_{i,1}}^2 + {V^*_{i,2}}^2 + {V^*_{i,3}}^2} \quad (15)$$

where, $V^*_{i,higher\ modes}$ is the storey shear force at the i^{th} floor of the building taking into account higher modes effects.
The proposed method can be used for the seismic design of limited ductile shear wall dominant regular or irregular buildings featuring transfer beams. In the derivation of the generalised mode shapes, the stiffness of the components making up the building has been modified in order that a more realistic estimate of global stiffness can be obtained. Accordingly, the proposed methodology is limited to the design of the aforementioned class of buildings that are often subject to low-to-moderate seismic damage during an earthquake event.

3.3 VALIDITY OF THE PROPOSED METHOD

13-storey and 18-storey buildings were used to check the validity of the proposed method. The plan view of the buildings is presented in Figure 6. f_c' is 40 MPa for the beam and column elements and 60 MPa for the walls. Amount of F_y for the rebars is 415 MPa. The live load was assumed to be 3kPa and the dead loads or permanent actions due to the partition weights was assumed to be 1 kPa. Two building heights (46.8 m and 64.8 m for the 13-storey and 18-storey building, respectively) were assumed in this study to check the accuracy of method for tall buildings. The buildings have been designed in accordance with the Australian Standards for gravity, wind and earthquake loadings (*AS/NZS 1170.0:2002*, *AS/NZS 1170.2:2011* and *AS/NZS 1170.4:2007*). The buildings were assumed to be located on the soil type C in Melbourne (k_p Z is 0.08 g). Finite element models of the buildings were constructed using program ETABS. (Computers and Structures Inc, 2017)

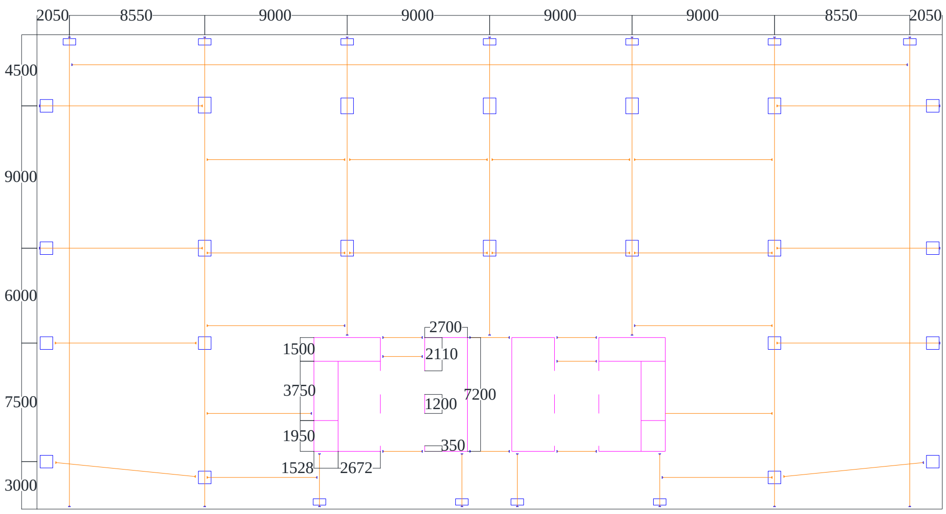


Figure 6 Framing plan

Using the method provided in Sections 3.1 and 3.2 (GLFM of analysis), the periods for the first three modes of vibration were calculated and compared with the periods obtained by dynamic analysis. The modal periods from the GLFM and the analysis are presented in Table 1.

Table 1 Comparison of periods according to the finite element models and GLFM of analysis

Period	13-storey structure		18-storey structure	
	GLFM	FE model	GLFM	FE model
T_1	0.836 s	0.854 s	1.417 s	1.418 s
T_2	0.209 s	0.199 s	0.354 s	0.324 s
T_3	0.084 s	0.165 s	0.142 s	0.151 s

In addition to the proposed method provided in section 3.2.1, the fundamental mode can be estimated using the code formulae for the fundamental natural period of the building. This can be used for a rapid and rough estimation for the displacements of floors and inertia forces without creating a finite element model and conducting any structural analysis. In this rapid estimation method, the amount of displacement and shear force of each storey for the fundamental mode can be computed using Equations 16 and 17.

$$\delta^*_{i,1} = [\Gamma.\phi]_{i,1}.[RSD(T_1)] \quad (16)$$

$$F^*_{i,1} = [\Gamma.\phi]_{i,1}.[RSA(T_1)].m_i \quad (17)$$

Equations 9-11 and 12-15 can be applied to find the displacement and shear force of each storey taking the three modes of vibration into account.

Figure 7 compares the displacement profile obtained by dynamic analyses, the Generalised Lateral Force Method (GLFM) and the Code Equivalent Static Method. The proposed method is generally shown to be able to provide accurate estimation of displacement profile.

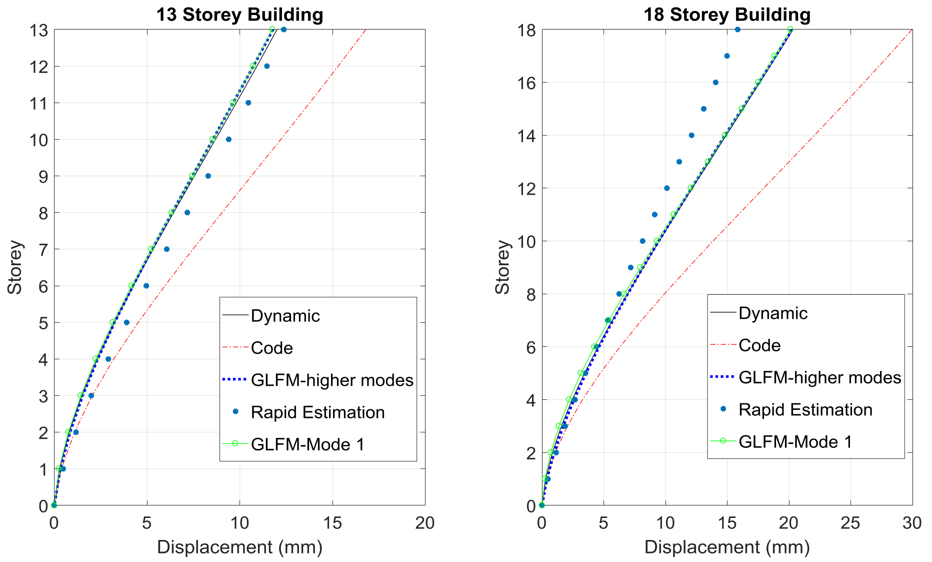


Figure 7 Displacement profile based on different analysis methods

The storey shear forces obtained from the Response Spectrum Analysis method, the *Generalised Lateral Force Method* (GLFM) and the Code *Equivalent Static Method* are presented in Figure 8. The proposed method reveals a reasonable estimation of the storey shear forces.

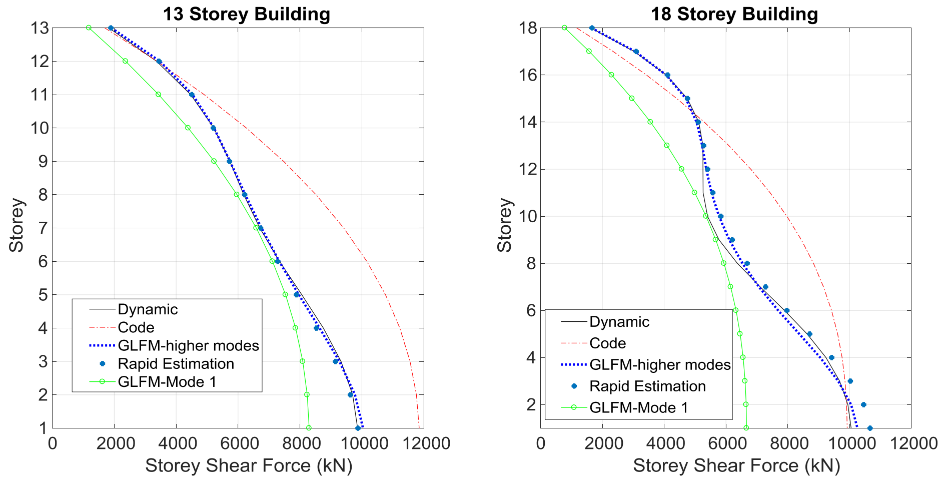


Figure 8 Storey shear force profile based on different analysis methods

In general, with the increase in height, the effects of higher modes become significant. The effects of higher modes are shown to be considerable for these buildings. It is noted that the effects of higher modes are not significant on the displacement profile but can be more tangible in the shear force profile. The proposed method has been demonstrated to be able to provide reasonable estimates of displacements and storey shear forces for these buildings.

4 CONCLUSION

The *Generalised Lateral Force Method (GLFM)* of Analysis is introduced in this paper. The method uses a simple rational approach to estimate maximum values of displacement and storey shear at the floor levels. Results calculated by the proposed method were compared with the computations based on dynamic analysis and the code stipulated *Equivalent Lateral Static Analysis Method*. The robustness of the proposed

method in estimating the seismic demands on shear wall dominant RC buildings with significant higher modes contribution has been demonstrated.

5 ACKNOWLEDGEMENT

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