Three-corner representation of earthquake source spectra at Kamchatka

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Abstract

Properties of source spectra of local shallow earthquakes of Kamchatka for the magnitude range M_w=3.5-6.5 were studied using 460 records of S-waves by station PET. To describe their shapes we use a three-cornered template that includes a common corner at f_{c1}, a second one at f_{c2}, and also a third one, f_{c3}, identical to the "source-controlled f_{max}". f_{c3} manifests itself in most cases. The family of average source spectra is constructed. Using these spectra, the relationship is studied between M_w and the key quasi-dimensionless source parameters: "stress drop" $\Delta\sigma$ and "apparent stress" σ_a. It is found that the $\Delta\sigma$ parameter is almost stable, whereas σ_a grows pronouncedly with magnitude. Thus, the important qualitative differences are revealed in the behavior of these source properties. As is known, at sufficiently large M_w, the revealed phenomenon disappears: both parameters $\Delta\sigma$ and σ_a do not show any clear dependence on magnitude. The threshold value of magnitude, namely M_w≈5.7, is determined, where such a change of behavior occurs for the case of Kamchatka.

Keywords: earthquake, source, scaling, stress drop, similarity, Kamchatka, corner-frequency, source spectrum

Introduction.

Source time function (STF) of an earthquake bears important information that may help to clarify the nature of earthquake rupture. Usually seismic moment rate function $\dot{M}_0(t)$ is treated as STF; also, its amplitude spectrum, $\dot{M}_0(f)$, is studied, called source spectrum. Key source parameters are seismic moment $M_0 = M_0(t)\big|_{t\to\infty} = \dot{M}_0(f)\big|_{f=0}$ and/or moment magnitude M_w = 2/3 lg M_0 [N·m]-10.7. In an ideal case, the functions $\dot{M}_0(t)$ and $\dot{M}_0(f)$ are directly related to displacement time function of seismic body wave, and to the spectrum of this signal. Scaling properties of populations of source spectra is a known study area; in particular, determination of average dependence of source spectra on M_0 or M_w, (scaling law of source spectrum) is of wide interest. A major component of this law is the relationship of the characteristic or corner frequency f_c of the spectrum with M_0. The scaling behavior of f_c is well studied. In the range of large magnitudes (M_w = 6-9) it is close to $f_c \sim M_0^{-1/3}$. Scaling of this kind corresponds to the assumption of geometrical and kinematic similarity of the sources of different sizes [1, 2]. If similarity holds, $M_0 \sim E_s \sim L^3 \sim B^3 \sim T^3 \sim f_c^{-3}$, where E_s is seismic energy, L is the size of a rupture, B is average slip (dislocation), and T is the duration of rupturing. In this case, main quasi-dimensionless parameters of the source, such as stress drop $\Delta\sigma \approx 2\mu B/L$ (μ is the shear module) and "apparent stress" $\sigma_a = \mu E_s/M_0$ show no systematic dependence on M_0. At lower magnitudes (M_w <6), situation differs. Some researchers [3, 4, etc.] believe that $\Delta\sigma$ and σ_a are stable even in this M range, and the property of similarity keeps to hold for small earthquakes as well. Another group [5, 6 etc.] believes that with decreasing M_0, the $\Delta\sigma$ and σ_a parameters also decrease, and the similarity hypothesis is violated here. Although the point of discussion is important for understanding physics of fault processes, the question keeps to be unresolved for decades. The problem is significant also for applied research, because the true variant of the answer defines how one should correctly extrapolate source spectra (and related ground motions) from frequent small-to-moderate (M_w =3-5) to rare and destructive strong (M_w =7-9) earthquakes.

In the presented work, an attempt is undertaken to clarify the listed questions by means of the study of 460 spectra of S waves of local earthquakes on Kamchatka. The prerequisite of this research is digital recording and satisfactory established properties of attenuation of S waves that allows one to recover source spectra with certain level of reliability. The basic results of the study are as follows:

(1) the family of average source spectra for the magnitude range M_w=3.5-6.5 has been constructed and the degree of scatter of individual spectra with respect to averages is determined.

(2) the presence in spectra of three characteristic (corner-) frequencies f_{c1}, f_{c2} and f_{c3} is confirmed; each of these three scale differently.

(3) the dependence of the stress drop parameter, $\Delta\sigma$, on M_w is found to be almost imperceptible, and in this particular aspect, the hypothesis of similarity of the sources is quite justified.

(4) on the contrary, the "apparent stress" parameter, σ_a, significantly (ten times and more) varies over the studied M_w range, demonstrating an expressed violation of similarity.

(5) it is established for the case of Kamchatka, that the described mode of behavior holds up to certain threshold magnitude, around M_w=5.5-6.0; at M_w>6 another behavior takes place, and the similarity hypothesis becomes valid.

2. Data and their processing. Recovery of source spectra

As an initial data set, 460 records of Kamchatka earthquakes with magnitudes M_L = 4 6.5 (M_w=3.5-6.5) and source depths 0-70 km are used, at hypocentral distances r = 80-220 km. The accelerograms with sampling frequency 80 or 100 sps were obtained in 1993-2012 by seismic station "Petropavlovsk" (PET). Amplitude spectra of acceleration over a selected S wave interval were calculated using tapered window, then averaged over two horizontal channels and over discrete FFT spectral values within log-spaced bins of the width equal to 0.1 decade (1/3 octave). The smoothed observed spectrum was then reduced to r=1км through applying corrections for geometrical spreading and for along-ray frequency-dependent attenuation (loss). Recently [7] a refined determination of S-wave loss parameters in the medium around PET has been performed. It was found, in particular (manuscript in press), that two radically different methods for loss estimation, applied in parallel, produced quite similar estimates. One of the methods used the change of spectral shape with distance; another was based on the distance decay of amplitudes measured for signals passed through a comb of band-pass filters. With such results at hand, one could believe that reducing spectra to the source could be considered, on the average, relatively reliable. The following attenuation parameters were accepted: geometrical spreading by $1/r$, where r - the hypocentral distance; and the following loss parameters: κ_0=0.034 s, inverse quality factor

$Q^{-1}(f, r) = Q_0^{-1} f^{\gamma} \left(1 + q\dfrac{r - r_0}{r_0}\right)$, where Q_0=164, γ=-0.59, r_0=100 km, q =-0.17.

The obtained reduced acceleration spectra *A* (*f*) were converted to velocity spectra *V*(*f*), and displacement spectra *Ω* (*f*). From *Ω* (*f*) spectrum, source spectrum was calculated as $\dot{M}_0(f)$ = C_1 *Ω* (*f*); where the constant $C_1 = 3.24 \cdot 10^{18}$ was derived and further used in calculation. The numerical values $\lg \dot{M}_0(f)\big|_{f=0}$ = lg M_0 for individual earthquakes were found to be close to the estimates of lgM_0 determined by the authoritative global world agency GCMT; the average misfit M_w - M_{wGCMT} = -0.17 was considered acceptable. To each of the observed spectral curves, plotted in log-log scale, a piecewise-linear approximation was determined in interactive mode. This near-optimal approximation of the spectral shape allows one to determine, for each individual $\dot{M}_0(f)$, $\ddot{M}_0(f)$ and $\dddot{M}_0(f)$ spectrum, the levels of near-maximum spectral plateaus $\lg \dot{M}_0(f)\big|_{f=0}$, $\lg \ddot{M}_0(f)_{\max}$ and $\lg \dddot{M}_0(f)_{\max}$ (correspondingly), and, simultaneously, the values of corner frequencies f_{c1}, f_{c2} and f_{c3}. Some of maxima do not show a plateau like seen on Fig. 1, rather, they are peaked; in other words, a coincidence may appear of the kind f_{c1}=f_{c2}, f_{c2}=f_{c3}, and even, in rare cases even f_{c1}=f_{c2}=f_{c3} (ω^{-3} spectrum). The estimates of corner frequencies and levels of the maxima that arise in such cases were included in statistics on a par with others. Examples of this processing procedure are shown in [8].

3. Data analysis and its results.

On Fig. 1 we show the sketch of source spectral scaling according to the accepted general concept; it is presented graphically in three equivalent versions, as $\dot{M}_0(f)$, $\ddot{M}_0(f) = 2\pi f \dot{M}_0(f)$ and $\dddot{M}_0(f) = 2\pi f \ddot{M}_0(f)$. Like the standard «$\omega^{-2}$-style» $\dot{M}_0(f)$ models of [1, 2], the accepted model includes a flat shelf (~f^0) at low frequencies (below f_{c1}) and a segment with f^{-2} behavior at high frequencies. Unlike the standard model, however, the two segments with ~f^0 and ~f^2 behavior do not butt at a single corner at some $f = f_c$. Instead, entire intermediate segment appears, with the behavior of the kind $f^{-(1\text{-}1.5)}$, bounded on each side by corners at f_{c1} and f_{c2}[2]. Above f_{c2} there is a «traditional» segment of the f^{-2} kind between f_{c2} and f_{c3}. Further, above f_{c3}, $\dot{M}_0(f)$ falls as $f^{-(3\text{-}4)}$. This model of spectral scaling was already introduced in [9] on a conceptual level; later, a lot of additional observations supporting the "three-corner" spectral scaling appeared, see [10]. As for Kamchatka spectra, this model was generally confirmed in [11] (however, no good estimates of f_{c3} are present in [11]). The particular variant of the model displayed on Fig. 1 has an important feature, of variable, *M*-dependent character of scaling for the level of the $\lg \dddot{M}_0(f)$ plateau. At a certain critical M_w value, fixed at 5.5 on Fig. 1, the rate of growth of this level with M_w switches from a fast one at low M_w=3-5.5 to slower one at higher M_w=5.5-7.5. However as regards the most significant parameter of the source, namely f_{c1}, it is assumed that the relationship $f_{c1} \sim M_0^{-1/3} \sim 10^{-0.5M_w}$ remains valid over the entire discussed M_w range.

To construct an empirical model of scaling that could be compared to the idealized scheme of Fig. 1, the relationships with M_w were studied for all critical parameters of such a model: frequencies f_{c1}, f_{c2} and f_{c3} (Fig. 2a), and levels $\lg \ddot{M}_0(f)_{\max}$ (Fig. 2b) and $\lg \dddot{M}_0(f)_{\max}$. (As for the $\lg \dot{M}_0(f)\big|_{f=0}$ level, it is rigidly fixed by M_w). These relationships were approximated by linear functions; also, the parameters of scatter of individual observations around these average trends were determined. On Fig. 2a the divergence between the trends for f_{c1}, f_{c2} and f_{c3} is evident: if similarity were present, log-log trends of parameters with the same dimension (frequency) would be parallel. It should be also noted that specifically for f_{c1}, a trend of the kind f_{c1}~$M_0^{0.33\pm0.02}$ was determined via regression; the exponent here matches the value of 1/3 expected for the case of similarity.

4. Scaling law for spectra of moderate earthquakes and its uncertainty.

With average trends of the listed spectral parameters at hand, the family of average spectra was constructed (Fig. 3); on this plot, the ranges of scatter of individual spectra are also depicted. As an entire picture, Fig. 3 corresponds quite well to the bottom part of Fig. 1. The major numerical parameter here is the rate of growth of the acceleration spectral plateau $\lg \ddot{M}_0(f)_{\max}$ with lgM_0; it is determined by the slope of line 2 on Fig. 2b/ This results in $\lg \ddot{M}_0(f)_{\max}$ ~ $M_0^{0.52}$; the exponent here is 1.55 times above the value 1/3 expected from the similarity hypothesis.

One can see on Fig. 2b that the trend observed for the magnitude range of 3.5-6. comes into the obvious conflict with the trend for range M=6-7, which can be determined for Kamchatka earthquakes from average acceleration spectra previously found in [11]. However, in the M_w=5-6 zone, these trends predict comparable values. Similar partial disagreement is seen with respect to several trends that describe Japanese data sets. One can come to conclusion that close to the values M_w =5.5-6 there is a "crossover" where the fast increase of the level of high-frequency radiation stops. (For the σ_a parameter, soon to be discussed, a similar crossover can be suspected here.) Above the boundary value, around M_w≈5.7, the "strong earthquake regime" dominates, and the property of similarity approximately holds.

5. Different scaling behavior for two variants of stress drop measure

To clarify the matter of similarity, the study of quasi-dimensionless source parameters can help, namely of stress drop $\Delta\sigma$ and of apparent stress σ_a. We use the Brune's spectral approach to $\Delta\sigma$ determination, and define it as $\Delta\sigma=8.47\cdot M_0\,(f_{c1}/c_S)^3$[12], where c_S is S-wave velocity. Note however that this definition significantly deviates from the common one, when the f_c value, inserted in such a formula, is determined by rather different mode of fitting the observed spectra. In many cases, such f_c is determined following [13], and the result is close or comparable to $(f_{c1}\,f_{c2})^{0.5}$. When f_c is defined in this alternative and common way, the discussed behavior for f_{c1} that agrees with similarity, and observed, disappears; often, increase of $\Delta\sigma$ is observed in the moderate-magnitude range. To find σ_a, defined as $\mu E_s/M_0$, we set μ =70 GPa, and calculate energy by the formula E_s [J] = $1.04\cdot10^{18}\Delta f V_{\max}^2$, where Δf [Hz] and $V_{\max}$ [m] are bandwidth of the reduced spectrum $V(f)$ and its maximum amplitude, correspondingly. The relations of $\Delta\sigma$ and σ_a with M_w are illustrated at Fig. 4. One can see, that for $\Delta\sigma$, the presence of correlation is doubtful in general (the significance level is 4 %); thus, the correlation is marginal, if exists at all. In essence, $\Delta\sigma$ is stable. At the same time, the correlation between σ_a and M_w is well expressed; with the corresponding significance level below 0.1 % one can be sure about its reality.

These facts suggest a new view onto the old dispute. At least in part of cases, the question of validity of the similarity hypothesis for small-to-moderate (M_w =3-5) earthquakes may have a simple and unexpected answer: deviations from this hypothesis are rather insignificant for the stress drop parameter, and, simultaneously, are expressed with full clarity for the apparent stress, i.e. for the seismic energy to seismic moment ratio.

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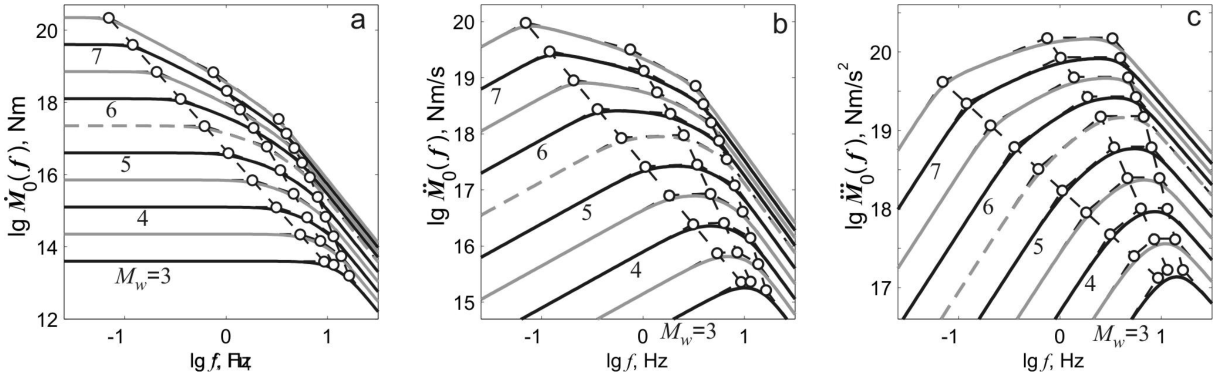

Figure 1. Idealized scaling law of earthquake source spectra, with three corner-frequencies f_{c1}, f_{c2} and f_{c3}, with each following its own trend. The presented scheme includes the switching of the growth rate of the level of high-frequency plateau, $\lg \dddot{M}_0(f)_{\max}$, from faster one at M_w <5.5 to slower one (~$M_0^{1/3}$) at M_w> 5.5. For the threshold value M_w=5.5, the spectrum curve is marked by dashed line. (a) - the family of spectra $\dot{M}_0(f)$ for a sequence of M_w values. (b) - similar set of $\ddot{M}_0(f)$ spectra. (c) - similar set of $\dddot{M}_0(f)$ spectra.

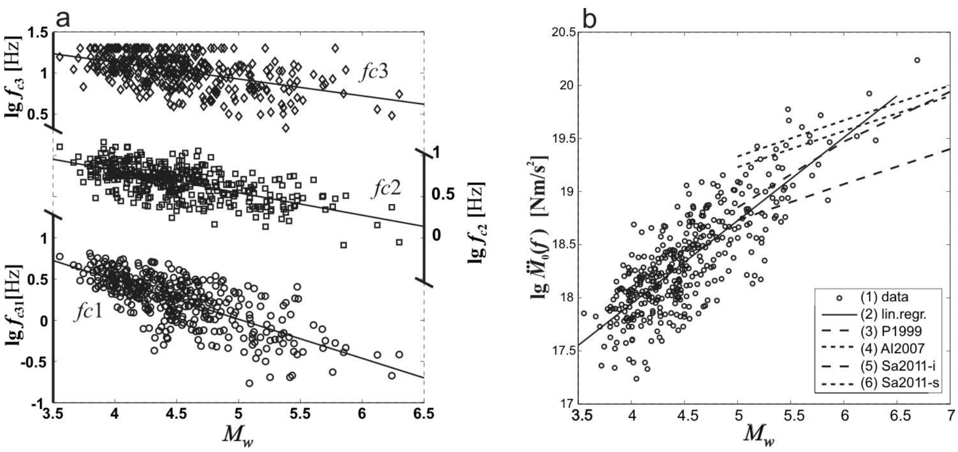

Figure 2. Relationships between spectral parameters and M_w. (a) - trends of corner-frequencies f_{c1}, f_{c2} and f_{c3}. (b) the trend for $\lg \dddot{M}_0(f)_{\max}$. (1) data; (2) linear approximation of the kind $\lg \dddot{M}_0(f)_{\max}$ ~ $10^{0.78}M_w$ (~ $M_0^{0.57}$); (3) the trend for the range M_w=5-7, derived from the average scaling of acceleration spectra determined in [11] for Kamchatka. (4) a similar trend for mantle sources near to Hokkaido. (5 and 6) similar trends for crustal and mantle sources near to Honshu, accordingly.

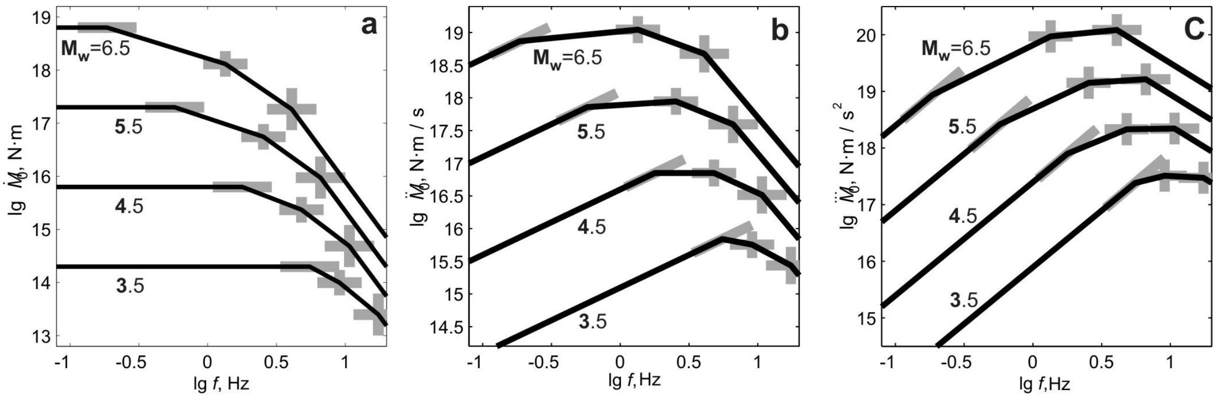

Figure 3. Families of average source spectra $\dot{M}_0(f)$ (a), $\ddot{M}_0(f)$ (b) and $\dddot{M}_0(f)$ (c) for earthquakes of Kamchatka (lines), and ±1σ ranges of scatter of individual spectra (grey bars): along ordinate - for their level, and along abscissa - for corner frequency position.

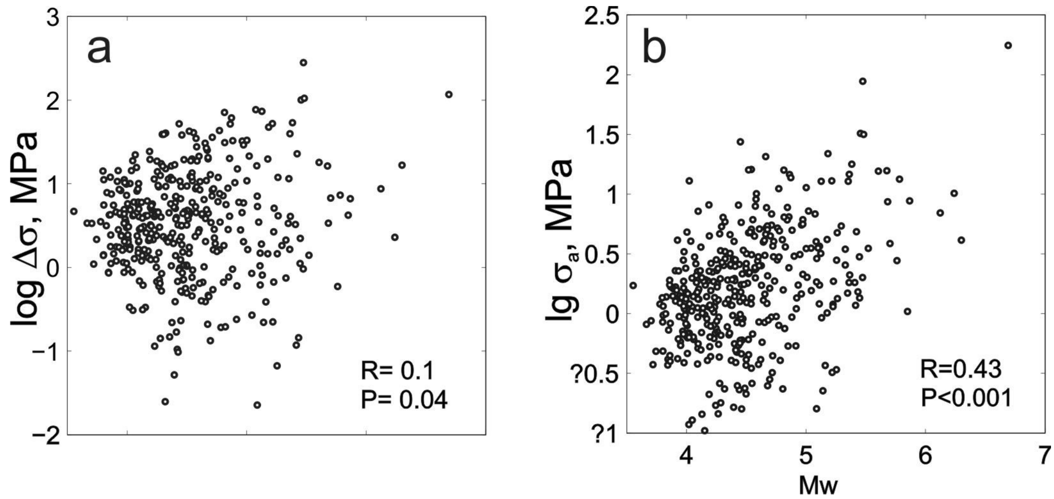

Figure 4. Dependence of quasi-dimensionless source parameters: stress drop $\Delta\sigma$ (a) and apparent stress σ_a (b) on M_w. One should keep in mind that a revision of particular values of the coefficients used in the calculation of the values of $\Delta\sigma$ and σ_a may only shift zero point on the ordinate axes of these plots; such a change can in no way affect the basic conclusion of work, namely the expressed qualitative distinction between scaling of these two parameters with respect to M_w.