Seismic loss optimization of nonlinear moment frames retrofitted with viscous dampers

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ABSTRACT: Viscous dampers are very effective in the realization of performance based design objectives, especially when it comes to seismic retrofitting. Consequently, various methods have been proposed for the optimal design of dampers and their distribution along the height of a building. The majority of these methods assume the parent frame to remain linear during the seismic event. In the event of a major earthquake, the assumption of a linear parent frame might demand a high quantity of initial damping material which might make the proposition of damper control economically non-viable. Also most of the formulated methods concentrate mainly on reducing the responses with no explicit consideration of their long-term economic impact. In this study, a generic optimization framework is formulated for optimally distributing viscous dampers along the height of a building by minimizing the initial cost subject to a constraint on the expected total seismic loss. The constraint on the expected total loss is assumed in such a way so as to also induce repairable nonlinearity in the parent frame by restricting the maximum of the structural system drift to a predefined value. An intensity based assessment is used for the computation of the expected total loss. A generic Sequential Linear Programming procedure is employed to solve the formulated optimization problem. The implementation scheme of the optimization procedure is outlined in detail. The efficacy of the proposed procedure is illustrated by applying it on a four story reinforced concrete frame.

1 INTRODUCTION

Viscous dampers are very effective in reducing seismic responses. This is mainly attributed to the fact that the damper force is linearly or nonlinearly proportional to velocity and is out of phase from the column displacements. As a result, the columns or foundations are not subjected to additional demand, and may not need to be strengthened (Constantinou *et al.*1993, Miyamoto & Scholl 1996, Lavan 2012). This paper is mainly concerned with the application of viscous dampers for seismic performance enhancement of existing frames.

In a retrofitting design using viscous dampers the main task that needs to be addressed by the engineer is the "sizing" and the allocation of the dampers. Both these tasks are coupled and a realistic optimum would be difficult to be achieved if both of them are treated as un-coupled. Nevertheless the majority of the optimal design methodologies for retrofitting using viscous dampers address the problem of distributing a given total added damping to achieve the best performance (minimize damage measures) which results in decoupling the two tasks. Some of the works in this direction presented very efficient methods (Zhang & Soong 1992, Tsuji & Nakamura 1996, Takewaki, 1997a, 1997b, 1998, 1999, Singh

& Moreschi, 2001, 2002, Garcia 2001). Note that in these works the total added damping is predetermined and the optimisation is mainly initiated for efficiently allocating the dampers only. The algorithms are thus used to determine the optimal positioning of this quantity along the height of the building. In some of these works, approaches to estimate a reasonable total added damping were also proposed.

From a different perspective, Lavan and Levy (2005, 2010) and Lavan (2015) minimized the total added damping subject to a constraint on the performance of the structure (allowable inter-story drifts). They also presented a practical analysis/redesign procedure for arriving at the optimal designs exploiting the advantage of the fully stressed characteristics of the optimal solution (Levy & Lavan, 2006, Lavan, 2015). While this formulation lends itself to the performance-based-design framework, the allowable inter-story drifts, or performance measures, are mainly determined based on code requirements. The performance measure are usually not determined explicitly based on the economic consequences.

The other important aspect is that majority of the optimisation strategies rely on the assumption of the linearity of the parent frames. An assumption of linearity on the parent frames might result in a huge economic consequence as it poses a heavy demand for higher quantity of initial damper material which would increase the initial cost tending the retrofitting option by viscous dampers nonviable.

In the present paper keeping all the above described shortcomings in purview a novel practical optimization scheme is developed in which the total initial cost is minimized while explicitly constraining the total expected loss. The constraint on the total expected loss is computed by restricting the peak drift to induce reparable nonlinearity. The total expected loss is the loss that the building incurs for a specific intensity of earthquake (Aslani & Miranda 2005). The formulated problem thus directly incorporates the economic impact, and the optimization scheme proposed reflects the economic aspect of the response optimization. The framework developed is generic and is easily extendable to other engineering demand parameters like floor accelerations.

2 PROBLEM FORMULATION

In the case of retrofitting, the mass and stiffness of the structure can be assumed to be constant as the structure is already existing and the initial cost can be considered to be equal to the quantity of damping material required. A component based loss computation methodology is adopted for calculating the total expected loss. For an economical retrofitting strategy, the parent frame should be allowed to incur a reasonable level of repairable damage in case of a major seismic event. So in the present study, optimisation is done by allowing nonlinearity in the parent frame. Nevertheless, seismic losses associated with downtime and injury are not accounted for in this study.

2.1 Equations of motion

The equations of motion of the nonlinear frame with added dampers is given as,

$$\begin{aligned} &\mathbf{M}\ddot{\mathbf{u}}(t) + \left[\mathbf{C} + \mathbf{C}_{damper}(\mathbf{c}_d)\right]\dot{\mathbf{u}}(t) + \mathbf{f}_s(\mathbf{u}(t), \dot{\mathbf{u}}(t)) = -\mathbf{M}\mathbf{i}\ddot{u}_g(t) \\ &\mathbf{u}(0) = 0; \dot{\mathbf{u}}(0) = 0 \end{aligned} \tag{1}$$

In eq. (1), $\mathbf{M}$ represents the mass matrix and $\mathbf{f}_s(\mathbf{u}(t), \dot{\mathbf{u}}(t))$ represents the restoring forces vector at time *t*. Similarly, $\mathbf{C}$ represents the inherent damping matrix, $\mathbf{C}_{damper}(\mathbf{c}_d)$ is the added supplemental damping matrix, $\mathbf{c}_d$ is the added damping vector, $\mathbf{i}$ represents the ground motion directional vector, $\ddot{\mathbf{u}}(t), \dot{\mathbf{u}}(t)$ and $\mathbf{u}(t)$ represent the relative acceleration, relative velocity and relative displacement, and $\ddot{u}_g(t)$ represents the ground acceleration.

2.2 Loss computation

In classical detail loss assessment framework, the expected annual loss or the loss expected over a period of time is computed as (Aslani & Miranda 2005, Bradley *et al.* 2009),

$$E[L_T] = \frac{1-e^{-\lambda t}}{\lambda}\int_0^{\infty} E[L_T \mid IM]\,dv(IM) \tag{2}$$

where $E[L_T]$ is the expected annual loss, λ is the discount rate (to convert the future loss to net present value), t is the period for which the rate is applied, $E[L_T \mid IM]$ is the expected loss conditioned on the intensity measure *IM*, and *v(IM)* is the mean annual rate of exceedance of the intensity measure. As the main focus of this study is to illustrate the optimization methodology, the expected loss is computed only at a single value of the intensity measure. Hence, the computed loss is independent of period *t* and hence eq. (2) is not readily usable. As addition of dampers reduces the probability of collapse, in the present study, the total expected loss is computed only for no-collapse scenario conditioned on the peak engineering demand parameter ($\overline{EDP}$) which in turn is conditioned on the selected intensity measure (IM_1) and is assumed to be (Aslani & Miranda 2005),

$$E\left[L_T \mid NC, \overline{EDP}_{IM_1}\right] = \sum_{j=1}^{N} a_j \left(E\left[L_j \mid NC, \overline{EDP}_{IM_1}\right]\right) \tag{3}$$

Over here, a_j is the cost of a new j^{th} component and *NC* refers to no-collapse state, *N* refers to the number of components. Eq. (3) is period independent and $\overline{EDP}$ is computed for the specific intensity measure (IM_1). In the case of a rare major seismic event, the optimal addition of dampers should reduce the nonlinear response to reparable level; hence if the optimisation is done for the envelope engineering demand parameter, the reliability of the performance of the structure increases in other seismic events which are less critical.

2.3 Optimization problem

The optimization problem is formulated as,

$$min\, J(\mathbf{c}_d) = \mathbf{c}_d^T \mathbf{1} \tag{4}$$

Subject to:

$$\sum_{i=1}^{N_d} (\phi_i) \le \Phi_{allowable} \tag{5}$$

Over here ϕ_i refers to the expected loss at the i^{th} degree of freedom computed based on the maximum peak response. Mathematically ϕ_i is given as,

$$\left.\begin{array}{l} \phi_i = \kappa(max(abs(\mathbf{d}_i(t)))) \\ \textit{where } \kappa \textit{ represents a function form and } \mathbf{d}_i(t) \textit{ is the response vector} \\ \textit{and satisfies the following equation,} \\ \mathbf{M\ddot{u}}(t) + [\mathbf{C} + \mathbf{C}_{damper}(\mathbf{c}_d)]\mathbf{\dot{u}}(t) + \mathbf{f}_s(\mathbf{u}(t), \mathbf{\dot{u}}(t)) = -\mathbf{Mr\ddot{u}}_g(t) \\ \mathbf{u}(0) = 0; \mathbf{\dot{u}}(0) = 0 \\ 0 \le \mathbf{c}_d \end{array}\right\} \forall \ddot{u}_g(t) \in \text{ whole ensemble} \tag{6}$$

Eq. (5) can be re-written as,

$$\Pi = \frac{\sum_{i=1}^{N_d}(\phi_i)}{\Phi_{allowable}} \leq 1.0 \qquad (7)$$

where Π is called the performance index and $\Phi_{allowable}$ is the allowable (i.e. acceptable) total expected loss. In the present study $\Phi_{allowable}$ is computed by restricting the interstory drift so as to produce a level of nonlinearity in the parent frame which can be deemed to be repairable.

3 OPTIMIZATION ALGORITHM

Stepwise implementation scheme of the optimization procedure is outlined in this section. An ensemble of ground motions is selected to match the target mean spectrum corresponding to the specific intensity level of interest. It has to be noted that the procedure outlined in this section is equally applicable to time based assessments in which case the constraint will be on the expected annual loss instead of the total expected loss used in the present study. Following steps are involved in the proposed optimization scheme.

Step 1 Selection of *active ground motions*

A methodology to identify the active ground motions is given by Lavan and Levy (2006). For the whole ground motion ensemble matching the target mean spectrum, spectral response curves of a single degree of freedom having the same fundamental natural frequency as that of the parent structure is generated with response amplitudes on the y-axis and the increasing damping coefficients on the x-axis. In the present study the maximum displacement is taken as the response quantity. The ground motion which produces the largest spectral response curve for a reasonable range of damping is taken as the critical active ground motion. This significantly reduces the analysis effort and makes the scheme more appealing for practical application.

Step 2 Computation of envelope responses of the nonlinear frame

Solve eq. (1) using any of the time integration schemes available in literature with an initial amount of damping vector $\mathbf{c}_d$ for all the active ground motions and compute the responses. In the present study modified Newmark constant average acceleration method is used. In order to reduce the iterations and to make the scheme more practical, the initial amount of $\mathbf{c}_d$ vector can be computed using any of the approximate methods available in the literature (Liang *et al.* 2012).

Step 3 Evaluation of the total expected loss and performance index Π

Compute the total expected loss and performance index Π by using eqs. (3), (6) and (7).

Step 4 Compute the gradient of the performance index Π and the objective function *J*

Gradient for the objective function *J* is trivial as it is a direct function of the damping vector $\mathbf{c}_d$ and the sensitivity will return a vector **1.** But the gradient of the performance index Π is not trivial. In this study the gradients are derived using the classical finite difference scheme as given below,

$$\frac{\partial \Pi}{\partial c_{dj}} \approx \frac{\Pi_{new} - \Pi}{\Delta c_{dj}} \qquad (8)$$

where Π_{new} is the new performance index computed with perturbed $\mathbf{c}_d + \Delta\mathbf{c}_{dj}$ where $\Delta\mathbf{c}_{dj}$ is the perturbed vector with Δc_{dj} at the j^{th} location and zero elsewhere in the vector.

Step 5 Estimate a new $\mathbf{c}_d$ for the optimal design using Sequential Linear Programming (SLP)

The original optimization problem is given by eqs. (4) and (7) satisfying eq. (6). This is a nonlinear programming problem. SLP is chosen to solve this problem as other nonlinear schemes require the estimation of Hessian matrices which can pose serious difficulties in the vicinity of the optimum solution. So linearizing the objective function given by eq. (4) at the i^{th} iteration gives,

$$J^i(\mathbf{c}_d) = J(\mathbf{c}_d^i) + \{\nabla_{\mathbf{c}_d} J(\mathbf{c}_d^i)\}(\Delta\mathbf{c}_d^i) \quad (9)$$

and linearizing the constraint at the i^{th} iteration satisfying eq. (6) is given as,

$$\Pi^i(\mathbf{c}_d) = \Pi(\mathbf{c}_d^i) + \{\nabla_{\mathbf{c}_d} \Pi(\mathbf{c}_d^i)\}(\Delta\mathbf{c}_d^i) \quad (10)$$

In order to solve eqs. (9) and (10), an additional side constraint of '*move limits*' limiting the damper step size has to be introduced. So the linearized optimization problem for the i^{th} iteration is given as,

$$\left.\begin{aligned} &min\ J^i(\mathbf{c}_d) \\ &Subject\ to: \\ &\Pi^i(\mathbf{c}_d) \le 1.0 \\ &\Delta\mathbf{c}_d^{low} \le \Delta\mathbf{c}_d^i \le \Delta\mathbf{c}_d^{upper} \end{aligned}\right\}\forall i \quad (11)$$

Solving eq. (11) gives the $\Delta\mathbf{c}_d$ required for the next iteration. Update the damping vector $\mathbf{c}_d$ as,

$$\mathbf{c}_d + \Delta\mathbf{c}_d \quad (12)$$

Step 6 Check for termination condition

The iteration is terminated if the change in added damper vector $\Delta\mathbf{c}_d$ is less than the tolerance or maximum number of iterations has been reached. Otherwise, update the iteration number as $i=i+1$ and proceed to step 1.0. The optimization algorithm when $\Delta\mathbf{c}_d^i \ge \varepsilon_{tolerance}$ is illustrated in Fig. 1.

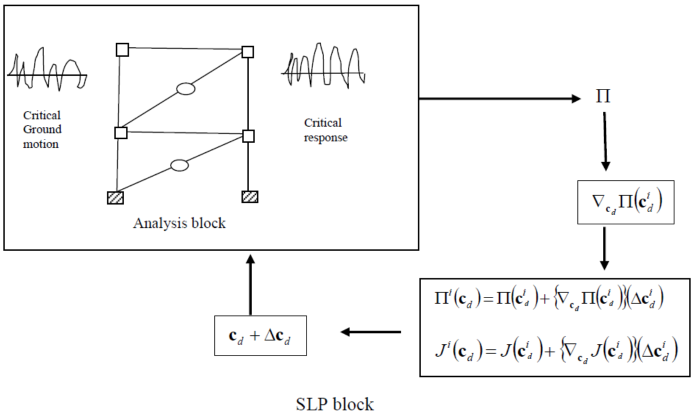

Figure.1 Schematic representation of the optimization procedure for $\Delta\mathbf{c}_d^i \ge \varepsilon_{tolerance}$

Step 7 Check for all other ground motions in the ensemble whether the present optimal damper distribution violates the constraint on the expected loss. If it violates then the ground motion gets added into the active set of ground motions and the optimisation steps are repeated.

4 NUMERICAL STUDY

A four story reinforced concrete frame described in (Arede, 1997), designed in accordance with Eurocode 8 (EC8) and Eurocode 2(EC2) is used to illustrate the proposed optimization procedure. The frame is designed for high seismicity assuming a PGA of 0.3g. The geometric and material properties of the frame and the arrangement of the pre-located dampers are given in Appendix A. A suite of 7 artificial ground motions scaled to match a Eurocode 8 design spectra with PGA adopted as 2.0 times the design PGA is used for the present study. A scale factor of 2 is intentionally used to generate a conservative very rare earthquake scenario and as the main focus of the paper is to demonstrate the optimization scheme, no further effort is invested in this aspect and is deemed to be sufficient. Uncontrolled frame analysis has revealed that this level of ground motion intensity can incur significant inelastic excursions in the parent frame (Arede, 1997). As incorporation of supplemental damping in the frame reduces the risk of collapse, no second order effects are incorporated in the analytical model. A simple lumped point plasticity model based on Giberson one component element is used for the modelling of the frame. A strength and stiffness non-degrading bilinear hysteresis with 1% strain hardening is used as hysteretic model for the plastic hinge. Theoretically the choice of this hysteresis over simplifies the hysteretic performance of the concrete frame; but as the main focus is to illustrate the optimisation scheme for the damper quantification and distribution, the choice of this simple hysteresis is deemed to be sufficient. *Critical ground motions* are identified by the method described in step 1.0 in the optimisation scheme. For the present case study building only one ground motion is active. The allowable expected loss in eq. (5) is computed by assuming an allowable inter-story drift of 1.0% which is assumed to ensure repairable inelastic frame behaviour. Only drift sensitive structural loss is accounted in the present study. Figs. 2a and 2b illustrate the performance index and the constraint error. As SLP is used, in order to ensure better convergence an adaptive *move limits scheme* had to be introduced. An initial *move limit* of 0.1% of the design damping vector is used for the iterations. In the neighbourhood of the optimum the initial *move limit* is adaptively updated for better convergence.

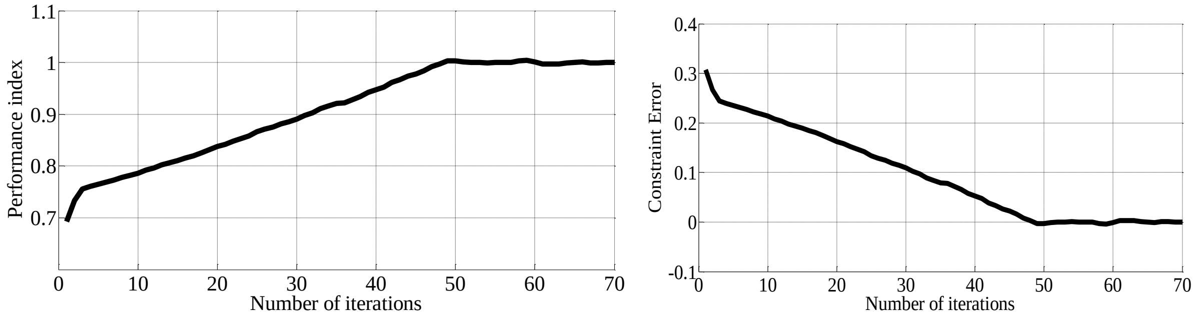

Figure. 2 (a) Performance index plot illustrating convergence to optimum, (b), Constraint error plot

Fig. 3a gives the optimum distribution of the damper coefficients along the height of the building. The initial uniformly distributed total damping vector used in this study for starting the optimization procedure is 225 kN-sec/m per story. The total damping material after optimisation is obtained as 517kN-sec/m distributed as shown in figure 3a.

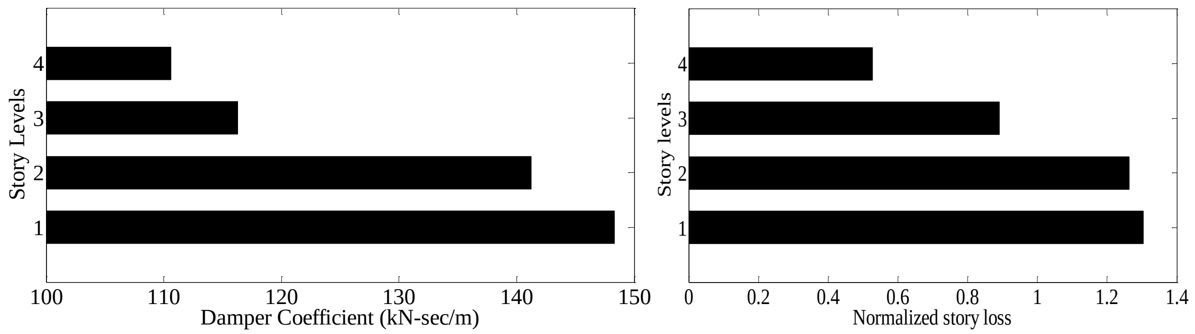

Figure. 3 (a) Optimum damper distribution (b).Disaggregation of the expected loss per story

In order to understand the localized effect of this optimal distribution of dampers achieved in Fig. 3a, a disaggregation of the total expected loss per story is also conducted. Fig. 3b shows the story level expected normalized losses. Normalized loss is obtained by dividing the computed story level loss to the allowable story level loss. In a realistic scenario, the allowable loss at a story level should be determined based on the performance criterion to be satisfied for that story. Since Fig. 3b is plotted simply to understand the distribution of the expected loss per story, the allowable value of the loss at story level is assumed to be obtained by dividing the total system allowable value of the loss by the number of stories.

The present optimization problem is formulated in such a way that the total expected loss of the building system is less than a certain allowable value. In this study *no story level constraints* on the expected loss is considered. So from Fig. 3b, it could be clearly seen that with the optimum damper distribution shown in Fig. 3a, the first and second story losses exceeds the assumed allowable limit while the third (at ~88%) and fourth story (at ~55%) is well within the allowable limit. *This is to be expected as no story level constraint on the expected loss is applied in the optimization procedure*. So Fig. 3b suggests that in the present study if we had adopted a constraint on the allowable story level loss limit, in addition to the constraint on the total allowable expected loss for the whole system, more damping would have to be allocated to the first and second story and lesser damping to the third and fourth story. Intuitively this would mean that the damping material would be removed from the third and fourth story and distributed to the first and second story. But, as the total system expected loss is of more concern for engineers and other stakeholders, only this is considered as the constraint in the present study. However, if desired the story level constraints can be easily added by specifying the allowable story loss based on the specific requirements of the building. This gives more flexibility in the algorithm and allows the functional requirements of different stories of the building to be accounted for.

5 CONCLUSIONS

A gradient based sequential linear programming (SLP) optimization methodology is adopted to optimally quantify and optimally position added viscous dampers in nonlinear multi-story frames. The optimization problem addressed is to minimize the amount of added damping subject to a constraint on the total expected seismic loss. Details of the optimization algorithm are presented and its application is illustrated using a four story reinforced concrete frame subjected to a set of critical ground motions selected from a suite of ground motions matching the mean target spectra. It is shown that the proposed procedure is capable of arriving at an optimal quantity of dampers and also simultaneously optimally distributes the dampers along the height of the building.

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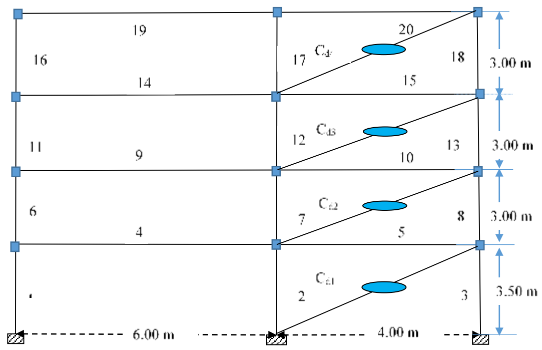
APPENDIX A: DETAILS OF THE FOUR STORY FRAME



C_{di} refers to added dampers and $i=1....4$

Material Property

Dynamic Young's modulus $= 3.5 \times 10^{10}\ N/m^2$

Geometric Properties

Member number	Width (mm)	Depth (mm)
1,6,11,16,17,12,7,2,3,8,13,18	450	450
4,5,9,10,14,15,19,20	300	450

Nodal Mass

Floor level	Mass per node (kg)
1st floor	29800
2nd -4th floor	29500

REFERENCES

Arede A.J.C.D. 1997. Seismic assessment of reinforced concrete frame structures with a new flexibility based element. *Research Thesis, Universidade Do Porto*.

Aslani, H. & Miranda E. 2005. Probabilistic earthquake loss estimation and loss disaggregation in buildings. *Research report No:15, University of Stanford*.

Bradley B.A., Dhakal R.P., Cubrinovski M., MacRae G.A., Lee S.D. 2009. Seismic loss estimation for efficient decision making. *Bulletin of the New Zealand Society for Earthquake Engineering*. 42 (2). 96-110.

Constantinou MC. Syman MD. Tsopelas P. Taylor DP. 1993. Fluid viscous dampers in applications of seismic energy dissipation and seismic isolation. *ATC-17-1*.581-591

Garcia, D.L. 2001. A simple method for the design of optimal damper configuration in MDOF structures. *Earthquake Spectra.* 17(3). 38-398.

Lavan, O. & Levy, R. 2005. Optimal design of supplemental viscous dampers for irregular shear frames in the presence of yielding. *Earthquake Engineering & Structural Dynamics.* 34(8). 889-907.

Levy, R. & Lavan, O. 2006. Fully stressed design of passive controllers in framed structures for seismic loadings. *Structural Multidisciplinary Optimization.* 32. 485-498.

Lavan, O., & Levy, R. 2010. Performance based optimal seismic retrofitting of yielding plane frames using added viscous damping. *Earthquakes & structures.* 3. 307-326.

Lavan O.2012. On the efficiency of viscous dampers in reducing various seismic responses of wall structures. *Earthquake and Structural Dynamics.* 41. 1673-1692.

Lavan, O. 2015. Optimal Design of Viscous Dampers and Their Supporting Members for the Seismic Retrofitting of 3D Irregular Frame Structures. *Journal of Structural Engineering.*

Lavan, O. 2015. A methodology for the integrated seismic design of nonlinear buildings with supplemental damping. *Structural Control and Health Monitoring*. 22 (3). pp 484–499.

Liang Z.,Lee G.C.,Dargush G.F., Song J. 2012. Structural Damping: Applications in seismic response modification. *CRC press*.

Miyamoto HK, Scholl R.E.1996. Case study: seismic rehabilitation of non-ductile soft story concrete structure using viscous dampers. *11th World Conference on Earthquake Engineering* , Acapulco, Mexico.

Takewaki, I. 1997a. Optimal damper placements for minimum transfer functions. *Earthquake Engineering & Structural Dynamics,* 26 (11). 1113-1124.

Takewaki, I. 1997b. Efficient redesign of damped structural systems for target transfer functions. *Computer Methods in Applied Mechanics & Engineering.* 147(3-4). 275-286.

Takewaki, I. 1998. Optimal damper positioning in beams for minimum dynamic compliance. *Computer Methods in Applied Mechanics & Engineering.* 156(1-4). 363-373.

Takewaki. I. 1999. Displacement-acceleration control via stiffness-damping collaboration. *Earthquake Engineering & Structural Dynamics*, 28. 1567-1585.

Tsuji, M., Nakamura, T.1996. Optimum viscous dampers for stiffness design of shear buildings. *Journal of the Structural Design of Tall Buildings,* 5**.** 217-234.

Singh M.P. & Moreschi, L.M. 2001. Optimal seismic response control with dampers. *Earthquake and Structural Dynamics,* 30 (4). 553-572.

Singh M.P. & Moreschi, L.M. 2002. Optimal placement of dampers for passive response control. *Earthquake and Structural Dynamics,* 31 (4). 955-976.

Zhang, R.H., Soong T.T.1992. Seismic design of visco-elastic dampers for structural applications. *Journal of Structural Engineering*. 118 (5). 1375-1392.