Local strain of reinforcement and tension stiffening in reinforced concrete walls

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ABSTRACT: This paper presents the development of a mathematical model for determining the ratio between the global strain of an RC wall and the local tensile strain in the vertical reinforcement. It was found that the parameters affecting the ratio of local to global strain are the ultimate tensile strength of the concrete and percentage of vertical reinforcement. A comparison of the different tensile strength models in various concrete codes (i.e. AS 3600, NZS 3101, EN 1992-1-1 and ACI 318) was undertaken and a recommendation for the mean ultimate tensile strength of concrete is presented. Elastic and inelastic bond stress values for D500N reinforcement is included, however it was found that while the level of bond stress between the reinforcement and concrete affects crack widths, it does not affect the ratio of local to global strain. The mathematical model developed was validated against recent experimental testing performed in the Smart Structures laboratory, with very good correlation observed. Charts have been produced for determining the local strain in the reinforcement from the global strain and vice versa. The paper concludes with a brief discussion on how the developed model could be used to account for tension stiffening effects in RC walls.

1 INTRODUCTION

The authors are currently undertaking a long time research project into the performance of limited ductile reinforced concrete (RC) walls in areas of lower seismicity. The majority of low, mid and high-rise buildings in areas of lower seismicity, such as Australia, use limited ductile RC walls as the lateral load resisting system of the structure. These walls typical have low axial load ratios (i.e. the axial load on the wall divided by the concrete strength times the area of concrete: $n = N/A_0/(f_cA_g)$) and as such are capable of developing significant tensile strains in the end regions of the walls when subject to lateral load from extreme earthquake or wind events. This paper outlines the author’s preliminary works for developing robust tensile strain limits for limited ductile RC walls.

Under cyclic lateral load, such as during an earthquake, the end regions of RC walls are subject to cyclic axial load. Depending on different factors, such as the geometry and shape of the wall cross section and the level of axial load on the wall, the end region of a wall will be subject to cyclic axial compression-compression or tension-compression. A series of experimental studies have been performed by the authors to study the cyclic axial tension-compression behaviour of the boundary elements of limited ductile RC walls (i.e. Menegon et al. (2015b) and the experimental works outlined in this paper). It has been observed that a major factor affecting the cyclic axial performance of RC is the relationship between level of local plastic strain developed in the vertical reinforcement and global strain of the element, i.e. the crack spacing and how the inelastic behaviour is distributed along the length of the element. A model for predicting the ratio of global strain to local strain of reinforcement in RC elements based on the percentage of reinforcement and the ultimate tensile strength of concrete has been developed. The process of developing this model is presented in the subsequent sections.
2 CRACK WIDTH AND LOCAL STRAIN THEORETICAL MODEL

The developed model was adapted from the crack width and yield penetration model presented by Sezen and Moehle (2004). The adapted model was generalised for a limited ductile RC element with continuous cracking along its length. The model can be seen conceptually in Figure 1. The crack width \( x \) is determined using Equations 1 to 3.

![Figure 1. Generalised crack width and local strain model.](image)

\[
\begin{align*}
&\text{if } \varepsilon_{s2} \leq \varepsilon_{s1} \leq \varepsilon_{sy}: \\
&x = 2 \times \int_0^{L_s} \varepsilon \, dL = 2 \times [0.5(\varepsilon_{s1} + \varepsilon_{s2})L_s] \quad (1) \\
&\text{if } \varepsilon_{s1} \leq \varepsilon_{sy} \leq \varepsilon_{s2}: \\
&x = 2 \times \left[ \int_0^{L_s} \varepsilon \, dL + \int_{L_s}^{L_s + L'_s} \varepsilon \, dL \right] = 2 \times \left[ 0.5(\varepsilon_{s1} + \varepsilon_{sy})L_s + 0.5(\varepsilon_{sy} + \varepsilon_{s2})L'_s \right] \quad (2) \\
&\text{if } \varepsilon_{sy} \leq \varepsilon_{s2} \leq \varepsilon_{s1}: \\
&x = 2 \times \int_0^{L'_s} \varepsilon \, dL = 2 \times [0.5(\varepsilon_{s1} + \varepsilon_{s2})L'_s] \quad (3)
\end{align*}
\]

Where: \( \alpha \) is the average crack spacing; \( L_s \) is the length of elastic bond; \( L'_s \) is the length of inelastic bond; \( \varepsilon_{ub} \) is the average ultimate elastic bond strength; \( \varepsilon_{ub}' \) is the average ultimate inelastic bond strength; and \( \varepsilon_{sy} \) is the yield stress of the reinforcement.

The local strain in the reinforcement and global strain of section is determined using Equations 4 and 5 below.

\[
\begin{align*}
\varepsilon_{local} &= \varepsilon_{s2} \quad (4) \\
\varepsilon_{global} &= \frac{x}{\alpha} \quad (5)
\end{align*}
\]

2.1 Average crack spacing – \( \alpha \)

The average crack spacing is dependent on the minimum crack spacing (i.e. \( a_{min} \)) of the end region of the wall (i.e. the boundary element of the wall). For a rectangular wall the boundary element can be defined as the portion of wall extending a minimum of \( 0.15L_w \) and \( 0.5b_w \) from the end of the wall (European Committee for Standardization (CEN) 2004b). Alternatively for walls with engaged columns or box-shaped lift cores, the boundary elements would be the column and flange section of the wall respectively. For the context of this study, the boundary elements of various wall cross sections are illustrated in Figure 2.
The minimum crack spacing of an RC element is the minimum distance required to transfer sufficient force through bond stress into the concrete such that the tensile capacity of the concrete is exceeded. This is determined by equating the maximum ultimate tensile capacity of concrete and the surface area of reinforcement multiplied by the average ultimate bond stress (i.e. Equation 6).

\[ A_c f'_t = a_{min} u_b d_b \pi \Rightarrow a_{min} = \frac{A_c f'_t}{u_b d_b \pi} \]  \hspace{1cm} (6)

Where: \( A_c \) is the net area of concrete; \( f'_t \) is the ultimate tensile capacity of concrete; and \( d_b \) is the diameter of the reinforcement. The net area of concrete is equal to the gross area of concrete minus the area of vertical reinforcement. This value can be expressed in terms of the percentage of vertical reinforcement (i.e. \( p_v = A_{st}/A_{gross} \)), i.e. Equation 7.

\[ A_c = \frac{A_{st}}{p_v} - A_{st} = A_{st} \left( \frac{1}{p_v} - 1 \right) = \frac{\pi d_b^2}{4} \left( \frac{1}{p_v} - 1 \right) \]  \hspace{1cm} (7)

Using the expression for the net area of concrete developed in Equation 7, the minimum crack spacing can be expressed in terms of the ultimate tensile strength of concrete, reinforcement bar diameter, elastic bond stress and percentage of vertical reinforcement.

\[ a_{min} = \frac{f'_t d_b}{4u_b} \left( \frac{1}{p_v} - 1 \right) \]  \hspace{1cm} (8)

Equation 8 represents an equation for the minimum crack spacing in an RC element, however the average crack spacing may be somewhat larger than this value. When RC is loaded in tension it initially cracks at discrete irregular locations, with subsequent cracks occurring at a distance \( a_{min} \) away from these initial cracks. When the spacing of the initial cracks (i.e. \( a \) in Figure 3) is greater than \( 2a_{min} \) an additional crack will form as indicated in Figure 3. Alternatively, when the spacing of the initial cracks is less than \( 2a_{min} \) an additional crack will not be able to form. As such, Park and Paulay (1975) suggest the “crack spacing [of an RC element in tension] can be expected to vary between \( a_{min} \) and \( 2a_{min} \), with an average spacing of approximately \( 1.5a_{min} \)”. For this study an average crack spacing of \( 1.5a_{min} \) has been adopted, resulting in Equation 9. The readers should note that the horizontal reinforcement in RC walls can act as crack propagators and in effect change the average crack spacing from what is discussed here. Irrespective of this, the crack spacing should still be between \( a_{min} \) and \( 2a_{min} \). The spacing of horizontal reinforcement in an RC wall would vary greatly on a case by case basis and as such including it as a parameter in a generalised model would add significant layers of complexity.

\[ a = 1.5 \frac{f'_t d_b}{4u_b} \left( \frac{1}{p_v} - 1 \right) \]  \hspace{1cm} (9)
2.2 Tensile strength of concrete – $f'_c$

An important factor in determining the crack spacing of an RC element is the ultimate tensile strength of the concrete. A brief summary of the different equations provided by various codes of practice around the world is presented in Table 1 and Table 2. This summary includes: The Australian Standard for Concrete Structures, AS 3600 (Standards Australia 2009); The New Zealand Standard and Commentary for Concrete Structures, NZS 3101:Part 1 and NZS 3101:Part 2 respectively (Standards New Zealand 2006a; Standards New Zealand 2006b); Eurocode 2, EN 1992-1-1 (European Committee for Standardization (CEN) 2004a); and the American Building Code Requirements for Structural Concrete, ACI 318 (American Concrete Institute 2014). The recommendations made in NZS 3101:Part 2 are a summary of what was proposed in the FIB-CEB Model Code 1990 (Comite Euro-International Du Beton 1993).

Typically the ultimate tensile strength of concrete is expressed as either the direct tensile strength or the flexural tensile strength. The latter relates to members where the tensile stresses are due to bending/flexure actions, where concrete can sustain a somewhat higher level of tensile stress due to the varying strain gradient across the depth of the section. This is typically the case for not very deep sections. As the section depth (i.e. wall length $l_w$) increases the flexural tensile strength approaches and eventually equals the direct tensile strength. The Eurocode 2 model proposes this occurs when the section depth is 1600 mm or greater. It would be unusual for walls to have a length less than 1500 mm and as such, for a generalised model, the authors are recommending the direct tensile strength be used. Equation 10 has been adopted for determining the ultimate tensile strength of concrete as the model is being developed for RC structures constructed in Australia. The mean value, as opposed to lower characteristic value, is being used because this study is looking at the actual in-situ performance and response of RC structures instead of the codified characteristic (i.e. “worst case”) response.

$$f'_c = 1.4\sqrt{f'_c} = 0.5\sqrt{f''_c}$$

Table 1. Comparison of direct tensile strength models.

<table>
<thead>
<tr>
<th>Standard</th>
<th>$f'_c$ Mean</th>
<th>$f_{ct,m}$ Mean</th>
<th>$f_{ct,u}$ Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>AS 3600</td>
<td>0.36$f'_c$</td>
<td>1.4$f'_c$</td>
<td>1.8$f'_c$</td>
</tr>
<tr>
<td>NZS 3101:Part 1</td>
<td>0.36$f'_c$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>NZS 3101:Part 2</td>
<td>0.68$f_{ct,m}$</td>
<td>$1.4 \left(\frac{f'_c}{10}\right)^{2/3}$</td>
<td>1.32$f_{ct,m}$</td>
</tr>
<tr>
<td>EN 1992-1-1</td>
<td>0.7$f_{ct,m}$</td>
<td>$0.3(f'_c)^{2/3}$; $f'_c \leq 50$ MPa</td>
<td>1.3$f_{ct,m}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$2.12 \ln\left(1 + \frac{f_{cm}}{10}\right)$; $f'_c &gt; 50$ MPa</td>
<td></td>
</tr>
<tr>
<td>ACI 318</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Table 2. Comparison of flexural tensile strength models.

<table>
<thead>
<tr>
<th>Standard</th>
<th>$f_{ct,f}$</th>
<th>$f_{ct.f.m}$</th>
<th>$f_{ct.f.u}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lower characteristic</td>
<td>Mean</td>
<td>Upper characteristic</td>
</tr>
<tr>
<td>AS 3600</td>
<td>0.6$\sqrt{f_c'}$</td>
<td>1.4$f_{ct,f}$</td>
<td>1.8$f_{ct,f}$</td>
</tr>
<tr>
<td>NZS 3101:Part 1</td>
<td>-</td>
<td>0.6$\sqrt{f_c'}$</td>
<td>-</td>
</tr>
<tr>
<td>NZS 3101:Part 2</td>
<td>0.68$f_{ct,f.m}$</td>
<td>$f_{ct.m}/\left[\frac{1.5(l_w/100)^{0.7}}{1+15(l_w/100)^{0.7}}\right]$</td>
<td>1.32$f_{ct,f.m}$</td>
</tr>
<tr>
<td>EN 1992-1-1</td>
<td>0.7$f_{ct,f.m}$</td>
<td>$\max\left(1.6 - \frac{l_w}{1000}, f_{ct.m}; f_{ct.m}\right)$</td>
<td>1.3$f_{ct,f.m}$</td>
</tr>
<tr>
<td>ACI 318</td>
<td>-</td>
<td>0.62$\sqrt{f_c'}$</td>
<td>-</td>
</tr>
</tbody>
</table>

2.3 Elastic and inelastic bond strength of reinforcement – $u_b$ and $u_b'$

The elastic bond strength model selected is the model used by AS 3600 for calculating development lengths of D500N reinforcement. The commentary to the Australian concrete standard, AS 3600 Supp1 (Standards Australia 2014), provides a detailed explanation of the bond strength model used in AS 3600. The bond strength model is presented in Equation 11. The upper limit of bond stress expressed in Equation 11 is calculated using the equation developed by Reynolds (1983) for the minimum develop length of a deformed bar.

$$u_b = \frac{k_2}{\phi k_1 k_3 k_4 k_5} (0.5\sqrt{f_c'}) \leq \frac{1}{\phi 0.232 k_1}$$

(11)

Where: $k_1$ is a factor to account for settlement of fresh concrete, typically equals 1.0 for walls; $k_2$ is a factor to account for the increase in average bond stress as the bar diameter decreases and is equal to $(132 - d_b)/100$; $k_3$ is a factor to account for the area of undisturbed concrete around the bar being anchored and is equal to $1.0 - 0.15(c_d - d_b)/d_b$, for walls $c_d$ can conservatively be taken as 30 mm; $k_4$ and $k_5$ are factors to account for transverse reinforcement and transverse compressive pressure respectively and can be taken as being equal to 1.0 for walls; $\phi$ is a capacity reduction factor equal to 0.6; and $f_c'$ is the characteristic compressive strength of the concrete.

The characteristic compressive strength of concrete used in Equation 11 is replaced by the mean strength of concrete ($f_{cm}$). AS 3600 and AS 3600 Supp1 suggest Equation 12 for calculating the mean strength of concrete from the characteristic compressive strength. The mean strength is used because the actual in-situ performance of RC walls is being considered.

$$f_{cm} = (1.2875 - 0.001875 f_c') f_c'$$

(12)

Sezen and Moehle (2004) suggest the ratio of inelastic to elastic bond stress to be 0.5. For this study the inelastic bond stress has been assumed to equal $0.5 \times u_b$. Table 3 summaries proposed elastic bond stress values for D500N reinforcement for various standard grades of concrete used in industry for RC walls. The proposed inelastic bond stress values for D500N reinforcement can be calculated by multiplying Table 3 by 0.5. It is important for the reader to note that while the bond stress value chosen will directly affect the calculated crack widths, it does however, have no effect on the final ratio of global to local strain. As decreasing the amount of elastic bond stress will increase both the local strain and average crack spacing at the same rate and vice versa. This means changing the initial elastic bond stress values from what is proposed in Table 3 will have no effect on the final results of the study summarised by Figure 6 and Figure 7.
Table 3. Proposed elastic bond stress values of D500N rebar.

<table>
<thead>
<tr>
<th>Bar size</th>
<th>Concrete grade: N32</th>
<th>Concrete grade: N40</th>
<th>Concrete grade: N50</th>
<th>Concrete grade: S65</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_{cm} = 39.3 \text{ MPa}$</td>
<td>$f_{cm} = 48.5 \text{ MPa}$</td>
<td>$f_{cm} = 59.7 \text{ MPa}$</td>
<td>$f_{cm} = 75.8 \text{ MPa}$</td>
</tr>
<tr>
<td>N12</td>
<td>7.18 MPa</td>
<td>7.18 MPa</td>
<td>7.18 MPa</td>
<td>7.18 MPa</td>
</tr>
<tr>
<td>N16</td>
<td>6.97 MPa</td>
<td>7.18 MPa</td>
<td>7.18 MPa</td>
<td>7.18 MPa</td>
</tr>
<tr>
<td>N20</td>
<td>6.32 MPa</td>
<td>7.03 MPa</td>
<td>7.18 MPa</td>
<td>7.18 MPa</td>
</tr>
<tr>
<td>N24</td>
<td>5.86 MPa</td>
<td>6.51 MPa</td>
<td>7.18 MPa</td>
<td>7.18 MPa</td>
</tr>
<tr>
<td>N28</td>
<td>5.49 MPa</td>
<td>6.10 MPa</td>
<td>6.77 MPa</td>
<td>7.18 MPa</td>
</tr>
</tbody>
</table>

Note: values to the right of the line correspond to the upper limit of bond stress in Equation 11.

2.4 Minimum reinforcement ratio – $p_{v.min}$

An underlying assumption of the model is that cracking occurs prior to the development of inelastic deformation of the vertical reinforcement. That is, in the boundary element (refer Figure 2), the area of concrete times the ultimate tensile capacity (i.e. $A_Cf'_C$) has to be less than or equal to the area of vertical reinforcement times by the yield stress of the reinforcement (i.e. $A_{st}f_{sy}$). Assuming the area of concrete ($A_C$) approximately equals the gross area of the section ($A_{gross}$), an equation for the minimum percentage of reinforcement can be developed, i.e. Equation 13.

$$p_{v.min} = \frac{f'_C}{f_{sy}}$$

(13)

In addition to ensuring the assumptions of this model are met, providing enough vertical reinforcement such that the condition of Equation 13 is met will ensure the RC wall being designed will be capable of developing distributed cracking, ideally allowing the formation of a plastic hinge. A major assumption of force-based seismic analysis is that structures are capable of yielding and developing plastic hinges such that the assumed level ductility in the analysis can be developed in the structure. If less reinforcement is provided than that of Equation 13 it is possible for the formation of discrete irregular cracking in the plastic hinge region with concentrated plasticity in these locations; possibly resulting in poor performance or unexpected failure during an earthquake due the structure being unable to develop the assumed level of ductility used in the analysis.

For the model being developed in the paper, the ultimate tensile stress of concrete was assumed to equal the mean direct tensile stress given in AS 3600. Given the high variability in concrete tensile strengths (e.g. AS 3600 suggest a factor of 1.8 for the ratio of lower to upper characteristic strengths) and that the concrete strength can commonly be somewhat higher than that specified in the design (e.g. precast sub-contractors using higher early strength concrete to decrease production times), the upper characteristic direct tensile strength of $1.8 \times 0.36\sqrt{f'_C}$ given in AS 3600 is recommended when using Equation 13. While the outputs of the model presented later use mean properties of reinforcement, it is recommended to use the characteristic yield stress of reinforcement in Equation 13. Table 4 summarises a set of proposed minimum reinforcement percentages to be adopted in regions of RC walls required to develop a plastic hinge. Low and mid-rise structures utilising cantilever RC walls as the principal lateral load resisting system would require these minimum reinforcement ratios at the base region of walls (i.e. ground floor).

Table 4. Proposed minimum reinforcement ratios for RC walls.

<table>
<thead>
<tr>
<th>Concrete grade</th>
<th>Concrete grade: N32</th>
<th>Concrete grade: N40</th>
<th>Concrete grade: N50</th>
<th>Concrete grade: S65</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{v.min}$</td>
<td>0.7 %</td>
<td>0.8 %</td>
<td>0.9 %</td>
<td>1.0 %</td>
</tr>
</tbody>
</table>
3 EXPERIMENTAL VERIFICATION

Experimental verification of the proposed model was performed using laboratory testing performed recently in the Smart Structures Laboratory at Swinburne University of Technology. The experimental test specimens and loading region were similar to that outlined by Menegon et al. (2015b). The reader is directed there for a more detailed explanation of the test setup and loading protocol. These test specimens differed slightly from those presented by Menegon et al. (2015b) in that they had (a) a crack propagator and (b) post yield strain gauges installed on the vertical reinforcement; both located at mid height. An overview of the test specimens used for the experimental verification is presented in Table 5.

Table 5. Experimental test specimen overview.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Thickness (mm)</th>
<th>Length (mm)</th>
<th>Height (mm)</th>
<th>Vertical reinf.</th>
<th>Vertical reinf. ratio</th>
<th>Horizontal reinf. ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall type 5</td>
<td>130</td>
<td>450</td>
<td>800</td>
<td>6-N10*</td>
<td>0.0080</td>
<td>0.0028</td>
</tr>
<tr>
<td>Wall type 6</td>
<td>130</td>
<td>450</td>
<td>800</td>
<td>3-N16†</td>
<td>0.0103</td>
<td>0.0014</td>
</tr>
<tr>
<td>Wall type 7</td>
<td>130</td>
<td>450</td>
<td>800</td>
<td>6-N16*</td>
<td>0.0206</td>
<td>0.0028</td>
</tr>
</tbody>
</table>

* Vertical reinforcement placed with 3 bars per face (i.e. 2 grids of reinforcement).
† Vertical reinforcement placed with 3 bars centrally (i.e. 1 central grid of reinforcement).

Due to the design of the test specimens the initial cracking occurs at the interface to the boundary element and at mid height where the crack propagator is located. Subsequent cracking would follow at a distance of $a_{min}$ away from these locations (Figure 4(a)). For wall types 5, 6 and 7 $a_{min}$ was calculated to be 133, 160 and 81 mm respectively. The calculated crack distribution and spacing for each test specimen is shown in Figure 4. This crack distribution approximately matched that which was observed during the tests.

The test specimens were applied under a cyclic axial tension-compression loading regime. The proposed model was used to calculate the cracks widths, local strains and reinforcement stress for the global displacement of each tension load cycle. The accuracy of the model was assessed by comparing the local strain of the reinforcement at mid height, where the post yield strain gauges were installed, and the tension force, expressed in terms of reinforcement stress. A very good correlation between the experimental tests and the proposed model was observed – Figure 5.

![Figure 4](image-url)
4 PARAMETRIC STUDY

A parametric study was performed to determine a generalised relationship between the global strain of an RC element and the local strain of the reinforcement. The study was performed for 500 MPa class N bars (i.e. normal ductility) and 500 MPa class L mesh (i.e. low ductility) to AS/NZS 4671 (Standards Australia and Standards New Zealand 2001). The study was conducted using expected in-situ mean material properties: for D500N and D500L reinforcement respectively, the yield stress was taken as 550 and 585 MPa, the ultimate stress as 660 and 620 MPa and the ultimate strain as 9.5 and 3.3 per cent (Menegon et al. 2015a). The Priestley, Calvi and Kowalsky (2007) stress-strain model for reinforcement, modified to have no yield plateau, was adopted. The distinct yield plateau seen in uncoiled N grade bars is not present when the bars are subject to cyclic post yield loading. L grade reinforcement does not exhibit an observable yield plateau irrespective of previous loading cycles. The stress-strain curve for reinforcement adopted in the study can be expressed by Equations 14 and 15.

\[ f_s = E_s \varepsilon_s \]

where: \( \varepsilon_s \leq \varepsilon_{sy} \)  

(14)

\[ f_s = f_{su} - (f_{su} - f_{sy}) \left[ \frac{\varepsilon_{su} - \varepsilon_s}{\varepsilon_{su} - \varepsilon_{sy}} \right]^2 \]

where: \( \varepsilon_{sy} < \varepsilon_s \leq \varepsilon_{su} \)  

(15)
The results of the parametric study are shown in Figure 6 and Figure 7. Figure 6 can be used to calculate the local strain in the reinforcement based on the global strain of the section and the ratio of its percentage of vertical reinforcement \( (p_v) \) to mean ultimate direct tensile strength \( (f_{ct.m}) \). Figure 7 can be used to calculate the maximum global strain a section can undergo while ensuring the local strain of the reinforcement is less than (a) the characteristic ultimate strain to AS/NZS 4671 (i.e. 1.5 and 5 per cent for L and N grade reinforcement respectively) or (b) mean ultimate strain (i.e. 3.3 and 9.5 per cent for L and N grade reinforcement respectively).

**Figure 6.** LEFT: Global to local strain chart for N grade reinforcement. RIGHT: Global to local strain chart for L grade reinforcement.

\[
k_1 = \frac{p}{f_{ct.m}}
\]

**Figure 7.** LEFT: Maximum global strain to limit local strains to less than characteristic ultimate strains. RIGHT: Maximum global strain to limit local strains to less than mean ultimate strains.
5 TENSION STIFFENING

When undertaking a moment-curvature analysis of an RC element using a fibre element approach, typically the tensile capacity of the concrete is ignored. The stress block is balanced based off this assumption, meaning the tension strain in the reinforcement calculated, is the local strain in the reinforcement. This would result in the strain diagram denoted by the dashed line in Figure 8 and possibly an overestimated section curvature. It is hypothesised that the equation expressed in Figure 8, in conjunction with the tables in Figure 6, could be used to calculate the reduced section curvature taking into account the tension-stiffening phenomenon of cracked concrete.

![Figure 8. Tension-stiffening of RC box-shaped lift core.](image)

6 CONCLUSIONS

This paper has outlined the development of a theoretical model for calculating the local and global tensile strains in RC walls. An input parameter for the model is the ultimate tensile capacity of concrete. A summary of the different expressions used by various concrete standards (i.e. AS 3600, NZS 3101, EN 1992-1-1 and ACI 318) has been included. Average elastic and inelastic bond stress values of D500N (i.e. N grade normal ductility) reinforcement for commonly used concrete grades in RC walls is presented.

A discussion about the minimum percentage of vertical reinforcement required to allow for the development of a plastic hinge at the base of RC walls is presented. An equation based on the ultimate tensile strength of concrete and the yield stress of reinforcement is proposed for ensuring sufficient cracking in RC walls can occur. This will ideally allow for enough distributed plasticity for the development of a plastic hinge. Providing 1 per cent vertical reinforcement at the base of cantilever RC wall should allow for the development of a plastic hinge in walls with concrete grades up to S65 (i.e. $f'_{t} = 65 \text{ MPa}$).

The proposed theoretical model was validated against recent experimental testing undertaking by the authors in the Smart Structures Laboratory at Swinburne University of Technology. The theoretical model showed good agreement with the experimental works. A parametric study was performed to produce charts for determining the local strain in reinforcement based on the global strain of the section and vice versa. The charts are based on the ratio of percentage of vertical reinforcement in the boundary element of the wall to the ultimate tension capacity of the concrete. This ratio suggests that higher concrete strengths are not necessarily better for the performance of RC walls approaching collapse. This is illustrated by considering two concrete walls with the same content of vertical reinforcement but with different concrete grades, one being high strength concrete and the other normal strength. At the same level of displacement demand and so presumably equal global strains at the base of the walls, the wall with the high strength concrete will likely have a higher risk of bar fracturing given that the $k_1$ ratio of the wall is much lower.
7 ACKNOWLEDGEMENTS

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8 REFERENCES

American Concrete Institute, 2014. Building Code Requirements for Structural Concrete (ACI 318-14), American Concrete Institute, Farmington Hills, MI.


