An equivalent truss method for the analysis of timber diaphragms

D. Moroder, T. Smith, S. Pampanin, & A.H. Buchanan

University of Canterbury, Christchurch, New Zealand

ABSTRACT: Recent years have seen more architects and clients asking for tall timber buildings. In response, an ambitious timber community has been proposing challenging plans and ideas for multi-storey commercial and residential timber buildings. While engineers have been intensively looking at gravity-load-carrying elements as well as walls, frames and cores to resist lateral loads, floor diaphragms have been largely neglected.

Complex floor geometries and long span floor diaphragms create stress concentrations, high force demand and potentially large deformations. There is a lack of guidance and regulation regarding the analysis and design of timber diaphragms so structural engineers need a practical alternative to simplistic equivalent deep beam analysis or costly finite element modelling.

This paper proposes an equivalent truss method capable of solving complex geometries for both light timber framing and massive timber diaphragms. Floor panels are discretized by equivalent diagonals, having the same stiffness as the panel including its fasteners. With this method the panel unit shear forces (shear flow) and therefore fastener demand, chord forces and reaction forces can be evaluated. Because panel stiffness is accounted for, diaphragm deflection, torsional effects and transfer forces can also be assessed.

1 INTRODUCTION

There is a growing interest in engineered multi-storey timber buildings around the world. A number of tall timber buildings have already been built in Europe and Northern America, with New Zealand and Australia following this global trend (BSLC 2014; MPI 2014).

The local availability of glued laminated timber (glulam), Cross Laminated Timber (CLT), Laminated Veneer Lumber (LVL) as well as prefabricated Light Timber Frame (LTF) elements are encouraging the use of timber for new multi-residential, commercial and industrial buildings, supported by the actual and the soon to be released revision to the NZ Timber Structures Standard NZS3603 as well as upcoming changes to the Australian National Construction Code (ABCB 2015).

This new interest in medium to high-rise multi-storey timber buildings creates the need for more rigorous design. Notable research regarding gravity structures and lateral load resisting systems has been carried out (CUREE 2000; Ceccotti et al. 2006; van de Lindt et al. 2010; Gagnon et al. 2011) but little research is available on the behaviour of diaphragms. Irregular floor geometries, variable floor setups (i.e. staggered nailing patterns) and non-uniform loading conditions make traditional simplistic diaphragm analysis methods, like the deep beam analogy, inadequate. In addition, there is a large knowledge gap regarding the performance and design of both LTF and massive timber diaphragms under earthquake loading (Moroder et al. 2014). Feedback from the designers of recent timber structures in New Zealand and a recent cost comparison (TDA 2015) indicated the lack of diaphragm analysis tools as a limitation in the uptake of modern timber buildings.

Substantial effort has recently been dedicated to the dissemination of improved design methods for concrete diaphragms like the strut-and-tie analysis (Schlaich et al. 1987; Bull et al. 2014). This paper provides a similar truss method for the design of LTF floors and massive timber floor panels (CLT, glulam, LVL or similar).

1.1 Role of diaphragms

Independent of the construction material used, diaphragms have multiple roles to play in the
structural behaviour of a building. Aside from acting as slabs under gravity loads, diaphragms tie all other structural elements together, creating the three-dimensional framework of the structure. Horizontal forces from wind and earthquake actions are transferred by the diaphragm to the vertical elements of the Lateral Load Resisting System (LLRS). Because of the sometimes incompatible deflected shape of the LLRS under load, diaphragms also need to resist transfer forces (Park et al. 1997). The diaphragms also provide buckling restraint to gravity carrying columns and beams and resist the thrust of inclined columns. As the first element to resist most gravity and horizontal forces, a loss of diaphragm action will likely compromise the behaviour of the whole structure.

A number of damaged concrete diaphragms in the 1994 Northridge Earthquake and the 2011-2012 Canterbury earthquake series have highlighted shortcomings in code provisions and a lack of understanding of diaphragm behaviour. Underestimation of diaphragm accelerations, disregard of diaphragm flexibility in the structure’s behaviour, displacement incompatibilities between the floor diaphragm and the LLRS and incomplete or inappropriate diaphragm analysis can compromise the internal load paths and result in unconservative diaphragm designs (Nakaki 2000; Fleischman et al. 2001; Bull 2004; Canterbury Earthquake Royal Commission 2012). Although this paper focuses on seismic design, similar principles regarding force distribution and connection detailing apply to the design of diaphragms for wind loads or any other lateral loads (for example stability, soil/water pressure) on buildings. All forces created within the diaphragm, or deriving from load applications along the diaphragm edges, need to be transferred to the LLRS. Concentrated forces, floor openings or re-entrant corners create stress concentrations, which need to be accounted for to prevent premature failures. All components of floor diaphragms (chord beams, collectors and strut beams, panel elements and the various connections) need to resist the anticipated forces guaranteeing a clearly defined load path through the structure, from the points of load application to the foundations.

1.2 New developments in timber diaphragm systems

Traditionally, timber diaphragms consist of wooden sheathing on light timber framing (see Figure 1). This construction type has been used for floors and for large panelised roof systems for industrial buildings. Recent innovations however have opened the possibility of using larger massive timber floor panels made of LVL, CLT or glulam, as well as pre-assembled Structural Insulated Panels (SIPs), allowing the design of large floor geometries in a cost-effective manner.

![Figure 1 Light timber frame and massive timber diaphragm examples, with schematic cross sections](www.continuingeducationconstruction.com)

Diaphragms built from massive timber panels have not yet been codified internationally and design of CLT structures is often simplified by assuming rigid diaphragms. This is normally justified by the limited size of residential floor spans and by using sufficient overstrength for the diaphragm design (Ceccotti 2008; Dujic et al. 2010; Follesa et al. 2013). Regardless of a flexible or rigid floor diaphragm assumption, the forces in the panels, beams and connections need to be verified. The increasing use of massive timber floor systems for multi-residential and commercial buildings with larger floor spans and multiple rows of panels will require the calculation of deformations in order to verify the rigid...
diaphragm assumption and the resulting force distribution.

2 EQUIVALENT TRUSS METHOD (STRUT-AND-TIE FOR TIMBER)

Even though the deep beam analogy has been successfully used for the design of LTF diaphragms (Smith et al. 1986) and has been shown to be adequate for massive timber diaphragms (Waller-Novak et al. 2013; Moroder et al. 2015), it is not sufficiently accurate for the design of modern floors with irregular geometries. Floor openings, re-entrant corners, setbacks and other irregularities create shear force concentrations, force components perpendicular to the panel edge and decreased diaphragm stiffness, all effects not accounted for in the deep beam analogy.

2.1 The truss analogy

With the increasing popularity of strut-and-tie analysis (Schlaich et al. 1987) for concrete diaphragms (Bull 2004; Moehle et al. 2010) the authors encourage the adoption of an equivalent truss method for the analysis of complex timber diaphragms. Similar methods for concrete diaphragms have also been proposed (Bull et al. 2014; Scarry 2014). Initially proposed by Hrennikoff (1941) for generic elastic materials, the truss method was applied to timber diaphragms with openings by Kamiya (1990). Kessel et al. (2001) derived the equivalent diagonal stiffness for LTF diaphragms based on the 'shear field analogy' (see Figure 2). This concept has been further refined and modified by the authors for use with LTF and massive timber diaphragms.

Because reinforced concrete strut-and-tie models rely on the tensile strength of reinforcement bars and the compression capacity of the concrete, the arrangement of the strut and ties is not unique. For timber diaphragms the size of the panels is well known and each panel is surrounded by fasteners possessing a known relative stiffness. The equivalent diagonals can therefore be placed across each panel element, automatically defining the truss grid.

Since the panel connections are the main source of diaphragm flexibility, they need to be accounted for in the calculation of diagonal stiffness. Even though some assumptions required by the shear field analogy are not satisfied for massive timber diaphragms (i.e. the sheathing panels are surrounded by framing elements, the longitudinal stiffness of the panels is negligible compared to the axial stiffness of the framing members, etc.), the panels mainly work in shear, with longitudinal stresses only induced by local irregularities or discrete forces.

According to solid mechanics theory, as shown on the left of Figure 2, the shear force \( F \) deforms a rectangular element into a parallelogram with a shear angle \( \gamma \). The flexibility of the panel fasteners further increases the panel deflection. This deflection behaviour can be reproduced by a four-bar linkage system with an equivalent diagonal as shown on the right of Figure 2. The stiffness of the equivalent diagonal is chosen to give the same panel deformation under a load \( F \) as the shear panel.

Depending on the type of connection between the individual panels and between the panels and the beams, the equivalent shear-through-thickness rigidity of the panel \( (Gd)_{sf} \) becomes

\[
(Gd)_{sf} = \frac{1}{\frac{1}{Gd} + \frac{s}{K_{ser}} \left( \frac{c_1 + c_2}{b h} \right)}, \tag{1}
\]

where \( G = \) shear modulus of the sheathing; \( d = \) sheathing panel thickness; \( s = \) fastener spacing; \( K_{ser} = \) slip modulus of the fastener parallel to the panel edge; \( c_1 = \) number of lines of fasteners between adjacent panels along the sheathing panel height \( h \); and \( c_2 = \) number of lines of fasteners between adjacent panels along the sheathing panel length \( b \). In LTF diaphragms the sheathing panels are connected to each other via the framing elements, requiring a line of fasteners for each panel and therefore resulting in \( c_1 = c_2 = 2 \). For massive timber panels no framing elements are necessary along the longitudinal panel edge, so \( c_1 \) is typically 1. The ends of the panels are normally sitting on a beam, requiring two lines of fasteners to transfer the forces and therefore \( c_2 = 2 \).

If the unit shear force (or shear flow) in the panel is set equal to the value of the axial stress \( \sigma \) then the
The equivalent modulus of elasticity of the diagonal $E_{ef}$ can be determined as

$$E_{ef} = \frac{(Gd)_{ef} l^2}{hb}.$$

(3)

It is worth noting that by equalising the unit shear force and the stress, which is incorrect from a solid mechanics point of view, the units of the equivalent modulus of elasticity and cross sectional area do not have their correct units. Conceptually this can be avoided if the stress in the diagonal is equalled to the unit shear force divided by a unit length.

2.2 Multiple diagonals per panel

Because of very high aspect ratios of massive timber panels (available panel sizes in New Zealand can be up to 1.2 x 18.0 m or longer), the angle of the equivalent diagonals becomes very high, resulting in a poorly working truss. It is therefore necessary to subdivide the panel in order to obtain diagonals with less pronounced angles (ideally 45°). The presence of floor irregularities requires force redistribution in the form of local reinforcement and strut beams. These elements collect the high forces from the disturbed areas and tie them back into other parts of the diaphragms. The additional truss members also might require a subdivision of the panels into a number of diagonals to provide more accurate force redistribution.

Because massive timber floor panels possess relatively high axial stiffness compared to LTF floors, normal stresses along the two main directions need to be accounted for. Additionally, fasteners will not only transfer forces parallel to the panel edges, but also perpendicularly to them. By dividing the panels into multiple diagonals as shown in Figure 3, the transversal truss elements can account for these effects by including the fastener stiffness perpendicularly to the panel edges.

The individual diaphragm panel can be formally subdivided in a regular pattern obtaining $m \times n$
equivalent diagonals or by diagonals with varying lengths according to Table 1 and Figure 4. Higher forces are often attracted close to stiffer elements like beams or supports. In such cases the average of all diagonals belonging to one panel element should be considered.

Table 1 Diagonal properties in case of regular and irregular panel sub-divisions

<table>
<thead>
<tr>
<th>m x n</th>
<th>Regular diagonals</th>
<th>Irregular diagonals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( E_{ef,\text{mn}} = mn \left( \frac{Gd_{ef}}{h} \right) \frac{l_{\text{max}}^2}{hb} )</td>
<td>( E_{ef,\text{ij}} = \left( \frac{Gd_{ef}}{h} \right) \frac{l_{ij}^2}{hb_{ij}} )</td>
</tr>
<tr>
<td></td>
<td>( A_{ef,\text{mn}} = l_{\text{max}} = \sqrt{\left( \frac{b}{m} \right)^2 + \left( \frac{h}{m} \right)^2} )</td>
<td>( A_{ef,\text{ij}} = l_{ij} = \sqrt{b_{ij}^2 + h_{ij}^2} )</td>
</tr>
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</table>

Figure 3 Massive diaphragm panel and its idealization in the equivalent truss model with multiple diagonals

All beams (collector, chord, strut beams) and framing elements as well as other reinforcing elements are modelled with their actual axial stiffness. The remaining longitudinal truss elements (along the panel height \( h \)) are to be modelled with the axial panel stiffness corresponding to the tributary width \( b' \) of the truss element as shown in Figure 3.

Figure 4 Multiple diagonals for a) regular m x n sub-divisions or for b) irregular sub-divisions
For the transversal truss element (along the panel width \( b \)) not corresponding to beams or framing elements, the stiffness of the tributary panel strip is summed (in series) with the fastener stiffness perpendicular to the panel edge (\( K_{ser,\perp} \)). Considering a common sub-division of two diagonals along the panel width, the equivalent stiffness (in force per length) of the transversal member for a LTF diaphragm can be calculated as

\[
K_{ef,(LTF)} = \frac{1}{\frac{1}{E_{90}A'} + \frac{1}{n'K_{ser,\perp}}} \cdot (8)
\]

Where \( E_{90} = \) is the panel stiffness perpendicular to the panel direction (transversal stiffness); \( A' = h' d \) is the tributary cross section of the transversal truss element; \( d = \) is the panel thickness; \( b' = (b_i + b_{i+1})/2 \) is the tributary width of the longitudinal truss element; \( h' = (h_i + h_{i+1})/2 \) is the tributary width of the longitudinal truss element; \( n' = \) is the number of fasteners along \( h' \); and \( K_{ser,\perp} = \) is the slip modulus of the fasteners perpendicular to the panel edge.

In the case of a massive timber diaphragm the axial stiffness of the transversal truss element is much bigger than the stiffness of the fasteners and can normally be ignored. For a sub-division of two diagonals along the panel width, the equivalent stiffness of the transversal member in a massive timber panel can be calculated as

\[
K_{ef,(massive)} = \frac{1}{\frac{1}{E_{90}A'} + \frac{1}{2n'K_{ser,\perp}}} \cdot (9)
\]

The factor of 2 accounts for the fact that the fasteners are shared between two adjacent panels.

2.3 Validation of the truss analogy on an irregular massive timber diaphragm

Figure 5 shows FEM analysis carried out in SAP2000 (CSI 2004) and the equivalent truss method for an irregular floor layout made of 1.05 x 3.9 m CLT panels, with loads applied in the direction up the page. CLT panels of 1.2 x 4.8 m were modelled as orthotropic membrane elements (\( E_1 = 8 \) GPa, \( E_2 = 4 \) GPa, \( E_3 = 0.5 \) GPa, \( G_{12} = 600 \) MPa, \( G_{13} = 500 \) MPa, \( G_{23} = 100 \) MPa, \(\nu_{12} = 0.07\), \(\nu_{13} = \nu_{23} = 0.35\)) (Ashtari 2009) and the panel connections made of Ø6 mm screws at 150 mm along a lap joint, were modelled with linear elastic link elements (slip modulus \( K_{ser} = 3000 \) N/mm parallel and perpendicular to the panel edge). A seismic force of 3.5 kN/m² was applied as a surface load. The magnitude of this load is as would be expected on a floor diaphragm in a mid-rise timber building in a seismic area.

Each diaphragm panel has been modelled with 8 equivalent diagonals. Chord, strut and collector beams have been modelled with their actual sections. Loads have been applied as line loads to the truss elements parallel to the loading direction. A comparison of Figures 5a and 5c show that the unit shear forces can be predicted accurately with the equivalent truss. In average the truss method overpredicted the results by 15%, a value which is acceptable considering all assumptions and simplifications involved in the design of timber diaphragms. Similar accuracy was found by a comparison carried out with a very similar method by Kamiya et al. (1998).

The diaphragm flexibility can also be obtained in a very precise fashion, as well as the reaction forces (not shown). Figures 5b and 5d show the compression and tension forces in chord and collector beams from the FEM and the equivalent truss analysis, respectively. Once the tension and compression forces are obtained, the assumed section of the chords and collectors can be verified and their connections or splices can be designed. Knowing the unit shear, the fastener capacity, the panel shear capacity and the buckling strength (although the latter two seldom govern the design of massive timber diaphragms) can be verified.

This comparison between a sophisticated finite element analysis and a comparatively simple truss method for a massive timber diaphragm shows that the latter can predict all key aspects for a
diaphragm design in a sufficiently accurate manner. Some axial forces in the chord beams are underpredicted, so some conservatism in their section design and splice design should be adopted. This procedure is even more accurate for LTF diaphragms, since the sheathing panels almost exclusively work in shear. Independently from the type of diaphragm panels adopted, the equivalent diagonal stiffness needs to be calculated, taking into account panel shear stiffness and fastener stiffness.

The proposed equivalent truss method assumes linear elastic behaviour of the fasteners, so stress redistributions due to localized fastener yielding cannot be accounted for. If the non-linear load-displacement curve of the fasteners is known, the equivalent modulus of elasticity can be derived as a function of the panel displacement, allowing for non-linear behaviour of the diaphragm. Unlike strut-and-tie applications for concrete diaphragms, the diagonal stiffness must be re-calculated and re-assigned if the timber panel subdivision is changed, or if the fastener spacing or type is altered.

2.4 **Diaphragm flexibility**

Depending on the relative stiffness of the diaphragm compared to the LLRS, horizontal forces are distributed to the LLRS according to tributary areas in the case of flexible diaphragms or according to the relative stiffness of the LLRS elements in the case of rigid diaphragms. Rigid diaphragms also require the analysis of torsional effects.

To define whether the diaphragm is rigid or flexible, the horizontal deflection calculation for both the vertical LLRS and the diaphragm is required. The New Zealand Earthquake Loading Standard NZS1170.5 (Standards New Zealand 2004) for example defines a diaphragm as flexible when the maximum lateral deformation of the diaphragm itself is more than twice the average inter-storey drift of the vertical lateral load resisting elements of the associated storey. For irregular diaphragms the
irregular diaphragms have an essential role in the lateral load resistance of buildings and special care should be taken in their design. There is an increasing number of multi-storey timber buildings with irregular floor plates where the existing analysis tools, like the girder (or deep beam) analogy, are in most cases inadequate. Irregular floor diaphragms often incur stress concentrations and require additional collector beams or strut beams to transfer these forces into other parts of the diaphragm further away from the irregularity.

The equivalent truss method is a valid approach to quantify panel stresses and beam forces for irregular diaphragms, for both LTF and massive timber diaphragms. Based on the deflection analogy of a single shear panel, an equivalent diagonal can be defined by the use of simple equations. Because of floor irregularities and high aspect ratios, panels need to be subdivided into a number of diagonals. Additional truss elements resemble longitudinal stresses and fastener forces perpendicular to the panel edge which might become important for massive timber diaphragms. The use of a truss model with beams, frame members and the diagonals allows for a simple and intuitive analysis of diaphragms by determining panel unit shear forces, member forces, and diaphragm deflections.

Since the diaphragm flexibility is directly accounted for in the analogy, the method automatically distributes the lateral forces to the respective lateral load resisting elements, as long as they are modelled with their correct stiffness. Geometric torsional effects are also accounted for.

3 CONCLUSIONS

Floor diaphragms have an essential role in the lateral load resistance of buildings and special care should be taken in their design. There is an increasing number of multi-storey timber buildings with irregular floor plates where the existing analysis tools, like the girder (or deep beam) analogy, are in most cases inadequate. Irregular floor diaphragms often incur stress concentrations and require additional collector beams or strut beams to transfer these forces into other parts of the diaphragm further away from the irregularity.

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