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Frequency-dependent Strong Motion Duration Using Total Threshold Intervals of Velocity Response Envelope

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ABSTRACT: Spectrum representation of strong motion duration has been proposed using total threshold intervals of velocity response envelope calculated for viscously-damped single-degree-of-freedom oscillator of various natural periods. Numerical examples of spectral representation are shown using strong ground motion records observed during a shallow crustal earthquake and off-shore inter-plate earthquakes. It has been shown that the proposed method provides a convenient measure of frequency-dependent duration and the influence of strong shaking on various phenomena including structural response and human behaviour.

1 INTRODUCTION

Duration is one of the representative characteristics of a strong motion together with amplitude and frequency content. Given the same level of amplitude and frequency content, a strong motion with longer duration can cause severer damage to structures. Long duration may also hinder people from taking emergency actions, which may lead to increase in casualties. Various types of durations have been proposed. Bommer and Martinez-Pereira (2000) classified strong motion duration into four categories: 1) *significant duration*: the interval during which a certain portion of the total Arias intensity is accumulated, 2) *bracketed duration*: the interval between the first and last excursion of a particular threshold amplitude, 3) *uniform duration*: the sum of all of the time intervals during which the amplitude of the record is above the threshold, and 4) *structural response duration*: one of these three definitions above applied to dynamic response of a specific single-degree-of-freedom oscillator to an input ground motion, emphasizing on the *frequency-dependent* nature of duration. The thresholds to determine the duration can be either *absolute* or *relative* values (e.g. 0.2G or 10%PGA, respectively) of the associated shaking intensity.

Duration by itself is not a direct cause of damage. When adverse effects of duration to structural performance or human behaviour during strong shaking are discussed, the duration should be associated with its *absolute* amplitude level and with its *frequency* content as well. Trifunac and Westermo (1977) defined *frequency-dependent* duration based on the mean-square integrals of bandpass-filtered ground motion in terms of the sum of the time intervals corresponding to 90% of the total value, which can be interpreted as *uniform*/*relative* duration. Perez (1980) proposed spectra of amplitude levels sustained for a specific number of cycles using velocity response of SDOF oscillators with 5% critical damping. In narrow-band vibration, the number of cycles N can be approximately related to the duration D and the period T of the oscillator as $D \simeq NT$. Ishii (2008) defined response duration spectra using 5-95% *significant* durations of velocity response of SDOF oscillators with 1% and 5% critical damping, which can be regarded as *significant/relative* duration. Those proposed durations or duration spectra, however, are not directly associated with any *absolute* amplitude level. On the other hand, response spectrum is a useful expression of strong motions representing frequency-dependent response amplitude but inherently lose information on duration. As a whole, less attention has been paid to *uniform*/*absolute*/ *frequency-dependent* duration.

With this background, in this study, velocity response envelope duration D_{VRE} and its spectrum representation $S_{\mathrm{Dur\text{-}VRE}}$ are proposed as *frequency-dependent uniform* duration for *absolute* thresholds. First, time histories of velocity response are calculated for single-degree-of-freedom oscillators with various natural periods and damping ratio h=5% for a given accelerogram of strong ground motion. Next, an envelope waveform for each velocity response is calculated. Then the total of time intervals during which the envelope exceeds a specified threshold is calculated as velocity response envelope duration D_{VRE}. By plotting D_{VRE} for various natural periods and thresholds, the *frequency-dependent* durations are graphically represented as velocity response envelope duration spectra $S_{\mathrm{Dur\text{-}VRE}}$. Numerical examples are shown for a shallow crustal earthquake (the Hyogoken-Nambu Earthquake, Japan, January 17 1995, M_w=6.8) and off-shore inter-plate earthquakes (the off the Pacific Coast of Tohoku Earthquake, Japan, March 11, 2011, M_w=9.0 and the Tokachi-Oki Earthquake, Japan, September 26, 2003, M_w=8.0)

2 VELOCITY RESPONSE ENVELOPE DURATION SPECTRA

As stated above, this study employs the definition of *uniform* duration during which the velocity response of the SDOF oscillator exceeds a certain threshold. In evaluating duration of strong motion for an *absolute* threshold, both *uniform* and *bracketed* durations can be calculated. The *bracketed* duration has a drawback in evaluating temporally separated waveform with multi-peaks due to multiple sub-events of fault rupture in several strong motion generation areas (SMGA) (as shown in Figure 3(b) afterwards), resulting in substantial overestimation.

To avoid such overestimation, *uniform* duration is employed. Nonetheless, *uniform* duration has its own drawback. Velocity response is a dynamic motion in essence, taking both large and small absolute values alternately. In evaluating *uniform* duration, the time periods with small amplitude between peaks should be included in the duration. Otherwise the duration can be underestimated. An envelope waveform, therefore, must be defined for proper evaluation of uniform duration. There are various methods to determine envelopes such as complex envelope (Farnbach, 1975), peak connection (Trifunac, 1971), peak hold, and so on. In this study, velocity envelope defined by the total vibration energy of the oscillator is used.

2.1 Velocity response envelope duration and its spectra

The time history of the total energy, denoted by $Q(t)$, of a lightly-damped single-degree-of-freedom oscillator with a lumped mass m and stiffness k at time t is given as a sum of the kinetic energy and the potential energy.

$$Q(t) = \frac{1}{2}m\dot{y}^2(t) + \frac{1}{2}ky^2(t) \tag{1}$$

where $y(t)$ = relative displacement response and $\dot{y}(t)$ = relative velocity response of the oscillator at time t. Then, the envelope waveform for relative displacement response $E_{\mathrm{D}}(t)$ is given by:

$$E_{\mathrm{D}}(T,t) = \sqrt{\frac{Q(t)}{k/2}} = \sqrt{\frac{\dot{y}^2(t)}{\omega_0^{\ 2}} + y^2(t)} \tag{2}$$

where $\omega_0=(k/m)^{1/2}$ [rad/s] is undamped natural angular frequency and $T=2\pi/\omega_0$ [s] is undamped natural period. On this basis, by applying the multiple filter technique using the SDOF systems as narrow band-pass filters, Kameda (1975) defined evolutionary power spectrum. In the precedent study, Trifunac (1971) called the envelope of the relative displacement response, approximated by connecting the successive peaks of $y(t)$ by a straight line, as "response envelope spectrum" (RES) in order to investigate characteristics of the input ground motions. Kameda (1975) gave theoretical meanings to RES and its convenient evaluation method, Equation 2, on the basis of random vibration theory in order to develop a strong ground motion simulation model.

In this study, the envelope waveform for relative velocity response $E_{\mathrm{V}}(T,t)$ is adopted in order to measure the *uniform*/*absolute* duration of strong motions.

$$E_{\mathrm{V}}(T,t) = \sqrt{\frac{Q(t)}{m/2}} = \sqrt{\dot{y}^2(t) + \omega_0^{\ 2}y^2(t)} \tag{3}$$

The *uniform*/*absolute*/*frequency-dependent* duration using the velocity response envelope $E_{\mathrm{V}}(T,t)$ is calculated as total threshold intervals of $E_{\mathrm{V}}(T,t)$ for a given threshold E^* as follows:

$$D_{\mathrm{VRE}}(T,E^*) = \sum_i \delta(T,t_i \mid E^*)\Delta t \tag{4}$$

$$\delta(T,t_i \mid E^*) = \begin{cases} 1;\ E_{\mathrm{V}}(T,t_i) \geq E^* \\ 0;\ E_{\mathrm{V}}(T,t_i) < E^* \end{cases} \tag{5}$$

where Δt =time interval of accelerogram [s] and t_i = discrete time step.

Figure 1 illustrates the process of calculating velocity response envelope $E_{\mathrm{V}}(T,t)$ for the SDOF of natural period T=1.0s with damping ratio h=5%. Two terms in Equation 3, $\dot{y}(t)$ and $\omega_0 y(t)$ are also shown. Response envelope spectra can be obtained by plotting $E_{\mathrm{V}}(T,t)$ for various natural period T. Following Equations 4 and 5, $D_{\mathrm{VRE}}(T=1,E^*)$=16.5, 8.4, 5.7 and 0.56s for threshold values of E^*=20, 50, 100 and 200 cm/s, respectively. As seen around t=12s of E^*=100cm/s, time period not exceeding the threshold is not included in the uniform duration.

Duration spectrum $S_{\mathrm{Dur\text{-}VRE}}(T,E^*)$ can be obtained by plotting $D_{\mathrm{VRE}}(T,E^*)$ for various natural period T.

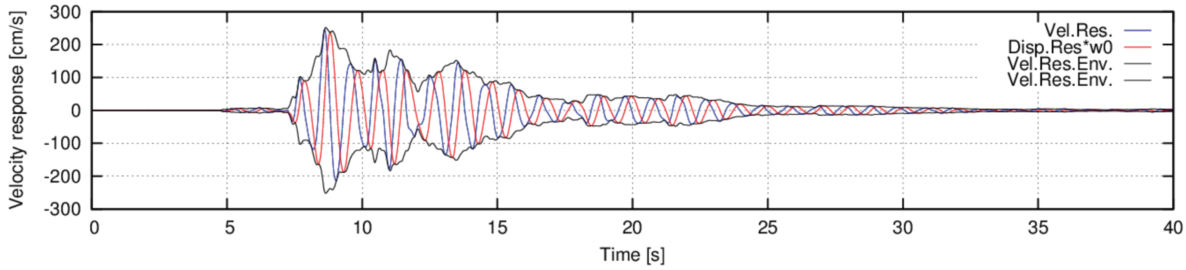


Figure 1 Velocity response envelope E_{V} and its uniform/absolute duration D_{VRE} (T=1s, h=5%).

2.2 Definition of significant duration

Although *significant*/*relative* durations are not of central interest in this study, they are also evaluated in numerical examples for comparison purpose. Let $x(t)$ represent a time history of an arbitrary measure of ground motion intensity. The Husid plot, denoted by $P_C(t)$, is calculated as a time history of normalized cumulative power of $x(t)$.

$$P_C(t) = \frac{100\int_0^t x^2(\tau)\,d\tau}{\int_0^{t_d} x^2(\tau)\,d\tau} \tag{6}$$

where t_d represents the total length of ground motion records [s]. *Significant*/*relative* duration, denoted by $D_{a\text{-}b}$, is then defined as consecutive time between $P_C(t)$ reaches a% and b% of the total power.

$$D_{a\text{-}b} = P_C^{-1}(b) - P_C^{-1}(a) \tag{7}$$

Figure 2 shows Husid plots and the results of significant duration $D_{5\text{-}95}$. Using cumulative power of ground acceleration and velocity response (T=1s), $D_{5\text{-}95}$ are obtained as 8.4s and 10.6s, respectively.

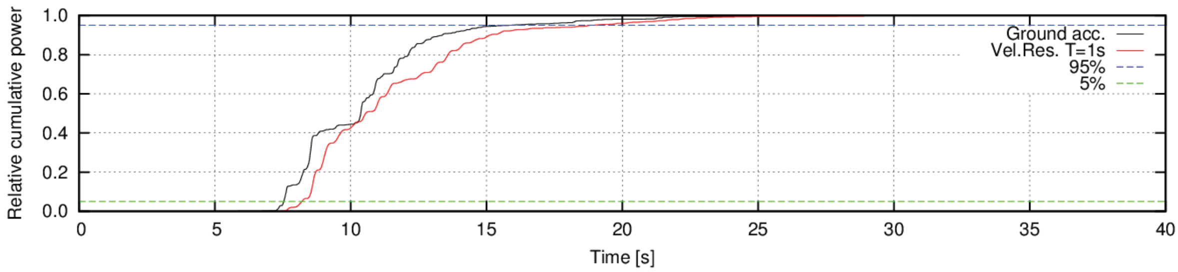


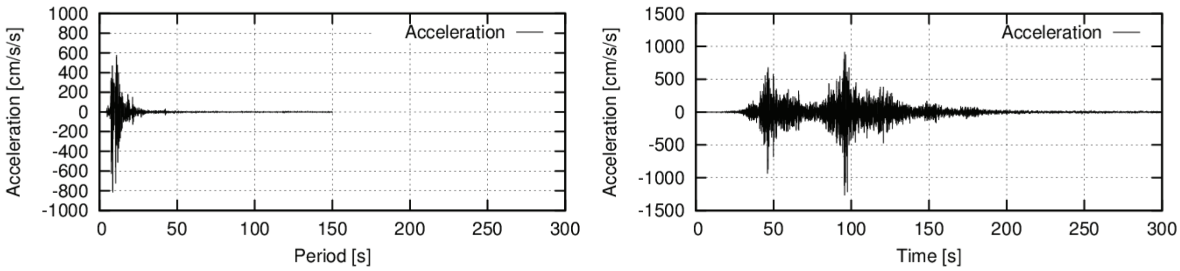
Figure 2 Significant duration $D_{5\text{-}95}$ based on acceleration and velocity response (T=1s, h=5%).

3 NUMERICAL EXAMPLES

3.1 Accelerogram, velocity response spectra and velocity response envelope spectra

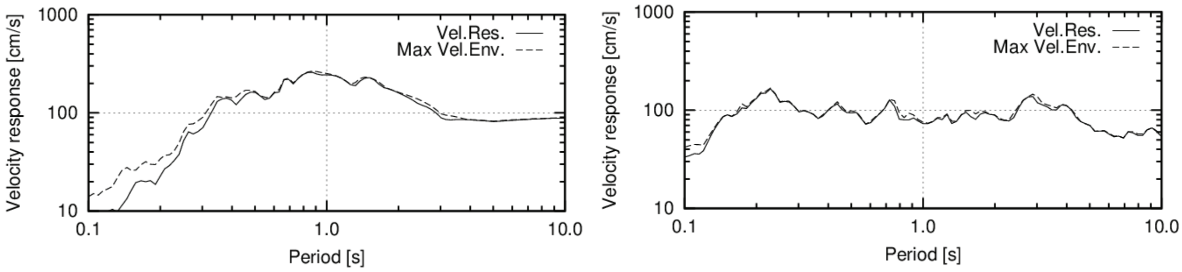
Examples of spectral representation of duration (*h*=5%) are shown, in a comparative manner, using two strong motions representative of a shallow crustal earthquake (the Hyogoken-Nambu Earthquake, Japan, January 17 1995, M_w=6.8) and an off-shore inter-plate earthquake (the 2011 off the Pacific Coast of Tohoku Earthquake, Japan, March 11, 2011, M_w=9.0). Figure 3 compares accelerograms of the NS component observed at JMA (Japan Meteorological Agency) Kobe Marine Observatory (KOB) in the 1995 Hyogoken-Nambu Earthquake and the EW component observed at K-NET (National Research Institute for Earth Science and Disaster Prevention) Tsukidate station (MYG004) in the 2011 off the Pacific Coast of Tohoku Earthquake. As for amplitude level, PGA=818cm/s^2 for KOB and 1268cm/s^2 for MYG004. In the accelerogram in MYG004, twin-peaks characteristics is prominent, which is caused by multiple sub-events of fault rupture. If low amplitude level is focused on, duration of KOB is around 20s, while that of MYG004 is longer than 120s. Obviously, the difference in the sizes of fault rupture, reflected in moment magnitude, accounts for the difference in duration.

In Figure 4, velocity response spectra S_V (*h*=5%) is shown by the solid line. In addition, the maximum value $E_{Vmax}(T)$ of velocity response envelope $E_V(T,t)$ throughout the entire waveform is shown by the dashed line. The latter is slightly larger than the former, depending on natural period. In essence, however, both are considered to be almost equivalent. In KOB, the predominant period is around T=0.8-0.9s and S_V exceeds 100cm/s in wide range of T=0.3-3s. On the other hand, in MYG004, sporadic peaks can be seen at several natural periods such as T=0.2, 0.4, 0.6 and 3s. Figure 5 shows velocity response envelope spectra $E_V(T,t)$ which visualize the dynamics of the SDOF oscillator reflecting the input ground motions characteristics in time-frequency domain.



(a) JMA KOB (1995) (b) K-NET MYG004 (2011)

Figure 3 Accelerograms for numerical examples.

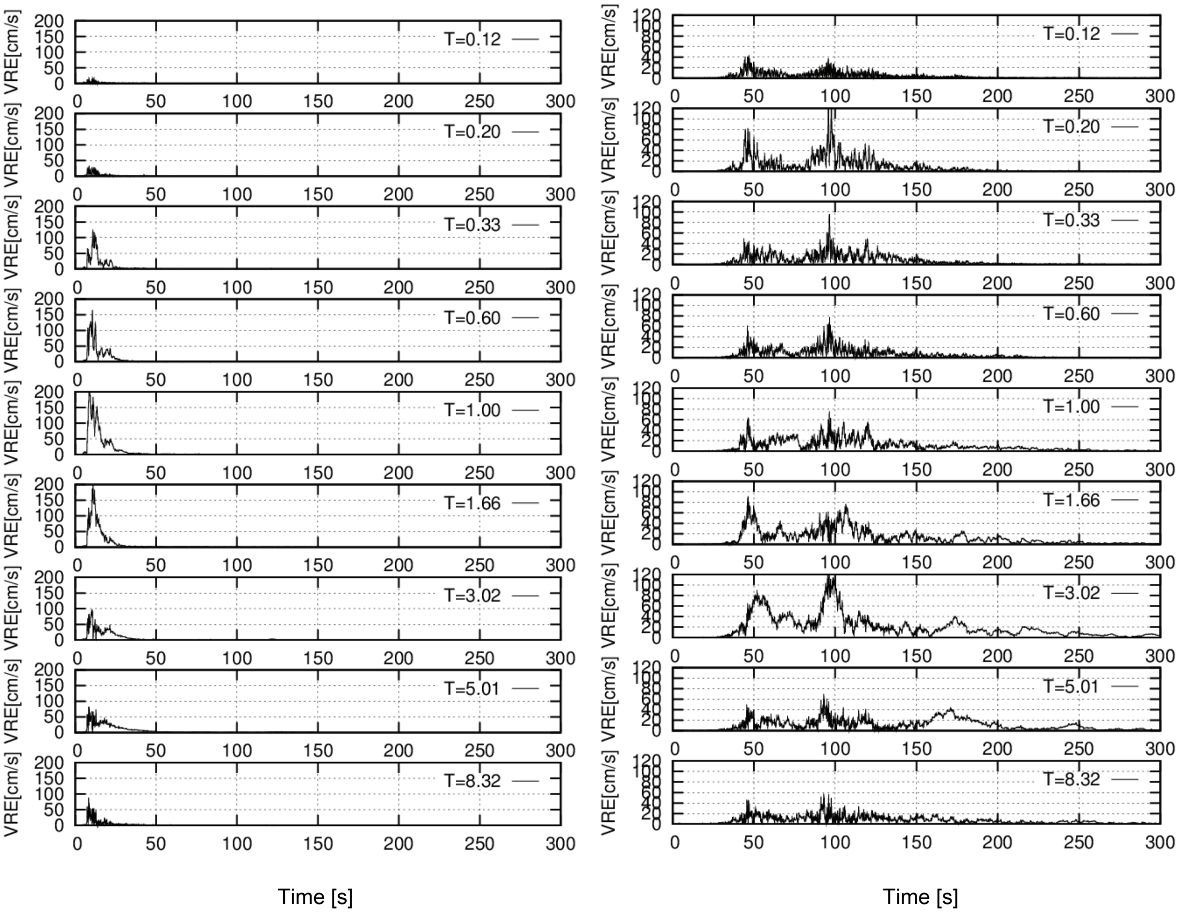


(a) JMA KOB (1995) (b) K-NET MYG004 (2011)

Figure 4 Velocity response spectra S_V and maxima of velocity response envelope E_{max} (*h*=5%).

3.2 Velocity Response Envelope Duration spectra

Figure 6 shows velocity response envelope duration spectra, $S_{Dur\text{-}VRE}$ for various values of E^*=5-200cm/s. Spectra for both *uniform* duration (upper) and *bracketed* duration (lower) are calculated. As mentioned earlier, *bracketed* duration tends to overestimate in particular for high thresholds. Durations longer than 50s for E^*=50cm/s in Figure 6(b) obviously include long intervals with low amplitude between twin peaks in Figure 5(b). Therefore, the discussion below focuses on *uniform* duration.

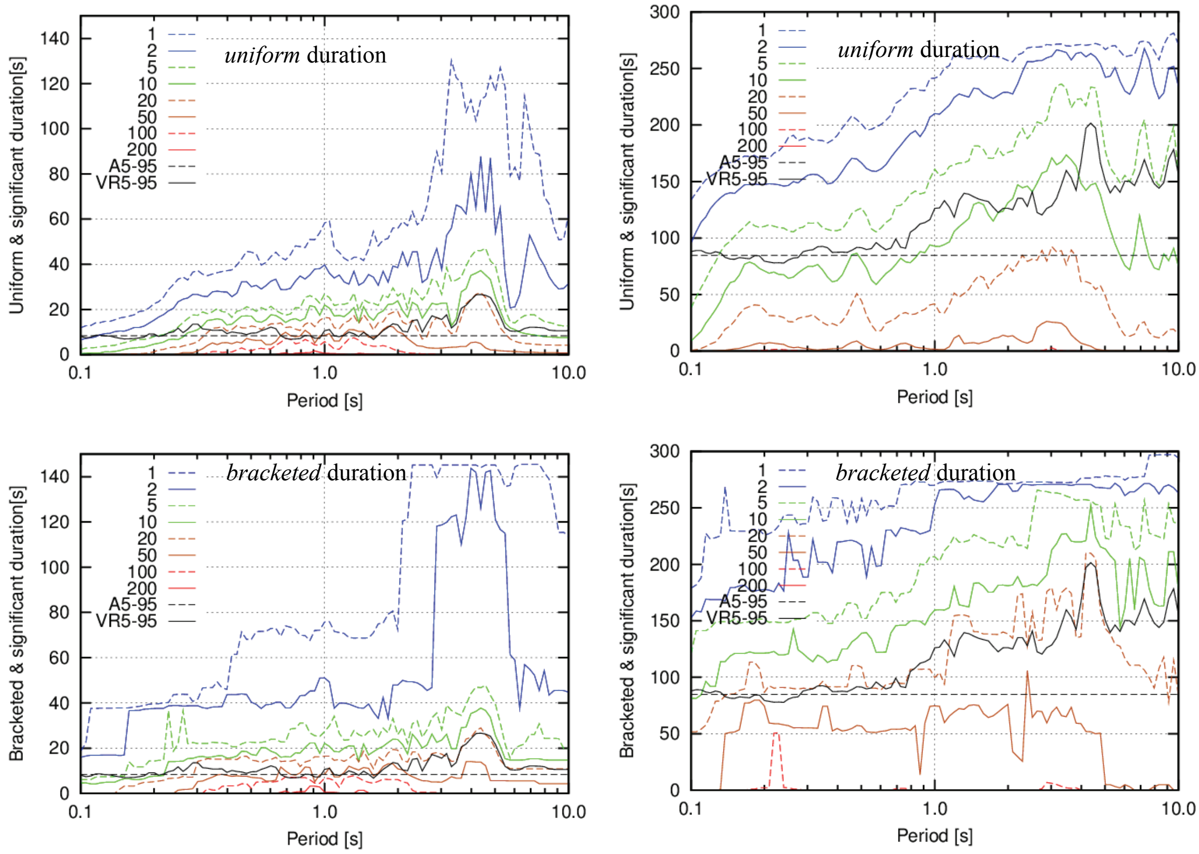


(a) JMA KOB (1995)　　　　(b) K-NET MYG004 (2011)

Figure 5 Velocity response envelope spectra E_V (*h*=5%).

For high threshold level such as E^*=50 or 100cm/s, in general, the shape of duration spectra is similar to that of response spectra exceeding the corresponding threshold level as shown in Figure 4. Durations of long period such as *T*=4-5s become longer with decreasing threshold, which implies the effect of surface waves. Velocity response durations of MYG004 substantially exceeds those of KOB for $E^*\leq$20cm/s as inferred from the accelerograms shown in Figure 3. For E^*=50cm/s, however, partially opposite tendency can be observed. Furthermore, velocity response envelope duration of MYG004 becomes extremely short for $E^*\geq$100cm/s and KOB substantially exceeds MYG004. These quantitative results of *uniform*/*absolute* duration defined by velocity response envelope properly reveal the *frequency-dependent* and threshold-dependent nature of durations which cannot be understood from visual inspection of accelerograms in Figure 3.

The horizontal dashed lines in Figure 6 represent frequency-independent significant duration $D_{5\text{-}95}$ calculated from cumulative power of accelerograms in Figure 3. While the significant duration is evaluated as $D_{5\text{-}95}$=8.4s for KOB (as also shown in Figure 2), that for MYG004 is $D_{5\text{-}95}$=84.7s, almost ten times as long as KOB. Although these values of $D_{5\text{-}95}$ are visually consistent with accelerograms in Figure 3, they cannot be related to either frequency content or amplitude level. On the other hand, black solid lines in Figure 6 represent *frequency-dependent* significant duration $D_{5\text{-}95}$ calculated from cumulative power of velocity response in Figure 5. In KOB, for *T*=1s, $D_{5\text{-}95}$=10s is as long as duration $S_{\text{Dur-VRE}}$ for E^*=50cm/s, which is called "equivalent threshold." For shorter or longer period, however, the equivalent threshold is E^*=2-20cm/s, significantly dependent on natural period *T*. In MYG004, the equivalent threshold is E^*=10cm/s for $T\leq$4s and E^*=5cm/s for $T\geq$5s. The *significant* duration $D_{5\text{-}95}$ based on velocity response is *frequency-dependent*, but their equivalent thresholds are inconsistent throughout the range of natural period. As opposed to this, the velocity response envelope duration spectra $S_{\text{Dur-VRE}}$ is prescribed by a consistent threshold E^*. Therefore, comparing $S_{\text{Dur-VRE}}$ between different natural periods is much more meaningful than comparing $D_{5\text{-}95}$.



(a) JMA KOB (1995) (b) K-NET MYG004 (2011)

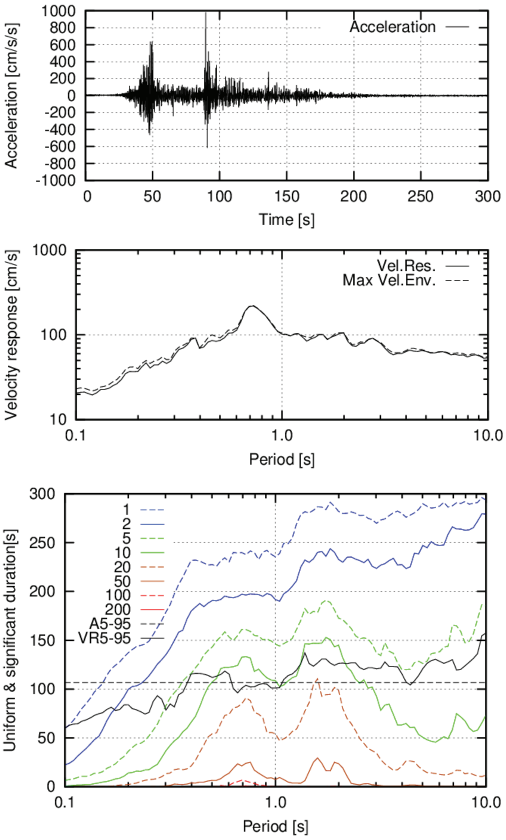
Figure 6 Velocity response envelope duration spectra $S_{\text{Dur-VRE}}$ according to uniform (upper) and bracketed (lower) definition of duration (h=5%).

Additional examples are shown for four ground motion records. Figure 7(a)-(c) are records observed at K-NET Sendai (MYG013), Hokota (IBR013) and Odawara (KNG013) in the off the Pacific coast of Tohoku Earthquake, and Figure 7(d) is a record observed at K-NET Tomakomai (HKD129) in the Tokachi-oki Earthquake, Japan, September 23, 2003 (M_w=8.0). Difference in predominant periods in each response spectrum affects velocity response envelope duration spectra $S_{\text{Dur-VRE}}$. However, $S_{\text{Dur-VRE}}$ do not necessarily reflect the tendency of response spectra S_{V}, especially at T=2s in MYG013 and at T=3s in IBR013. Velocity response envelope duration spectra $S_{\text{Dur-VRE}}$ provide vital information on frequency-dependent nature of duration which has been lost in response spectra S_{V}. Moreover, velocity response-based *significant* duration is negatively correlated with response spectra and also negatively with *uniform* duration for high threshold as clearly seen in Figure 7(b) at $0.2 \le T \le 0.6$s, in Figure 7(c) at $T \ge 2$s and Figure 7(d) at $T \ge 5$s. Concentration of power, leading to destructively strong motions, makes *significant* duration shorter and *uniform* duration longer for high threshold. These observations imply the advantage of *uniform* duration D_{VRE} over *significant* $D_{5\text{-}95}$.

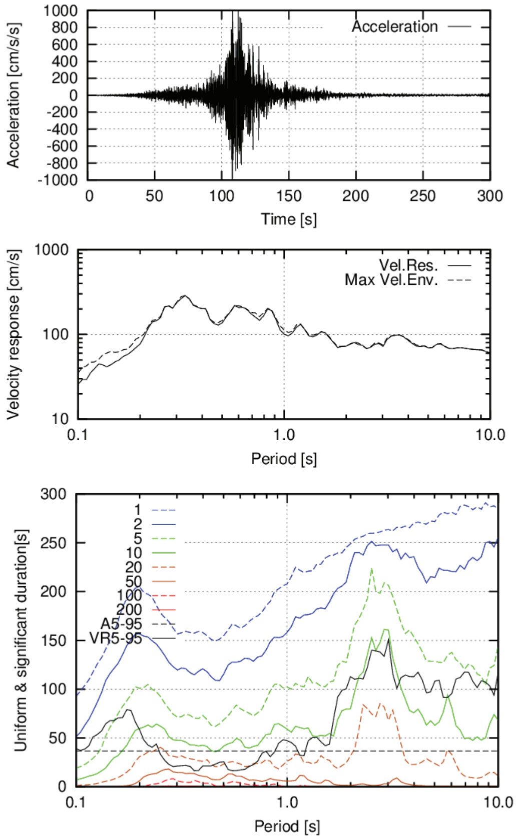
4 CONCLUSIONS

Major conclusions derived from this study can be summarized as follows:

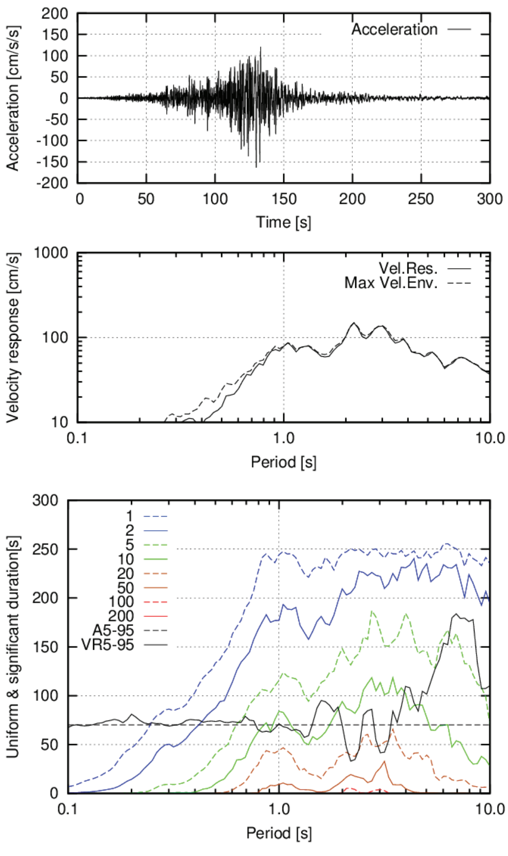
1) On the basis of *uniform*/*absolute*/*frequency-dependent* definition of duration, velocity response envelope duration spectra $S_{\text{Dur-VRE}}$ has been proposed using the envelope waveform of velocity response of SDOF oscillator with damping ratio h=5%. The duration of strong motion for various natural periods and thresholds were visually represented by a spectral form.
2) Numerical examples were shown for the JMA Kobe Marine Observatory (KOB) in the 1995 Hyogoken-Nambu Earthquake and the K-NET Tsukidate station (MYG004) in the 2011 off the Pacific coast of Tohoku Earthquake. Velocity response duration of MYG004 substantially exceeds that of KOB for $E^* \le 20$cm/s, while the latter substantially exceeds the former for $E^* \ge 100$cm/s.



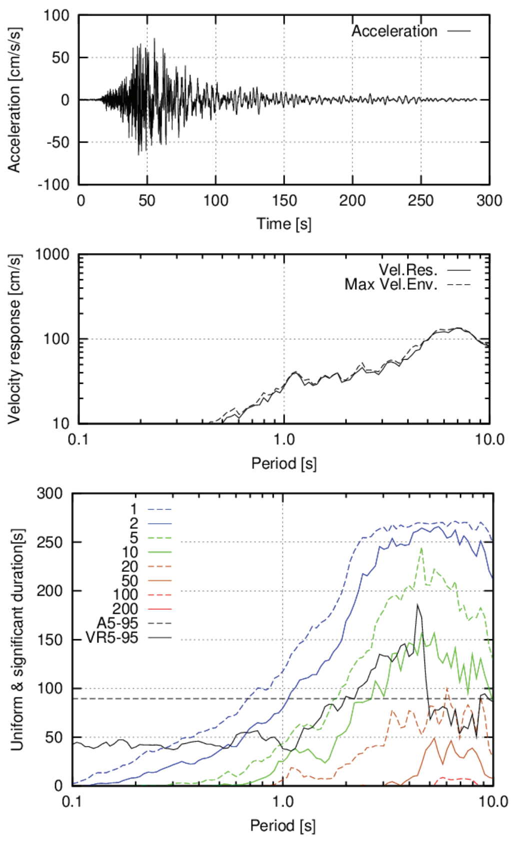
(a) K-NET MYG013 Sendai (2011)



(b) K-NET IBR013 Hokota (2011)



(c) K-NET KNG013 Odawara (2011)



(d) K-NET HKD129 Tomakomai (2003)

Figure 7 Accelerograms (upper), velocity response spectra S_V (middle) and velocity response envelope duration spectra $S_{Dur\text{-}VRE}$ and significant durations (lower) for various ground motions.

3) Acceleration-based *significant* durations $D_{5\text{-}95}$ were calculated as 8.4s and 84.7s for KOB and MYG004, respectively, which are visually consistent with accelerograms, but are not related to either frequency content or amplitude level. Velocity response-based *significant* durations $D_{5\text{-}95}$ were also calculated. Since thresholds are not specified, "equivalent threshold" was defined in comparison with velocity response envelope duration D_{VRE}. Equivalent thresholds of $D_{5\text{-}95}$ were found to be inconsistent throughout the range of natural period as opposed to D_{VRE}.
4) Through additional examples for four ground motion records, it was shown that velocity response envelope duration spectra $S_{\mathrm{Dur\text{-}VRE}}$ provide vital information on *frequency-dependent* nature of duration which has been lost in response spectra S_{V}. Velocity response-based *significant* duration was found to be negatively correlated with *uniform* duration when power is concentrated during short time, since *significant* duration becomes short whereas *uniform* duration becomes longer for high threshold. These observations imply the advantage of *uniform* duration D_{VRE} over *significant* $D_{5\text{-}95}$.

Although uniform thresholds (E^*=1, .., 200 cm/s) have been applied to all natural periods in this study, the proposed method can be extended to frequency-dependent threshold functions defined for various performance limits. One of practical applications is evaluation of duration during which emergency human response is hindered, since human behaviour depends on the predominant frequency as well as the intensity (Nojima, 2015). Another engineering application is evaluation of duration, or equivalently number of cycles, during which a structure is exposed to excessive response over spectral design criteria. These issues will be examined in future developments.

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