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Assessing simplified expressions for the deformation capacity of RC walls

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ABSTRACT: In order to obtain an accurate expression for the plastic hinge length of RC walls, several experimental investigations have been performed in the past. The objectives of this paper are to assess the performance of existing plastic hinge length equations, by comparing analytical estimates of deformation capacity with those observed for a number of RC walls, and quantify the associated dispersion. Experimental data was obtained from the SERIES database. This data was provided in terms of lateral displacement at the top of the wall, which allowed the yield and plastic curvature capacity of the walls to be determined from the experimental data using existing analytical relationships. Using different equations found in the literature, a set of curvatures was obtained and compared with the data from moment-curvature analyses. Results indicated that the expression by Berry *et al*. [2008] provided the best estimate of displacement capacity and by slightly increasing the dependence of plastic hinge length on the wall height, a better fit was obtained. It was noted that a lognormal distribution fit the ratios of experimental-to-analytical curvatures reasonably well, and that the curvature ratios were able to be predicted with a dispersion of 0.30. Finally, the relevance of the findings for performance-based earthquake engineering are discussed.

1 INTRODUCTION

Over the past decades it has been recognized that, in ductile systems, the control of seismic displacement demands is more important than the evaluation of force demands versus strength. This is due to the damage in ductile structures being better correlated with displacements than to forces (Priestley, 2000). In order to achieve the desired ductile behaviour, even during rare intense shaking, the application of capacity design is required. Furthermore, there is an increasing recognition that seismic design and assessment methods should strive to include uncertainty in demand and capacity, as part of a probabilistic assessment process (Cornell *et al*. 2002, Victorsson *et al*. 2014, Welch *et al*. 2014). For the displacement-based seismic design and assessment of reinforced concrete (RC) wall structures, Priestley *et al*. (2007) recommend that the flexural displacement capacity be estimated using the plastic-hinge concept, illustrated in Figure 1, in which the maximum displacement at the top can be computed as the sum of two components: an elastic component and a plastic component.

In line with the above, the total displacement at the top of the wall can be calculated as:

$$\Delta_u = \Delta_y + \Delta_p = \phi_y \cdot H_n^2 \cdot 1/3 + \phi_p \cdot L_p \cdot H_n \quad (1)$$

where Δ_u is the ultimate displacement at the top of the element, Δ_y is the yield portion of the ultimate displacement, Δ_p is the plastic portion, ϕ_y is the yield curvature, H_n is the height of the element, ϕ_p is the plastic curvature and L_p is the plastic hinge length.

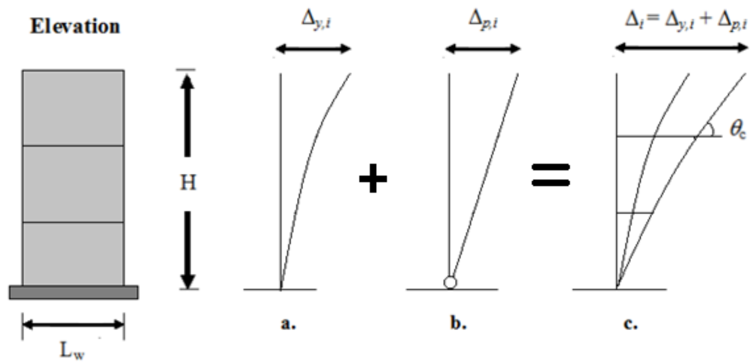

Figure 1. Displacement profile of a cantilever RC wall. a. Elastic displacement profile, b. Plastic displacement profile, c. Total displacement profile (adapted from Sullivan, 2000).

Prior to yield, Equation 1 assumes that the curvatures reduce linearly up the height of the wall, for reasons provided in Priestley *et al*. (2007). On the other hand, for the analysis of the inelastic response, the approach adopts the "lumped plasticity approach" (Park and Paulay 1975, Paulay and Priestley 1992, Priestley *et al.* 2007) which is based on the assumption that plastic strains are concentrated around the section that first yields, and the plasticity is spread over a certain region. In accordance with Park and Paulay (1975), the area of the actual inelastic distribution can be approximated as the area of a rectangle of a height equal to the plastic curvature, and a width that corresponds to the plastic hinge length. In this equivalent length (L_p), plastic curvature is assumed to be constant (refer to Priestley *et al*. 2007).

In order to obtain reliable estimates of the displacement capacity of structures using Equation 1 it is clearly necessary to have an accurate expression for the plastic hinge length (L_p), and that is why several investigations have been performed in the past (Priestley *et al.* 2007, Berry *et al*. 2008, Panagiotakos and Fardis 2001, Bae and Bayrak 2008), for RC walls and other elements. As the experimental data available on wall displacement capacities is increasing, and given the desire to incorporate uncertainty within design and assessment procedures, the objectives of this paper are to (i) assess the performance of existing L_p equations, comparing analytical estimates of displacement capacity with those observed experimentally for a number of RC walls and (ii) quantify the dispersion associated with such analytical expressions so as to permit their use within probabilistic design and assessment procedures in the future.

2 EXPRESSIONS FOR PLASTIC HINGE LENGTH IN THE LITERATURE

Several investigations have been performed in order to obtain the most accurate plastic hinge expression to be used in the estimation of the displacement capacity of structures. From the available equations in the literature, four of the most recent expressions are applied in order to ascertain which equation gives the best match between the experimental and analytical data collected in this work. The acquired experimental data is presented in Section 3.

2.1 Priestley *et al*. (2007)

This equation considers three terms, which were firstly addressed by Priestley and Park (1987):

1. The length of the region over which plasticity spreads due to the increase in capacity at first yield and ultimate moment, combined with the moment gradient.
2. The spread of plasticity caused by the inclined flexural-shear cracks, since the steel strains above the section of the largest moment will be increased (tension shift).
3. The strain penetration length (L_{sp}) of the longitudinal bars into the elastic base (foundation).

The general equation is:

$$L_p = C_1 \cdot H_n + C_2 \cdot L_w + C_3 \cdot d_b \tag{2}$$

where L_w is the length of the wall, d_b is the diameter of the longitudinal reinforcing bars and H_n was defined above.

Paulay and Priestley (1992) presented two different equations for the plastic hinge length of walls, where each of them only included two of the aforementioned terms. Finally, a revised form of the equation was presented in Priestley *et al.* (2007):

$$L_p = \left[0.2\left(f_u / f_y\right) \leq 0.08\right] \cdot H_n + 0.2L_w + 0.022 d_b f_y d_b \tag{3}$$

The term C_2 is taken as 0.2 which is the suggested value for assessment, instead of using the more conservative value of 0.1, recommended for design. The terms f_y and f_u correspond to the yield and ultimate strength of the steel of the reinforcing bars.

2.2 Berry *et al.* (2008)

Berry *et al.* (2008) evaluated the general expression provided by Paulay and Priestley (1992), and calibrated it using the results from RC bridge column monotonic tests. In this case, the term for tension shift was neglected and the strain penetration term was amended to include also the concrete strength.

$$L_p = 0.05 H_n + 0.1 \frac{f_y d_b}{\sqrt{f_c^{'}}} \tag{4}$$

where f'_c is the compression concrete strength, and the other terms were already defined previously.

2.3 Panagiotakos and Fardis (2001)

Again, the term for the tension shift is not included in the equation by Panagiotakos and Fardis (2001). In this case, the general equation was calibrated with RC columns, showing variations in the moment gradient and the strain penetration terms.

$$L_p = 0.12 H_n + 0.014 \alpha d_b f_y \tag{5}$$

where α is the confinement effectiveness, obtained in accordance to the Mander model for confined concrete (Mander *et al.*, 1988).

2.4 Bae and Bayrak (2008)

Bae and Bayrak (2008) undertook tests on four full-scale RC columns considering axial loading and reversed cyclic displacement excursions, in order to investigate the effect of the aspect ratio and the normalized axial load in their behaviour. It was found that the axial load influenced the length of the plastic hinge in all the cases, where at higher load levels, longer lengths were developed, leading to:

$$L_p = \max\left[\left(0.3 P / P_o + 3 A_{st} / A_{con} - 0.1\right) \cdot H_n + 0.25 L_w, 0.25 L_w\right] \tag{6}$$

$$P_o = 0.85 f_c^{'} \cdot \left(A_{con} - A_{st}\right) + f_y \cdot A_{st} \tag{7}$$

where P is the applied axial load, A_{st} is the total longitudinal reinforcing steel area and A_{con} is the concrete cross-section gross area.

3 SELECTED TEST SPECIMENS AND EXPERIMENTAL DISPLACEMENTS

Using the SERIES database and based on the available information found during the review of the corresponding references, 13 specimens were selected and are listed in Table 1. Those specimens were chosen considering: (i) aspect ratio (H_n/l_w) equal to or higher than 2.0 (intermediate and slender walls), (ii) flexural failure mode and (iii) available results: displacements for the different limit states and shear displacements.

Table 1. Geometric properties of selected specimens.

Specimen	Name	Reference source	Axial load ratio	Aspect ratio	H_n (mm)	ρ_h at BE* (%)	ρ_l** at BE*** (%)	ρ_l** at web (%)
1	R1	Oesterle *et al*. 1976	0.00	2.40	4572	1.47	0.31	0.25
2	R2	Oesterle *et al*. 1976	0.00	2.40	4572	4.00	0.31	0.25
3	B1	Oesterle *et al*. 1976	0.00	2.40	4572	1.11	0.31	0.29
4	B3	Oesterle *et al*. 1976	0.00	2.40	4572	1.11	0.31	0.29
5	WSH1	Dazio *et al*. 1998	0.05	2.28	4560	1.32	0.25	0.30
6	WSH2	Dazio *et al*. 1998	0.06	2.28	4560	1.32	0.25	0.30
7	WSH3	Dazio *et al*. 1998	0.06	2.28	4560	1.54	0.25	0.54
8	WSH4	Dazio *et al*. 1998	0.06	2.28	4560	1.54	0.25	0.54
9	WSH5	Dazio *et al*. 1998	0.13	2.28	4560	0.67	0.25	0.27
10	WSH6	Dazio *et al*. 1998	0.11	2.26	4520	1.54	0.25	0.54
11	RW1	Thomsen & Wallace 1995	0.10	3.00	3658	0.46	0.33	0.33
12	RW2	Thomsen & Wallace 1995	0.07	3.00	3658	0.46	0.33	0.33
13	RW-A20-P10-S63	Tran & Wallace 2012	0.10	2.00	2440	7.11	0.61	0.61

*ρ_h: transverse reinforcing ratio; **ρ_l: longitudinal reinforcing ratio
***BE: confined zone of the RC wall (boundary element).

In order to consider only displacements due to flexure, an indirect method is used to obtain the shear displacement measured during each test and subtract it from the reported values of yield and ultimate displacement. Table 3 shows the ratio of shear displacements (Δ_s) to flexural displacements (Δ_f) found in the literature (Beyer *et al.* 2008) for the different tests specimens.

Subsequently, the shear displacement for each test was obtained as:

$$\Delta_s = \frac{\Delta_s}{\Delta_f} \cdot \Delta_f = \frac{\Delta_s}{\Delta_f} \cdot \frac{\Delta_u}{\left(1 + \frac{\Delta_s}{\Delta_f}\right)} \tag{8}$$

This procedure can also be followed to obtain the flexural and shear portions of the yield displacements, by simply changing Δ_u by Δ_y in the equations. The obtained results from the different experimental tests are summarized in Table 2.

Table 2. Experimental displacement results.

Specimen	$\Delta_{y,total}$ (mm)	$\Delta_{u,total}$ (mm)	Δ_s / Δ_f	Δ_y (mm)		Δ_u (mm)	
				Δ_f	Δ_s	Δ_f	Δ_s
1	13.5	103	0.15	11.7	1.76	89.7	13.5
2	21.6	133	0.27	17.0	4.59	105	28.4
3	17.8	132	0.21	14.7	3.09	109	23.0
4	17.8	180	0.33	13.4	4.42	135	44.6
5	11.0	47.5	0.12	9.82	1.18	42.4	5.09
6	10.5	63.0	0.14	9.21	1.29	55.3	7.74
7	16.2	92.4	0.16	14.0	2.23	79.7	12.7
8	15.5	61.6	0.13	13.7	1.78	54.5	7.09
9	9.30	62.0	0.10	8.45	0.85	56.4	5.64
10	12.7	93.7	0.14	11.1	1.56	82.2	11.5
11	12.6	92.0	0.09	11.6	1.04	84.6	7.31
12	12.5	85.5	0.09	11.5	1.03	78.2	7.31
13	16.0	73.0	0.43	11.2	4.81	51.1	21.9

4 COMPARISON BETWEEN EXPERIMENTAL AND ANALYTICAL DATA

To evaluate the performance of the different plastic hinge length expressions described earlier, apparent experimental curvature capacities will be derived from the test results using the various plastic hinge length equations and then compared with those obtained from moment-curvature analyses. For this purpose, moment-curvature analyses of the wall sections were performed using the program Cumbia (Montejo *et al*. 2007), with strain limits based on recommendations made by Montejo *et al*. (2007), for concrete in compression, and Priestley *et al*. (2007) for steel in tension at the no-collapse limit state. The limits were calculated using reported material properties (where available) from the testing. Table 3 shows the yield and ultimate curvatures obtained.

As the experimental testing only provided data on total displacement capacities (with flexural components listed in Table 2), apparent curvature capacities were computed by rearranging Equation 1 as shown and trialling the different plastic hinge length equations from Section 2:

$$\phi_p = \frac{\Delta_u - \Delta_y}{L_p \cdot H_n} \tag{9}$$

$$\phi_y = \frac{3\Delta_y}{H_n^2} \tag{10}$$

In line with the above, ratios between apparent experimental to analytical ultimate curvatures are obtained and are presented in Figure 2(a). It can be seen that the different plastic hinge length expressions show similar variations with specimen number, and generally tend to underestimate the curvature capacity. One also notes that curvature estimates obtained using the Berry *et al*. (2008) approach appear to fit the experimental data best.

Table 3. Analytical curvature results.

Specimen	ϕ_y (1/m)	ϕ_u (1/m)
1	0.002	0.049
2	0.002	0.046
3	0.002	0.047
4	0.002	0.039
5	0.002	0.023
6	0.003	0.039
7	0.003	0.039
8	0.002	0.015
9	0.003	0.041
10	0.003	0.039
11	0.003	0.083
12	0.003	0.075
13	0.004	0.080

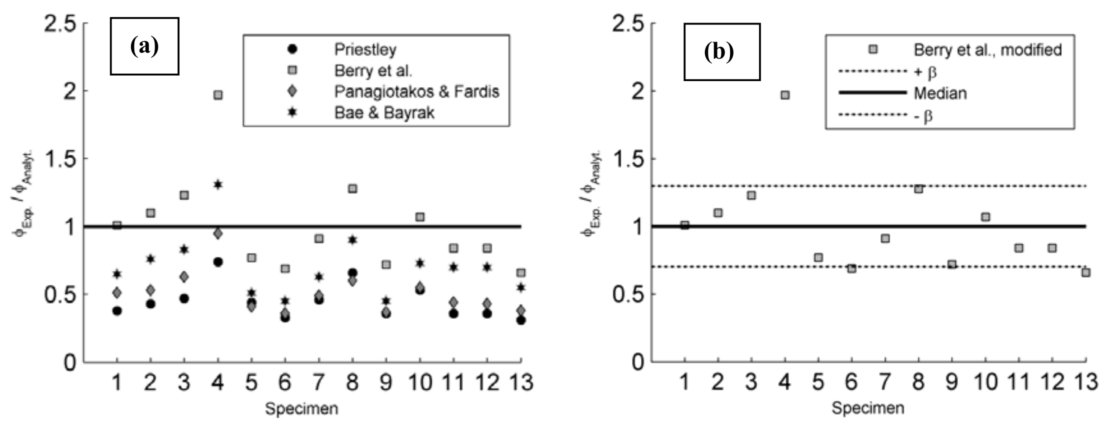

Figure 2. (a) Comparison between apparent ultimate curvatures obtained from experimental displacements using different plastic hinge length equations and curvatures obtained from moment-curvature analyses; (b) apparent experimental to analytical ultimate curvature ratios obtained using a modified version (Eq.11) of the Berry *et al*. (2008) equation.

Given these results, it was decided to refine the equation proposed by Berry *et al.* (2008), in order achieve a better fit of the data. As such, the factor related to the height of the wall (first term in Eq. 4) was adjusted so that the ratio between the experimental and the analytical ultimate curvatures had a median value of 1.0. The modified equation is given as Eq. 11 and new results are seen in Figure 2(b).

$$L_p = 0.058 H_n + 0.1 \frac{f_y d_b}{\sqrt{f_c^{'}}} \tag{11}$$

A goodness-of-fit test is performed for the results from Figure 2(b) and the outcome of this is shown in Figure 3, where it can be seen that the data fits a lognormal distribution. Statistical analysis of the data also permitted the dispersion to be computed as 0.299.

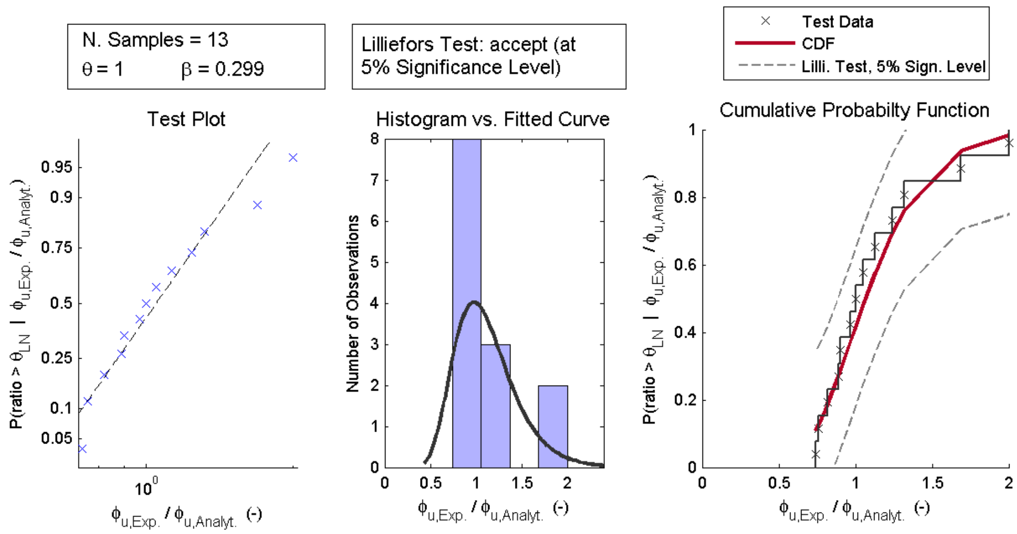

Figure 3. Lognormal PDF testing for ultimate experimental curvature ($\phi_{u,Exp.}$) to ultimate analytical curvature ($\phi_{u,Analyt.}$) ratios.

5 DISCUSSION

Considering further the results presented in Figure 2, it would appear that the ultimate deformation capacity of RC walls can be estimated with reasonable accuracy using simplified plastic hinge length expressions, such as that proposed by Berry *et al*. (2008) which depends only on the wall height, material strengths and longitudinal reinforcing bar diameter. Interestingly, while Bae and Bayrak (2008) found that axial load ratio and reinforcement contents should be included in the estimate of plastic hinge lengths of RC columns, the same result was not obtained for the RC wall specimens considered here. Admittedly, axial load ratios on RC walls do not tend to be as high as RC columns and this could explain why such a dependency was not observed. It is also recognized that despite the author's best efforts to obtain a large experimental dataset, results from only 13 specimens were identified and so one may expect that further refinements to the Berry *et al*. (2008) expression could be possible in the future as more experimental data becomes available.

Considering the statistical variation of predicted to observed ultimate curvature capacity, obtained using the Berry *et al*. (2008) plastic hinge length expression, it was noted that a lognormal distribution fits the data well, and by increasing the dependence on wall height, a median ratio of 1.0 could be obtained with dispersion of 0.299. This value of dispersion is quite high; however, given the simplicity of the prediction method, which could be adopted for either design or assessment, the observed value of dispersion is deemed acceptable. This new information on dispersion is expected to prove valuable for simplified performance-based earthquake engineering assessment procedures, such as those proposed by Welch *et al*. (2014).

6 CONCLUSIONS

This paper assessed the performance of existing equations for calculating plastic hinge lengths, by comparing apparent experimental curvature values with analytical curvatures for RC walls. The experimental curvatures were calculated based on top displacement data obtained from different experimental tests on RC walls, and reported in the SERIES database. For the computation of these curvatures, four different plastic hinge equations from the literature were used. On the other hand, the analytical curvature values were obtained from a moment-curvature analysis of the specimens (considering the geometric and mechanical properties reported). Finally, the ratios between experimental and analytical results were obtained, and it was observed that the equation from Berry *et al*. (2008) showed the best correlation between values. Then, considering the statistical variation of the ratios obtained using Berry *et al*. (2008) plastic hinge length expression, it was noted that a lognormal distribution fitted the data well, and that by increasing the dependence on wall height, a median ratio of 1.0 could be obtained with dispersion of 0.299. It is interesting to notice that this equation depends only on the wall height, material strengths and longitudinal reinforcing bar diameter. Hence, it can be concluded in this case study that the ultimate displacement capacity of RC walls can be estimated with reasonable accuracy using simplified plastic hinge length expressions.

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