Proceedings of the Tenth Pacific Conference on Earthquake Engineering
Building an Earthquake-Resilient Pacific
6-8 November 2015, Sydney, Australia

Seismic damage model based on fractal dimension of cracking

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ABSTRACT: cracking pattern on wall surface is associated to the experimented seismic demands. This paper proposes a damage index for estimating the damage level and the residual performance of thin RC walls subjected to seismic demands. The proposed damage index is based on the fractal dimension of the cracking observed during tests of 39 low-rise RC wall specimens constructed with typical variables of this type of housing, such as thin walls, low compressive strength of concrete, low axial loads, low reinforcement ratios, and web shear reinforcement made of deformed bars and welded-wire meshes. Variables of the experimental program were the type of concrete, aspect ratio of walls, web steel ratio and type of web shear reinforcement, and the testing method. The damage index removes the subjectivity and the variability associated with damage assessment based on visual inspection.

1 INTRODUCTION

Crack width is one of the main indicators of damage severity experimented by reinforced concrete, RC, structural components during an earthquake. Moreover, the cracking pattern on wall surface is associated to the experimented seismic demands, that is, to the variation of shear stresses in relation to high of the wall. Therefore, several studies has focused efforts to estimate the seismic performance level in terms of damage. For instance, Carrillo and Alcocer (2012a) have applied a damage index based on the relation between the damaged area (area of cracks) and the area of the façade of the RC wall. Carrillo (2015a) have proposed a damage index based on the stiffness degradation of walls. Such index depends on the story-drift ratio and the number of cycles experimented by the wall during a particular seismic event. In addition, Adhikari *et al.* (2013) proposes an integral model based on digital images processing for numerical quantification of cracks, pattern variations and neurological connection between cracks.

Fractals are geometric figures than cannot be described in terms of classical geometry. Such geometric figures are characterized by having copies of themselves at different scales randomly, i.e., they have a fine structure. The fractal dimension is a mathematical parameter that measures the geometric complexity level of a pattern further than evaluates the filling property of a particular geometric plane or space. The field of structural engineering has applied the approach of fractal theory for proposing alternative and innovative methodologies of damage evaluation. For instance, Chiaia *et al*. (1998) carried out fracture tests of concrete for assessing the failure modes using the fractal dimension of cracking patterns. Structural health monitoring has also been a field of application of the fractal analysis. Moustafa *et al*. (2013) monitored the propagation path of corrosion of post-tensioned structures by means of the fractal dimension of ultrasonic waves measured in steel tendons. Tzu-Kang *et al*. (2012) proposed a novel bridge health monitoring system and a safety index based on the fractal dimension of the correlation between the scour level and the fundamental period of the bridge superstructure. In a similar way, Hadjileontiadis and Douka (2007) proposed a cracking detection model based on the fractal dimension of the vibration mode shapes and the fundamental period of plate elements. Li *et al*. (1993) used the fractal theory for describing numerically the shape and distribution of aggregates for concrete. Miao *et al*. (2014) studied fractal and multifractal cgaracteristics of 3D asphalt pavement macro-structures in terms of its depth and friction coeeficient. Werner *et al*. (2013) used a fractal-based approach for assessing numerically the parameters related to fractured surface of concrete using laser scanning techniques.

The approach of fractal analysis for studying the cracking of concrete walls was initially used by Farhidzadeh *et al*. (2013). They implemented a theoretical model based on a damage index that depends on the fractal dimension of the cracking propagation recorded in concrete walls. A damage index removes the subjectivity and the variability associated with damage assessment based on visual inspection. Farhidzadeh *et al*. (2013) highlight the simplicity and effectiveness of such approach for evaluating the damage related to an eventual natural hazard. However, the damage index proposed by Farhidzadeh *et al*. (2013) fails in evaluating an actual condition of the structural integrity of the structural element subjected to seismic demands.

Construction of low-rise housing having thin concrete walls has increased in Latin America during last decade (Carrillo and Alcocer, 2012a). The particular characteristics of such walls are low concrete strength, low axial loads, low steel reinforcement ratios, and web shear reinforcement made of deformed bars and welded-wire mesh. A literature review reveals that a guideline for seismic damage evaluation of such houses is currently lacking. Guidelines for damage evaluation such as FEMA-306 (1998) and IAEA (2002) are based on visual inspection and are more directed to medium- or high-rise buildings. It is important to develop suitable parameters of damage levels that allow to evaluate the seismic performance of low-rise housing having thin concrete walls. This paper proposes a damage index for estimating the damage level and the residual performance of thin RC walls subjected to seismic demands. The proposed damage index is based on the fractal dimension of the cracking observed during tests of 39 thin RC wall specimens constructed with typical variables of this type of housing. Variables of the experimental program were the type of concrete, aspect ratio of walls, web steel ratio and type of web shear reinforcement, and the testing method.

2 DAMAGE INDICES BASED ON CRACKING

Several damage indices have been proposed in the literature on seismic damage assessment across a wide range of structures and loading types. There are various ways of categorizing the damage indices; one of the most fundamental distinctions is between local and global indices. Classification of damage indices can also be based on parameters from ground motion, as well as from linear and nonlinear response behavior. In case of nonlinear behavior, parameters are crack width, ductility factor, inter-story drift, plastic energy dissipated or absorbed in the hysteresis loops, parameters based on the accumulation of damage due to cyclic loading as cumulative ductility, and a combination of these parameters.

Cracking is a visible indicator of the severity of the earthquake-induced damage on reinforced concrete elements. Flexural cracks usually derived from diagonal tension stresses. These cracks form diagonally at an angle varying approximately between 35- degree and 70-degree. Major shear cracks cross the entire thickness of the wall and crack width at the two faces of the wall is similar. Crack width defines severity of damage. Maximum crack width can be significantly wider than the mean value of the width of parallel cracks. The mean value of the width of all cracks can be a good indicator of the mean value of strain at steel reinforcement. However, the maximum crack width is prized as a better indicator of the maximum value of strain at steel reinforcement and overall of the damage severity. Concentration of deformation in one or two wide cracks is frequently an indicator of an undesirable behavior mode related to excessive damage. Uniform distribution of deformation in several cracks is often an indicator of desirable seismic performance.

Some studies (see Carrillo and Alcocer, 2012a) have used a damage index based on the maximum width of residual cracks for assessing the damage stage of concrete elements. Such index is defined as the ratio between the area of all residual cracks (length times width) recorded on the façade at the end of an earthquake record. Such index, however, is time consuming to record all the cracks on the façade and fails to be an indicator of the remaining capacity of the structural element. To evaluate the earthquake-induced damage in terms of cracking on concrete elements, Farhidzadeh *et al*. (2013) have implemented a damage index based on the fractal dimension of the pattern and propagation of cracking recorded in concrete walls. Such damage index is computed using Eqn. 1.

$$DI = \frac{FD_i - FD_{\min}}{2 - FD_{\min}} \tag{1}$$

where FD_i is the fractal dimension of the current status of visible cracks (e.g., in the i^{th} inspection); and FD_{min} is the fractal dimension computed during the first inspection (e.g., once the cracks become visible for the first time). The constant "2" in the denominator is the maximum value of the fractal dimension for surface cracks, FD_{max}. DI varies between 0 and 1 and describes the difference between the current status of crack patterns and the baseline FD_{min}. When cracks cover the whole area of the concrete, Farhidzadeh *et al*. (2013) proposed that FD_{max} = 2. However, cracks patterns related to loss of lateral resistance limit state of thin RC wall specimens have demonstrated that area of cracks on façade is lower than 30% of the total area of façade (Carrillo and Alcocer, 2012a). For instance, for walls with web shear reinforcement made of welded-wire mesh and showing a diagonal tension failure, one or some inclined cracks are merely observed. For walls with web shear reinforcement made of deformed bars and showing a diagonal compression failure, significant cracks on façade are recorded. Although such crack patterns can be related to several damage stages, the total area of cracks is certainly not equivalent to the whole area of the concrete as supposed by Farhidzadeh *et al*. (2013). Therefore, the final value of the fractal dimension for surface cracks, FD_u, should be used instead of FD_{max} (FD_{uu}). In addition, previous studies (Carrillo, 2015b) have demonstrated that fractal dimension of final crack patterns depend on the geometrical and reinforcement characteristics of thin RC walls. In this study, initial (FD_{min}) and final (FD_u) values of the fractal dimension of cracks patterns observed in thin RC walls in terms of the particular characteristics of low-rise housing. Such two values of fractal dimension delimit the threshold of damage expected in thin RC walls for low-rise housing.

3 EXPERIMENTAL PROGRAM

Earthquake-induced damage of mid- and high-rise buildings has been widely assessed using tests of prototypes having various wall layouts. Damage will be symbolized by the pattern and distribution of cracks observed in 39 isolated thin RC walls tested under quasi-static cyclic load and under shake table excitations (Carrillo and Alcocer, 2012b). Variables studied are described in Table 1. The typical geometry and reinforcement layout of some of the full-scale wall specimens is shown in Figure 1. Behavior of walls was governed by shear deformation so that they exhibited a relatively fragile failure modes. For evaluating the observed wall behavior, three failure modes were identified: diagonal tension (DT) failure, diagonal compression (DC) failure, and a mixed failure mode (DT-DC).

Table 1. Variables studied

Variable	Description
Height-to-length ratio (h_w/l_w)	$h_w/l_w \approx$ 0.5, 1.0, 2.0 and also, wall with openings (door and window). Full-scale wall thickness (t_w) and clear height (h_w) were 100 mm and 2.4 m, respectively. Then, to achieve the height-to-length ratio, wall length was varied.
Concrete type	Normalweight (N), lightweight (L) and self-consolidating (S). Nominal concrete compressive strength, f_c', was 15 MPa (2175 psi).
Web steel ratio (vertical, ρ_v, and horizontal, ρ_h)	100% of ρ_{min} (0.25%), 50% of ρ_{min} (0.125%), 0% of ρ_{min} = without web shear reinforcement (plain concrete for reference). Minimum web steel ratio (ρ_{min}), is that prescribed by ACI-318 (2011). Wall reinforcement was placed in a single layer at wall mid-thickness, the same amount of horizontal and vertical reinforcement was used.
Type of web reinforcement	Deformed bars (D) and welded-wire mesh made of small-gage wires (W). Nominal yield strength of bars and wire reinforcement, f_y, was 412 MPa (for mild-steel) and 491 MPa (for cold-drawn wires).
Boundary elements	Thickness of boundary elements was equal to thickness of wall web (prismatic cross section). Longitudinal boundary reinforcement was designed and detailed to prevent flexural and anchorage failures prior to achieving the typical shear failure observed in RC walls for low-rise housing.
Axial compressive stress, σ_v	σ_v = 0.25 MPa (36.3 psi) was applied on top of the walls and kept constant during testing. This value corresponded to an average axial stress at service loads of first story walls of a two-story prototype house.
Type of testing	Quasi-static (monotonic and reversed-cyclic) and dynamic (shake table). In quasi-static reversed-cyclic testing, the loading protocol consisted of a series of increasing amplitude cycles. During shake table testing, the models were subjected to a series of base excitations from earthquake records associated with three limit states.

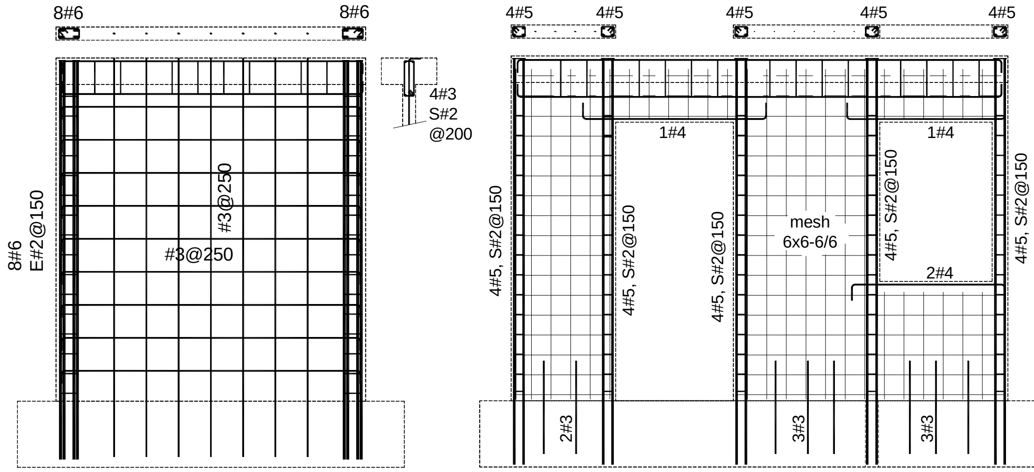

Figure 1. Typical geometry and reinforcement layout of wall specimens: (a) h_w/l_w = 1.0, 100% of ρ_{min} and using deformed bars (wall MCN100C); (b) wall with openings, 50% of ρ_{min} and using welded-wire mesh (wall MVN50mC) (Carrillo and Alcocer, 2012a).

Drawings of cracking patterns of walls were achieved by means of marking cracks and recording damage (notes and pictures) during tests of walls. Cracking drawings of the 39 walls were then converted to digital format. Shear force, V, and story-drift ratio, R, were also recorded for each characteristic loading stage. Story-drift ratio was expressed in percentage and was obtained by dividing the relative displacement measured at mid-thickness of the top slab by the height at which such displacement was measured. Drift ratios were related to four limit states namely diagonal cracking, maximum shear strength, loss of lateral resistance and failure of the wall or end of test (R_{cr}, R_{max}, R_u and R_{uu}) and to three performance levels namely immediate occupancy, life safety, and collapse prevention (R_{IO}, R_{LS} and R_{CP}). Limit states for PBSD of low-rise housing having thin RC walls were proposed by Carrillo and Alcocer (2012b). Diagonal cracking limit state is attained when inclined web cracking is observed. Maximum shear strength limit state corresponds to peak shear strength. Loss of lateral resistance limit state is associated to any of the two following scenarios: when a 20% drop to the peak shear strength is reached or when web shear reinforcement made of welded-wire meshes fractures. Performance levels are those recommended by Vision 2000 (SEAOC, 1995). Based on technical and economic facts, the IO, LS, and CP performance levels for low-rise concrete housing are related to initial inclined web cracking, to extension of web inclined cracks to wall edges without penetration into boundary elements, and to wall peak shear strength, respectively.

Carrillo (2015b) have correlated results of fractal dimension with story-drift ratio of walls. Based on such correlations, values of fractal dimension of cracking related to measured drift ratios at defined limit states (FD_{cr}, FD_{max}, FD_u and FD_{uu}) and performance levels (FD_{IO}, FD_{LS} and FD_{CP}) were proposed. A statistical analysis was then carried out for proposing values of fractal dimension in terms of the variables studied.

4 PROPOSED DAMAGE INDEX

To improve the quantitative analysis of structural damage, an empirical damage index, DI, was developed in this study. Taking into account the direct relationship between the damage level and the cracking pattern of RC walls for low-rise housing, the modified damage index is based on the fractal dimension of the pattern and propagation of cracking recorded in concrete walls. Such damage index is computed using Eqn. 2.

$$DI = \frac{FD_i - FD_{ini}}{FD_u - FD_{ini}} \tag{2}$$

where FD_i and FD_{ini} are defined similarly to the index proposed by Farhidzadeh *et al*. (2013), that is, FD_i is the fractal dimension of the current status of visible cracks (e.g., in the i^{th} inspection), and FD_{ini}

is the fractal dimension computed once the cracks become visible for the first time ("initial stage"). However, FD_u is the value of the fractal dimension for surface cracks related to loss of lateral resistance limit state of thin RC walls ("final stage"). Values of FD_{ini} and FD_u are proposed by Carrillo, 2015b. Similarly to index proposed by Farhidzadeh *et al*. (2013), the proposed damage index describes the difference between the current status of crack patterns and the baseline FD_{min}. As conventional, *DI* varies between 0 and 1. Some damage parameters based on cracking have been proposed; however, their expression in the form of a damage index ($0 \leq DI \leq 1$) requires the definition of the limiting value of cracking after which failure occurs. In this study, a damage index equal to zero indicates no damage or elastic behavior in the wall, and equal to 1.0 when the wall losses the lateral resistance.

As discussed earlier, in the damage index proposed by Farhidzadeh *et al*. (2013), the maximum value of the fractal dimension for surface cracks, FD_{max}, is 2 because the model assumes that cracks cover the whole area (100%) of the concrete surface. It can be inferred that the actual cracking stage is not evaluated when using $FD_{max} = 2$. However, such cracking condition is significantly high when compared with condition of cracks patterns related to loss of lateral resistance limit state of thin RC wall specimens (30% of the concrete surface). Therefore, the final value of the fractal for surface cracks, FD_u, should be used instead of FD_{max}. In addition, based on trends of experimental results, Carrillo (2015b) has proposed FD_{max} (FD_{uu}) values significantly lower than that proposed by Farhidzadeh *et al*. (2013); for instance, FD_{uu} values equal to 1.530 and 1.252 for walls with web shear reinforcement made of deformed bars and welded-wire mesh, respectively. Such noteworthy different values suggest that the damage index should be settled based on a particular value of FD_u in terms of the characteristics of the wall instead of a unique value of FD_{max} (FD_{uu}).

In addition, damage index computed with Eqn. 1 is related with a non-conservative estimation of the structural damage. For instance, for a wall having $h_w/l_w = 2$ and web steel ratio equivalent to 50% of ρ_{min} (0.125%) and using deformed bars, the fractal dimension of surface cracks related to the last cracking record at failure of the wall is $FD_i = 1.220$. Men value of FD_{ini} for walls having web shear reinforcement made of deformed bars is 0.927 (Carrillo, 2015b). Damage index computed using $FD_u = 2$ (as proposed by Farhidzadeh *et al*., 2013) is 27%. However, when using the mean value of FD_u proposed by Carrillo (2015b) for walls with deformed bars ($FD_u = 1.530$), the damage index is 51%. The damage index computed using the model proposed in this study ($DI = 51\%$) is significantly higher than that computed using the model proposed by Farhidzadeh *et al*. ($DI = 27\%$). Thus, it is demonstrated that the model of Farhidzadeh *et al*. can provide a non-conservative estimation of damage stage of walls having the particular characteristics of low-rise housing.

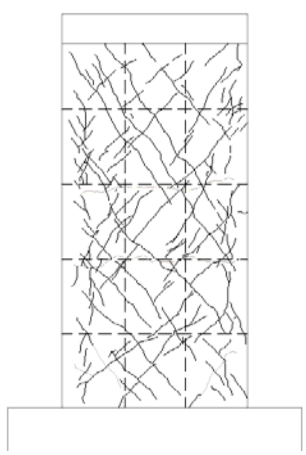

Figure 2. Cracking drawing of wall MEN50C at a drift of 1.4%.

Damage and corresponding cracking propagation was scarce for walls with web shear reinforcement made of welded-wire mesh because of the limited elongation capacity of the wire mesh itself. Therefore, the lowest value of FD_u found by Carrillo (2015b) was associated for walls having such type of web reinforcement. All these findings suggest that the damage index should be settled based on a specific value of FD_u depending on the characteristics of the wall instead of using a unique value of FD_{max} (FD_{uu}).

In Eqn. 2, FD_i is the fractal dimension of the current status of visible cracks related to a particular limit state or performance level. Therefore, the proposed damage index is an effective tool for explaining the seismic behavior of a structure based on the performance of the walls. Table 2 shows the expected damage index at defined limit states (DI_{cr}, DI_{max}, DI_u and DI_{uu}) and performance levels (DI_{IO}, DI_{LS} and DI_{CP}). Damage index were arranged in terms of aspect ratio of walls, type of concrete, web steel ratio, type of web shear reinforcement, and type of testing. For the IO (immediate occupancy) performance level, walls reinforced with deformed bars and welded-wire mesh have attained 20% of the performance capacity (DI_{IO} = 0.20). For the LS (life safety) performance level, walls reinforced with deformed bars and walls with welded-wire mesh have attained 46% (DI_{LS} = 0.46) and 56% (DI_{LS} = 0.56) of the performance capacity, respectively. For the CP (collapse prevention) performance level, walls reinforced with deformed bars and walls with welded-wire mesh have attained 69% (DI_{CP} = 0.69) and 99% (DI_{CP} = 0.99) of the performance capacity, respectively. Although two types of walls have comparable shear strength capacities (Carrillo and Alcocer, 20xx), residual capacity at CP performance level of walls with welded-wire mesh is scarcely 1% (1-0.99) while such capacity of walls with deformed bars is 31% (1-0.69). These significant differences of residual capacity is directly related with lower costs of seismic rehabilitation of walls with deformed bars when compared with wall with welded-wire mesh.

Table 2. Damage index associated to limit states and performance levels in terms of different variables

Variable		DI_{ini}	Limit states				Performance levels		
			DI_{cr}	DI_{max}	DI_u	DI_{uu}	DI_{IO}	DI_{LS}	DI_{CP}
Aspect ratio (h_w/l_w)	h_w/l_w = 2.0		0.13	0.79		1.38	0.09	0.46	0.87
	h_w/l_w = 1.0	0.00	0.26	0.75	1.00	1.29	0.21	0.47	0.71
	h_w/l_w = 0.5		0.21	0.75		1.31	0.19	0.48	0.79
Type of concrete	Normalweight		0.24	0.77		1.34	0.21	0.47	0.78
	Lightweight	0.00	0.24	0.76	1.00	1.22	0.18	0.49	0.73
	Self-consolidating		0.21	0.72		1.35	0.14	0.40	0.63
Web steel ratio	0% ρ_{min}		0.20	0.82		1.19	0.05	0.20	0.25
	50% ρ_{min}	0.00	0.22	0.81	1.00	1.28	0.18	0.49	0.85
	100% ρ_{min}		0.26	0.69		1.34	0.22	0.47	0.69
Type of web reinfor-cement	Deformed bars		0.24	0.69		1.40	0.20	0.46	0.69
	Welded-wire mesh	0.00	0.23	0.91	1.00	1.09	0.20	0.56	0.99
	No reinforcement		0.20	0.82		1.19	0.05	0.20	0.25
Type of testing	Quasi-static monot.		0.18	0.67		1.17	0.11	0.29	0.43
	Quasi-static cyclic	0.00	0.25	0.76	1.00	1.31	0.20	0.47	0.76
	Shake table		0.23	0.83		1.38	0.27	0.63	0.98

5 CONCLUSIONS

Damage quantification based on visual inspection of cracking pattern is a subjective estimate because the damage criterion depends on the expertise of the inspector. Although characteristics of cracks (length, maximum width, residual width) are a key indicator of structural damage, pattern and distribution of cracks of the damaged structural component should also be considered. The damage observed in thin reinforced concrete (RC) walls for low-rise housing subjected to seismic demands is evaluated in this study by means of fractal dimension of the cracking propagation. The study was aimed at establishing a new evaluation method of seismic damage based on the fractal dimension of cracking. Variables of the experimental program were the aspect ratio of the wall, type of concrete, steel ratio and type of web shear reinforcement, and type of testing. To improve the quantitative analysis of structural damage under a particular seismic excitation, an empirical damage index was developed in this study. It was demonstrated that other models proposed in the literature can provide a non-conservative estimation of damage stage of walls having the particular characteristics of low-rise housing.

ACKNOWLEDGEMENT

The author gratefully acknowledges the financial support from the Research Office (Vicerrectoría de Investigaciones) at Nueva Granada Military University (UMNG, Colombia), through project number

IMP-ING-1574. The views expressed in this paper are solely those of the authors and do not necessarily reflects the views of the sponsor.

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