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Modelling non-linear behaviour of lightly reinforced concrete walls

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ABSTRACT:

Reinforced concrete shear walls are prevalent lateral load resisting systems for buildings, and more so in earthquake-prone regions. Elaborate design and detailing provisions are required to ensure ductile behaviour of shear walls and to avoid premature shear failure. A substantial number of existing (particularly old) reinforced concrete buildings in regions of low to moderate seismicity have only been designed to withstand wind and gravitational loads and have not been checked for seismic resistance. To mitigate the risks of earthquake-induced collapse of a building, an effective assessment procedure which provides realistic estimate of the imposed seismic demand on the lateral resisting elements in the building (including non-ductile shear walls) would need to be developed.

Non-linear time history analyses are often required in the assessment of certain critical structures given its ability to mimic the dynamic response of a structure. However, the predicted response is highly reliant on assumptions of the nonlinear behaviour of its members. These assumptions include: backbone curve parameters (effective stiffness, ultimate curvature and plastic hinge length) as well as stiffness degradation parameters on unloading and reloading. In this paper theoretical models that have been developed to model the backbone curve and the stiffness degradation parameters for reinforced concrete walls are evaluated by comparisons with results from experimental testing of nine lightly reinforced wall specimens.

1 INTRODUCTION

Shear walls are a common form of lateral load resisting systems in medium- to high-rise buildings. Ductile detailing of the walls is essential for satisfactory performance in an earthquake, especially in regions of high seismicity. However, a significant number of existing (particularly old) reinforced concrete buildings in regions of low to moderate seismicity have only been designed to withstand wind and gravitational loads and have not been checked for seismic resistance and hence non-ductile detailing is to be expected in such buildings. To mitigate the risks of earthquake-induced collapse of a building, an effective assessment procedure which provides a realistic estimate of the imposed seismic demand on the lateral resisting elements in the building (including non-ductile shear walls) would need to be developed.

Simplified static procedures generally provide conservative estimates of the seismic demand which is desirable for new buildings; however, such conservatism might not be affordable when assessing existing buildings. Alternatively, dynamic non-linear time history analysis is the preferred technique, owing to its ability to simulate the dynamic response behaviour of a structure. This procedure can be used to predict the performance behaviour of a structure in an earthquake, given that the member's non-linearity and the dynamic properties of the structure are properly modelled.

Non-linear behaviour of structural walls is typically modelled through beam elements (e.g. Giberson one-component beam element (Sharpe 1974)) which can be called up for use in the library of most software, e.g. RUAUMOKO and OPENSEES. This type of elements utilises the concentrated plasticity approach wherein plastic rotational springs are allowed to form at either, or both, ends of the member (e.g. walls are generally modelled with one plastic hinge at the base) with elastic internal section properties. A large number of hysteretic rules that can be assigned to the member plastic hinges have been proposed in the literature. The Modified Takeda rule (Otani 1974) which is considered representative of RC flexural members encompasses the bi-linear moment-curvature relationship (backbone curve) as well as the rate of stiffness degradation on unloading and reloading. Furthermore, plastic hinge length should be provided where rotational springs are assumed.

A significant number of models have been proposed in the literature to estimate the non-linear properties of RC structural walls. However, the applicability of the proposed models to the case of slender lightly reinforced walls that are common in regions of low to moderate seismicity is not generally confirmed. The objective of this paper is to review and evaluate some of the proposed models to estimate the bi-linear backbone curve parameters, stiffness degradation factors and plastic hinge length for the case of slender lightly reinforced RC walls. The evaluation process involves comparison with results from previous tests conducted on nine lightly reinforced concrete walls.

2 OVERVIEW OF THE EXPERIMENTAL DATA USED IN THE ANALYSIS

A set of nine wall samples was chosen from the previously tested wall specimens. The samples were selected to be as representative as possible of slender lightly reinforced walls in regions of lower seismicity. These wall specimens featured a shear span ratio of greater than two (to guarantee that the behaviour is dominated by flexure), low longitudinal reinforcement percentage (≤1.12%) and low axial load ratio, as detailed in Table 1.

Table 1. Details of the walls used in the analysis.

Researcher	Test unit	Shear span ratio (H_e/L)	Axial load ratio ($P/A_g f'_c$)	Gross longitudinal steel percentage	Total boundary transverse (confinement) steel percentage
Oesterle et al. (1976)	R1	2.4	0	0.49	0.37
Thomsen and Wallace (2004)	RW2	3	0.07	1.12	0.69
Han, Oh and Lee (2002)	W3	3	0.1	0.62	0.36
Dazio, Wenk and Bachmann (1999)	WSH1	2.3	0.05	0.54	0.5
	WSH2	2.3	0.06	0.54	0.5
	WSH3	2.3	0.06	0.82	0.5
	WSH4	2.3	0.06	0.82	0.25
	WSH5	2.3	0.13	0.39	0.74
	WSH6	2.3	0.11	0.82	1.12

3 BACKBONE CURVE MODELLING UP TO ULTIMATE

The bi-linear moment-curvature relationship is defined by four parameters: yielding and ultimate moment, effective stiffness and ultimate curvature. Further details in relation to these parameters are provided in the next sections.

3.1 Moment-curvature relationship

Fiber analysis was conducted with the aid of spreadsheets to obtain the required moment and curvature values at the yield and ultimate limit states. The steel stress-strain relationship model of Priestley, Calvi and Kowalsky (2007) was chosen for both the tensile and compressive steel. The Mander, Priestley and Park (1988) model (for confined and unconfined concrete) was chosen to prescribe the stress-strain relationship of the concrete under compression. The confined concrete model was used for the concrete surrounded by the boundary steel, whereas the unconfined concrete model was used for the wall cover and the web.

The effects of contributions by the concrete tensile strength are commonly ignored. However, the influence of tension stiffening can be quite significant, especially for lightly reinforced walls. As a result, the effects of tension stiffening have been incorporated in the analysis as per the modified compression field theory approach of Vecchio and Collins (1986).

The simulated moment-curvature was utilized to determine the point of yield and ultimate conditions. The point of yield was defined as the conditions of first yielding that corresponds either to a concrete strain of 0.002 or to the steel yielding strain, whichever occurs first (Priestley & Kowalsky 1998). However, defining the ultimate conditions with respect to the curvature can be misleading, as the moment-curvature curve becomes nearly flat after reaching the peak moment. Priestley and Kowalsky defined the ultimate curvature as the lowest curvature corresponding to a concrete strain of 0.018 or steel strain of 0.06, but this definition is valid only for well-confined concrete. As this definition is limited to the case of confined concrete, and the lightly reinforced walls are likely to be built with no ductile detailing, another more conservative definition similar to the one adopted by Inel and Ozmen (2006) was chosen with slight modifications, as shown by Equation 1.

$$\phi_u = \text{lesser curvature corresponding to} \begin{bmatrix} 0.95 M_{peak}; \\ \varepsilon_c = 0.004 + \dfrac{1.4 \rho_s f_{yh} \, (0.6\, \varepsilon_{uh})}{f'_{cc}}; \\ \varepsilon_s = 0.6\, \varepsilon_{su} \end{bmatrix} \tag{1}$$

where ϕ_u = the ultimate curvature of the wall; M_{peak} = the peak moment as obtained from the moment-curvature; ε_{su} = the ultimate strain of the longitudinal steel; ρ_s = the volumetric ratio of the confining steel; f_{yh} = the yield strength of horizontal reinforcement; ε_{uh} = the ultimate strain of the horizontal steel; and f'_{cc} = the peak confined concrete compressive strength.

3.2 **Effective stiffness**

The effective stiffness estimation of structural walls has been largely addressed in the literature, albeit with variations in terms of the comprehensiveness and complexity. The simplest form is given as a fraction of the gross un-cracked stiffness (I_g) to account for the overall cracked section behaviour. Although this estimation of stiffness is attractive due to its apparent simplicity, it is not reliable enough due to the large variations across the codes and researchers. As an example, the suggested value of 0.2 I_g (Panagiotakos & Fardis 2001) was derived from data of 963 specimens with only 63 samples of structural walls, but this value is almost half the effective stiffness (0.35 I_g) recommended by the ACI code (1995) for beams and walls. Alternatively, several expressions have been proposed in the literature to predict the effective stiffness of structural walls which take into account parameters such as the axial load ratio, steel and concrete grade, as given by Table 2. Furthermore, the effective stiffness of a structural wall can be derived directly from the slope of the moment-curvature relationship at the point of yield as shown by Equation 2.

$$E_c I_e = \frac{M_y}{\phi_y} \tag{2}$$

where E_c = the concrete modulus of elasticity; I_e = the effective moment of inertia of the wall section, and; M_y, ϕ_y = the moment and curvature at yielding, respectively.

The effective stiffness of the nine tested specimens (calculated at the yielding point) was compared to the theoretical stiffness, as calculated by the aforementioned models. The comparisons listed in Table 3 indicate that the effective stiffness values derived directly from the theoretical moment-curvature analysis, from Paulay and Priestley's (1992) model and from the lower-bound model of Adebar, Ibrahim and Bryson (2007) all match the experimental observations well. It was noted, however, that the former approach performs marginally better. The other three models significantly overestimate the effective stiffness for lightly reinforced concrete walls.

Table 2. Theoretical effective stiffness models.

No.	Researcher	Equation
1	Paulay and Priestley (1992)	$\frac{I_e}{I_g}=\frac{100}{f_y}+\frac{P}{f_c^{'}A_g}$
2	Adebar, Ibrahim and Bryson (2007)	$\frac{I_e}{I_g}=0.6+\frac{P}{f_c^{'}A_g}\leq 1 \text{ (Upper Bound)}$
		$\frac{I_e}{I_g}=0.2+0.25\frac{P}{f_c^{'}A_g}\leq 0.7 \text{ (Lower Bound)}$
3	Fenwick and Bull (2000)	$\frac{I_e}{I_g}=0.267+\left(1+4.4\frac{P}{f_c^{'}A_g}\right)\left(0.62+\frac{190}{f_y}\right)\left(0.76+0.005f_c^{'}\right)$
4	Fenwick, Hunt and Bull (2001)	$\frac{I_e}{I_g}=0.31+\frac{P}{f_c^{'}A_g} \text{ (for } f_y=500 \text{ Mpa)}$
		$\frac{I_e}{I_g}=0.44+1.2\frac{P}{f_c^{'}A_g} \text{ (for } f_y=300 \text{ Mpa)}$

where I_g = the gross moment of inertia of the wall section; f_y = the steel yielding strength; P = the applied axial load; A_g = the gross area of the wall section and; f'_c = the concrete compressive strength. Certain limitations exist in the models of both Fenwick and Bull (2000) and Fenwick, Hunt and Bull (2001) in regard to the amount of the longitudinal reinforcement and the applied axial load ratio.

Table 3. Comparison of the effective stiffness of various theoretical models versus experimental results.

Test unit	I_e/I_g						
	Measured	Slope of the moment-curvature	Paulay and Priestley (1992)	Adebar, Ibrahim and Bryson (2007)		Fenwick and Bull (2000)	Fenwick, Hunt and Bull (2001)
				Lower bound	Upper bound		
R1	0.17	0.15	0.20	0.20	0.60	N.A.*	N.A.*
RW2	0.16	0.42	0.34	0.23	0.70	0.39	0.46
W3	0.31	0.31	0.35	0.23	0.70	0.40	0.48
WSH1	0.19	0.23	0.23	0.21	0.65	0.31	0.37
WSH2	0.20	0.23	0.23	0.21	0.66	0.30	0.37
WSH3	0.18	0.26	0.22	0.21	0.66	0.30	0.37
WSH4	0.19	0.26	0.23	0.21	0.66	0.31	0.37
WSH5	0.26	0.26	0.30	0.23	0.73	0.37	N.A.*
WSH6	0.29	0.30	0.28	0.20	0.60	0.25	0.32

* The wall failed to comply with the model limitations with respect to the longitudinal reinforcement and the axial load ratio.

4 PLASTIC HINGE LENGTH

Plastic hinge length is a very critical assumption in the lumped plasticity approach, where plasticity is assumed to concentrate at specific locations of the member (e.g. base of the wall for medium-rise buildings). The inelastic displacement demand of the wall is determined from rotation in the anticipated plastic hinge region. However, plastic rotation is calculated based on the estimated hinge curvature and the assumed hinge length.

Although the modelling of plastic hinge length has been widely addressed in the literature, the majority of the studies have been conducted on beams and columns and later extended to apply to the case of structural walls. According to Bohl and Adebar (2011), the assumption of 0.5 to 1 of the wall section length to approximate the plastic hinge length has become a very common practice which was originally derived from data of beam tests. However, more realistic models for the wall plastic hinge length have been proposed in the literature. Such models are based on dividing the plastic hinge length into one or more components (e.g. flexure, strain penetration and/or shear). For the case of slender lightly reinforced structural walls, the dominant deformation components are the flexure and the strain penetration components.

The applicability of four plastic hinge length models (as listed in Table 4) which are believed to suit the case of slender lightly reinforced walls has been evaluated. The plastic hinge length was first calculated from the four models and then used to predict the ultimate displacement of the nine tested specimens, according to Equation 3. The predicted displacements were then compared with the measured displacements, as shown in Table 5. It can be observed from Table 5 that the models of both Paulay and Priestley (1992) and of Bohl and Adebar (2011) are in good agreement with the experimental results for the majority of the samples. However, the other two models may largely overestimate the plastic hinge length for slender lightly reinforced structural walls.

$$\Delta_u = \frac{1}{3}\phi_y H_e^2 + L_p \phi_p \left[H_e - \frac{L_p}{2} \right] \tag{3}$$

Table 4. Theoretical plastic hinge models.

No.	Researcher	Equation
1	Paulay and Priestley (1992)	$L_P = 0.08 H_e + 0.022\, d_b.f_y \geq 0.044\, d_b.f_y$
2	Bohl and Adebar (2011)	$L_P = (0.2L_w + 0.05H_e)\left(1 - \frac{1.5P}{f_c' A_g}\right) \leq 0.8L_w$
3	Priestley, Calvi and Kowalsky (2007)	$L_P = \min\left(0.2\,(\frac{f_u}{f_y} - 1)\,, 0.08\right) H_e + 0.2\,L_w + 0.022\,d_b.f_y$
4	Kazaz (2013)	$L_P = 0.27L_w \left(1 - \frac{P}{f_c' A_g}\right)\left(1 - \frac{f_y\,\rho_{sh}}{f_c'}\right)\left(\frac{H_e}{L_w}\right)^{0.45}$

where Δ_u = the ultimate displacement of the wall; L_p = the plastic hinge length; ϕ_p = the plastic curvature of the wall; H_e = the effective height of the wall; d_b = the boundary reinforcing bar diameter; L_w = the wall length; f_u = the steel ultimate strength and; ρ_{sh} = the ratio of web horizontal reinforcement to vertical cross section.

Table 5. Comparison of the ultimate displacement of various models versus experimental results.

Test unit	Ultimate displacement (mm)				
	Measured	Predictions from different plastic hinge models			
		Paulay and Priestley (1992)	Bohl and Adebar (2011)	Priestley, Calvi and Kowalsky (2007)	Kazaz (2013)
R1	76.0	78.0	95.7	126.0	111.9
RW2	85.0	59.8	57.8	87.7	70.0
W3	40.0	45.3	43.7	62.6	51.7
WSH1	30.0	42.4	47.7	50.8	55.2
WSH2	52.0	69.4	77.9	99.2	92.3
WSH3	93.0	71.9	77.1	94.4	91.2
WSH4	60.0	39.4	42.0	48.1	48.2
WSH5	50.0	41.4	43.5	53.9	51.3
WSH6	80.0	57.9	58.8	73.1	70.5

5 STIFFNESS DEGRADATION

The hysteretic behaviour of flexural reinforced concrete members is commonly represented by the Modified Takeda model. Stiffness degradation due to unloading and reloading is implemented through two factors (alpha and beta) in this model (Fig.1). Alpha controls the unloading stiffness while beta controls the reloading stiffness: these can vary, depending on the member type, from 0 to 0.5 and from 0 to 0.6 for the former and the latter, respectively.

The hysteretic behaviour of the nine wall specimens was calibrated through HYSTERESIS software (Carr 2007) in order to obtain the best combination of alpha and beta to match the actual behaviour as obtained from the tests. According to the calibrated specimens (Fig.2), a very good match was achieved for the majority of the samples by assigning a value of 0.5 and 0 for the unloading and the reloading factors, respectively. Furthermore, the authors conducted a parametric study on a range of slender walls to investigate those factors, which confirmed the findings. As a result, stiffness degradation of slender lightly reinforced walls can be reliably modelled by assuming a value of 0.5 for alpha and zero for beta.

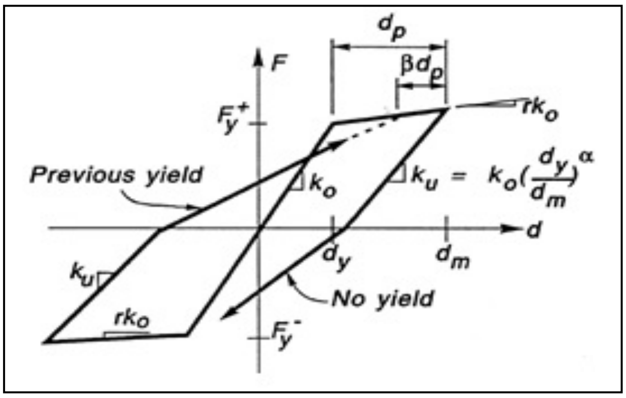
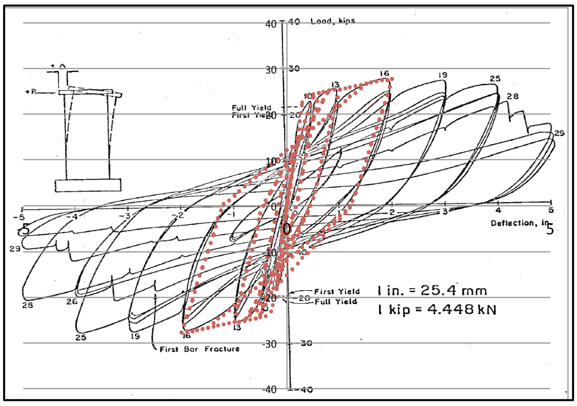
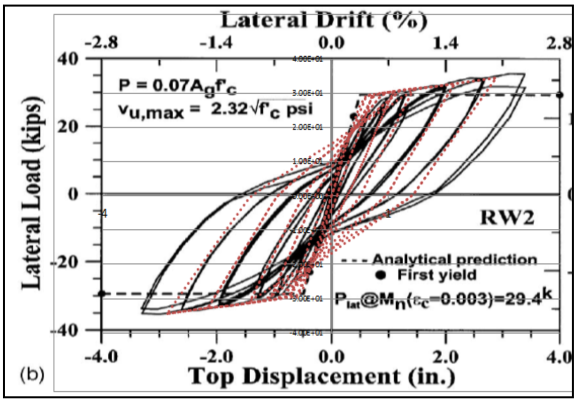


Figure 1 Modified Takeda model (cited from Carr 2008).

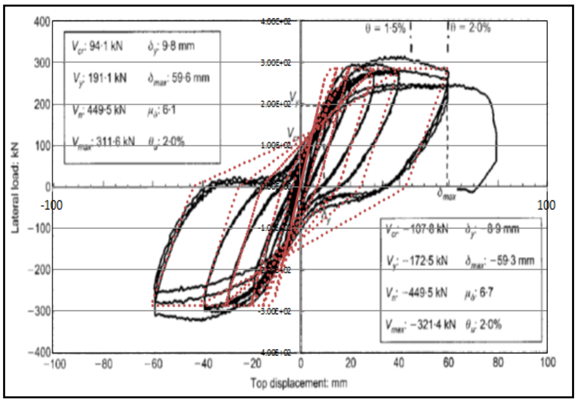
Calibrated Experimental



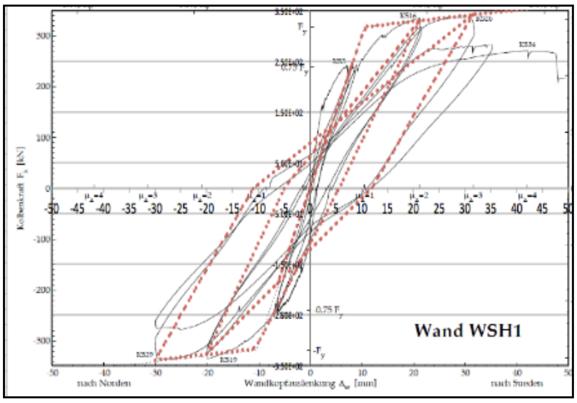
A) R1 (α = 0.4, β = 0.3)



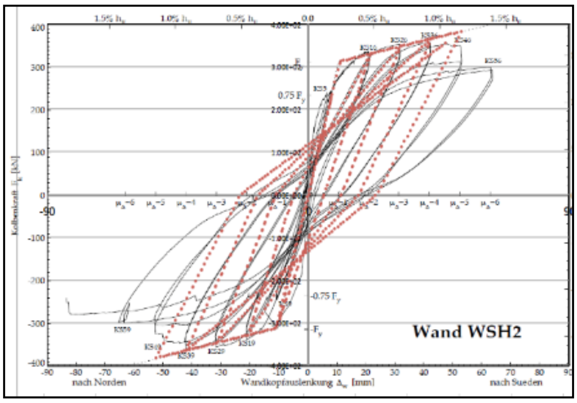
B) RW2 (α = 0.5, β = 0.6)



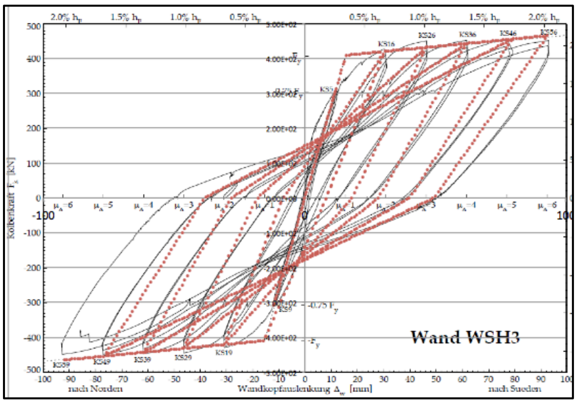
C) W3 (α = 0.25, β = 0.0)



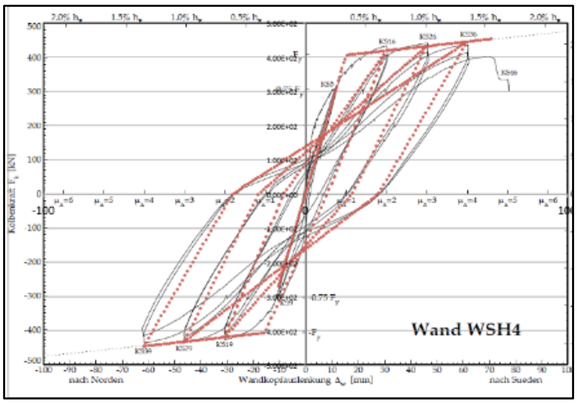
D) WSH1 (α = 0.5, β = 0.0)



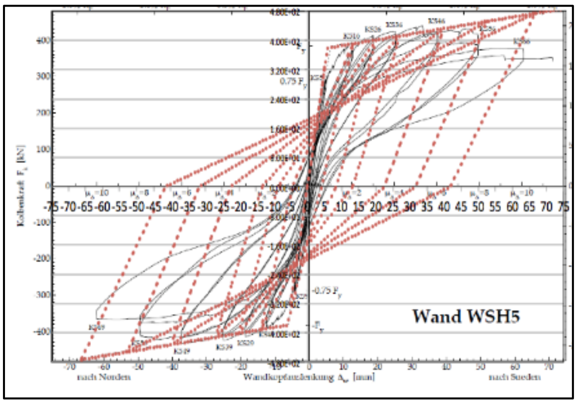
E) WSH2 (α = 0.5, β = 0.0)



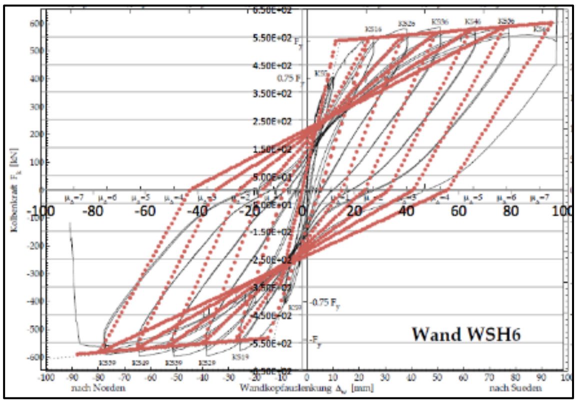
F) WSH3 (α = 0.5, β = 0.0)



G) WSH4 (α = 0.5, β = 0.0)



H) WSH5 (α = 0.5, β = 0.0)



I) WSH6 (α = 0.5, β = 0.0)

Figure 2 Stiffness degradation factors for nine calibrated samples.

6 CONCLUSION

- This paper reviewed some of the models proposed in the literature of reinforced concrete walls to estimate their non-linear properties: bi-linear backbone curve parameters, stiffness degradation factors and plastic hinge length. The suggested models were then evaluated for the case of slender lightly reinforced walls in regions of low seismicity through comparison with nine tested wall specimens.
- The initial stiffness (effective stiffness) as calculated from the slope moment-curvature at the point of yield was very close to the stiffness proposed by Paulay and Priestley (1992) and, both were in close agreement with the test results of the nine specimens which were of rectangular cross-sections.
- The plastic hinge model proposed by Paulay and Priestley (1992) as well as the model proposed by Bohl and Adebar (2011) also closely matched with the test results, while other models might have largely overestimated the plastic hinge length for the case of slender lightly reinforced concrete walls.
- It was observed from calibrating the stiffness degradation parameters of the Modified Takeda model with the nine specimens that a value of 0.5 for the unloading factor (alpha) and 0 for the reloading factor (beta) were appropriate for the case of slender lightly reinforced concrete walls.

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