

# Non-linear Analysis Method of Ground Response Using Equivalent Single-degree-of-freedom Model

K. Sakai & Y. Murono

*Center for Railway Earthquake Engineering, Railway Technical Research Institute, Tokyo, Japan.*

**ABSTRACT:** This paper proposes a simple method to evaluate surface ground waveforms using both the natural period of the ground and waveforms in the engineering bedrock. In this research, the "Equivalent Single Degree of Freedom method" is proposed and is based on the results of static push-over analyses of many grounds with various properties. A comparison was made of waveforms of multi-layered ground and that obtained from the proposed method using dynamic analysis. It was then confirmed that these waves are almost identical, and that the proposed method is applicable to the evaluation of the surface motion. Even if only a few data with regard to soil properties property is available, particularly in case of an existing railway facility, the proposed method enables economical dense calculation of trackside surface motion. These sets of waveforms will be utilized to identify locations where devastating damage is will be expected in the case of a strong earthquake.

## 1 INTRODUCTION

Social infrastructure facilities consist of not only those constructed according to earthquake resistance standards enacted following the Southern Hyogo Prefecture Earthquake, but also a large number of older structures. The earthquake resistance and the earthquake related risks of each structure differ greatly from one structure to another. An efficient method to reduce earthquake related risk of wide spread infrastructures is to locate weak structures where appropriate countermeasures are needed. Achieving this would require the comprehensive evaluation of the earthquake resistance of a very large number of structures, and the effective evaluation of earthquake motion of each of these points, followed by precise individual calculation of earthquake related risk for each structure.

The multi-dimensional or multiple-degree-of-freedom response analysis methods might used in order to evaluate the behaviour of the structure and the amount of damage to the structure under given earthquakes. However, it is not realistic to analyze such a large group of structures at once. For this reason, it is necessary to carry out a response evaluation using more simple method. For example, it has been confirmed that the accurate simulation is carried out for a relatively simple structure such as a viaduct, if parameters are given such as natural period, yield strength, and so on. In addition, these parameters can be estimated by only a few information available such as height of structure or ground condition (Murono and Sato 2007). If detailed data concerning the structure is available, it will be possible to calculate the natural period or the yield strength with higher accuracy by implementing simple static non-linear analysis (push-over analysis (Priestley *et al.* 1996)). In addition, a method for easily calculating the natural period and the yield strength from only a small number of parameters, such as the height of the viaduct and the ground conditions, has also been proposed. Likewise, simple modelling and response evaluation are being studied for embankments (Furukawa *et al.* 2013). Consequently, by using these methods, it is possible to evaluate the behaviour and the potential degree of damage of the large number of upper structures by simple and minimal calculations.

On the other hand, when evaluating the earthquake motion for these structures, it is important to consider ground behaviour. The most commonly used method for analyzing the ground response is sequential non-linear analysis where the ground is regarded as one-dimensional thin layers (Konder 1963) or equivalent linearization analysis (Schnabel *et al.* 1972). To apply this method, however, it is necessary to obtain the detail information such as shear wave velocity  $V_s$ , the unit weight  $\gamma$ , non-linear characteristics (the  $G - \gamma$  relationship, the  $h - \gamma$  relationship, the adhesion  $c$ , and the internal friction angle  $\phi$ ), and other parameters for the location where earthquake motion is being evaluated. In addition, such a dynamic analysis requires a multi-degree-of-freedom model. However, when

considering the evaluation of earthquake motion applied to an existing facility, very often adequate ground data is unavailable. Also, if the evaluation target consists of a group of many structures, it is assumed that this will require a vast number of calculations. Many researchers have been obliged therefore to evaluate ground behaviour using simple methods, most of which only allow evaluation of a single earthquake motion index or amplification factor of the response spectrum while other simple methods have been introduced on the assumption that the amplification factor is the same regardless of the input (Midorikawa *et al.* 1992, Nogami *et al.* 2012). Consequently, this situation called for the development of a simple method to evaluate the time history waveform of the ground surface taking into consideration the non-linearity of the soil.

In an attempt to meet this need, this study proposes a simple method to evaluate the surface waveform considering the complicated nonlinear behaviour of the strata of the ground. This paper proposes a new method for analyzing the static non-linearity of the overall ground system using a simple equivalent single-degree-of-freedom model. Despite the mass distribution of the ground is roughly uniform, the response characteristics change abruptly due to the localized non-linearization of the strata, which is clearly a contrast to the viaduct. That is to say, a static non-linear analysis method for bridges that applies a set of static load equivalent to the dynamic inertial force under earthquake, is not appropriate for estimating ground behaviour. Accordingly, the new static non-linear analysis method was proposed that is able to consider changes in the mode shape according to an increase in ground deformation. Furthermore, by using a dynamic analysis method which employs an equivalent single-degree-of-freedom model, it is possible to confirm that the earthquake motion of the ground surface is accurately evaluated.

The proposed static non-linear analysis can then be employed to evaluate the behaviour of grounds having different natural periods and strata compositions. In addition, the proposed method is employed to estimate the necessary parameters for the equivalent single-degree-of-freedom models using only the natural period of the ground, which is available in most situations. Finally the effectiveness of the response obtained using the proposed method was confirmed. It is found that this approach will permit the evaluation of the earthquake motion of the ground surface with just a simple investigation and response analysis.

## 2 METHOD FOR STATIC NON-LINEAR ANALYSES OF GROUND RESPONSE

This section first proposes a method for evaluating the change of stiffness and damping of the overall ground depending on the displacement of the surface soil. Here, in the case of a general bridge or viaduct, it is possible to roughly evaluate the load – displacement relation by a static nonlinear analysis, in which the acceleration acting on each node is equal. On the other hand, in case of evaluating ground motion, it is conceivable that the weight distribution is roughly uniform, but that the distribution and the response acceleration and response displacement vary in a complicated way according to the degree of non-linearity resulting from the changes in the strata composition and input level of earthquake motion. Consequently, it is not appropriate to carry out static non-linear analysis using the same procedure as that used for bridges or viaducts. For this reason, the evaluation of changes in stiffness and damping of the overall ground is necessary to consider changes in the mode shape along with deformation progression as shown in Fig. 1. The actual procedure is shown below.

- (1) Assuming the ground has  $k$  degrees of freedom and the values of the physical properties of each stratum (stratum thickness, weight, initial shear stiffness, damping, and non-linear characteristics) are given. At the initial step, the ground is assumed to be in an elastic region, and shear stiffness  $G_i^{(1)}$  is assumed to be equal to  $G_0^{(i)}$ . Here,  $G_i^{(1)}$  is the initial (1st step) shear stiffness of the  $i^{\text{th}}$  stratum, and  $G_0^{(i)}$  is the initial shear stiffness of the  $i^{\text{th}}$  stratum.
- (2), (3) Eigenvalue analysis is performed using the stratum thickness, weight and shear stiffness  $G_i^{(N)}$  of each stratum, and the primary natural frequency  ${}_1\omega^{(N)}$  and corresponding eigenvector  $\{{}_1u^{(N)}\}$  are calculated. Here, the eigenvector  $\{{}_1u^{(N)}\}$  is normalized so that the response at the ground surface position is 1.0. Also, superscript  $(N)$  indicates that the result is at the  $N^{\text{th}}$  step.
- (4) The displacement distribution is assumed so that the increment of deformation at the ground

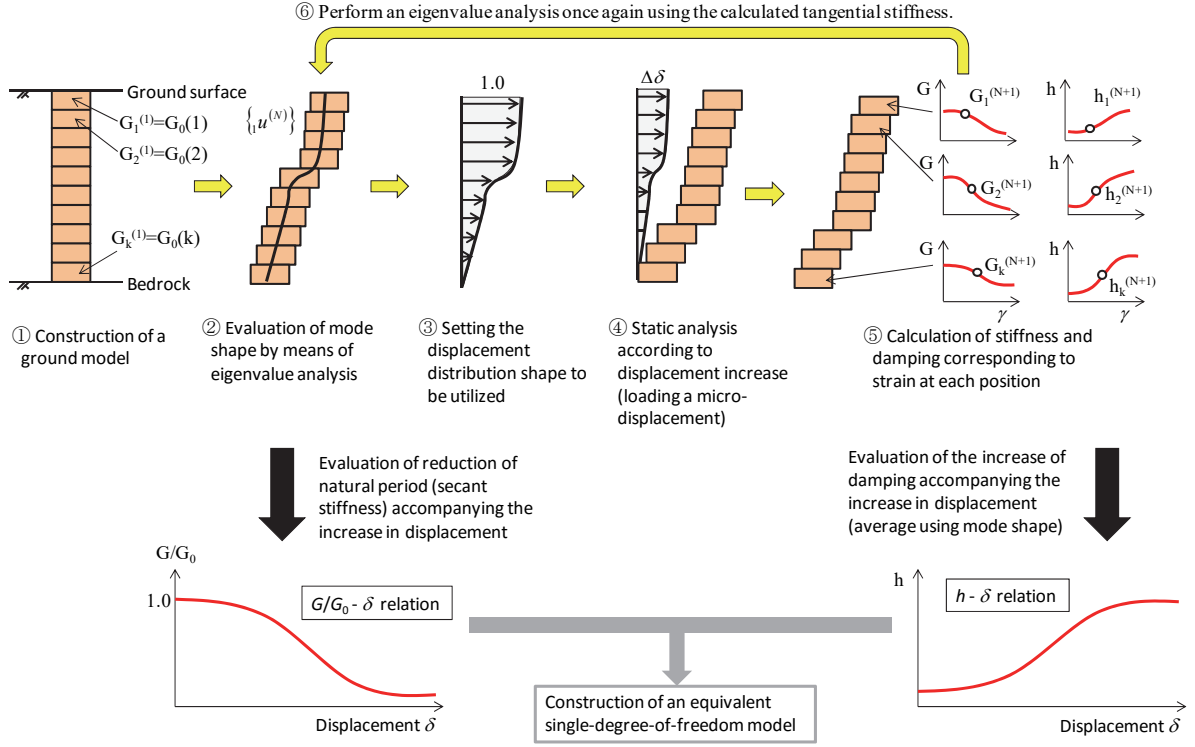


Fig. 1 Procedure for performing a static non-linear analysis of the ground

surface is  $\Delta\delta$ . Here, the top to bottom displacement distribution is assigned according to the primary mode shape obtained from (3) above, as shown in Eq.(1).

$$\{\delta^{(N)}\} = \{\delta^{(N-1)}\} + \Delta\delta \cdot \{1u^{(N)}\} \quad (1)$$

(5) The tangential stiffness  $G_i^{(N+1)}$  at each stratum and also the damping  $h_i^{(N+1)}$  are updated.

(6) The procedures (2) to (5) are repeated until the predetermined displacement is obtained at the ground surface position.

Also, as shown in Figure 1, the relationships  $G/G_0 - \delta$  and  $h - \delta$  for the overall ground system were obtained by using the aforementioned static non-linear analysis.

- $G/G_0 - \delta$  relationship: The reduction of the stiffness of the overall ground system is evaluated at each step from the change in the 1<sup>st</sup> mode natural frequency  $1\omega^{(N)}$  of the soil deposit, which is related to the increase in the ground surface displacement  $\delta$ , as shown in Eq.(2).

$$(G/G_0)^{(N)} = (1\omega^{(N)} / 1\omega^{(1)})^2 \quad (2)$$

- $h - \delta$  relationship: The average damping of the overall ground for each displacement condition is calculated from the weighted average of the damping  $h_i^{(N)}$  of each stratum corresponding to the strain distribution  $\{1\gamma^{(N)}\}$  in the primary mode. According to (5) of the above, it is given by following.

$$\begin{aligned} h_{eq}^{(N)} &= \{h_i^{(N)}\}^T \{1\gamma^{(N)}\} \\ &= \{h_1^{(N)}, \dots, h_{k-1}^{(N)}, h_k^{(N)}\} \left\{ \begin{array}{c} (1u_1^{(N)} - 1u_2^{(N)})/D_1 \\ \vdots \\ (1u_{k-1}^{(N)} - 1u_k^{(N)})/D_{k-1} \\ 1u_k^{(N)}/D_k \end{array} \right\} \end{aligned} \quad (3)$$

Where  $\delta$  is the surface displacement,  $D^i$  is the thickness of the  $i^{\text{th}}$  layer. In addition, it is necessary to take into consideration the participation function (PF) in order to convert the response of the primary mode into response at the ground surface position. For this reason, when performing the static non-

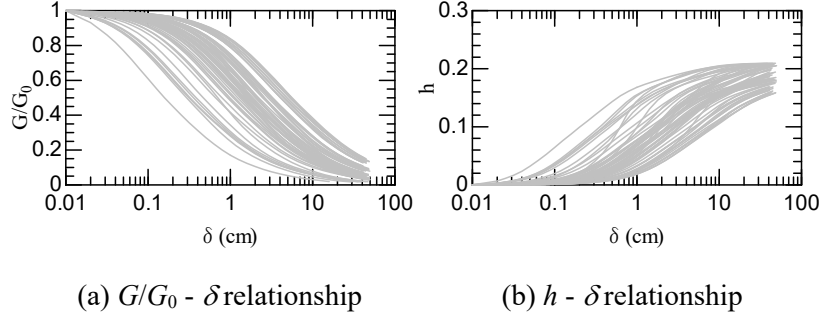


Fig. 2 Results of static non-linear analysis of multiple ground locations with different characteristics

linear analysis of the ground, the primary mode participation function for the displacement  $\delta$  at each step is to be also calculated.

### 3 EQUIVALENT SINGLE-DEGREE-OF-FREEDOM MODEL

#### 3.1 Standard parameters as a function of natural period of ground

The proposed static analysis method requires the detail information with regard to the target soil deposit as mentioned in Section 1. Such information, however, is not necessarily available particularly when evaluating the existing structures. In order to overcome the problem, the simple equivalent single-degree-of-freedom model is proposed to evaluate each ground location using only the natural period  $T_g$  of the ground. This  $T_g$  can be easily evaluated from microtremor measurements. In the study, 60 ground locations having a large variety of period characteristics and strata compositions were used, and a static non-linear analysis for each ground location was performed. Through the analysis, the  $G/G_0 - \delta$ , the  $h - \delta$ , and  $PF - \delta$  relationships were evaluated and an equivalent single-degree-of freedom model of the ground was constructed from the natural period  $T_g$  alone. Note that these ground locations are also used to evaluate the ground surface design earthquake motion of railway structures, whose natural periods are between 0.1 and 1.7 seconds. The non-linear characteristics of each ground location are expressed by GHE-S model, and the parameters assigned to each stratum of each ground location are changed according to the soil quality classification such as sandy soil or cohesive soil.

Static non-linear analysis was carried out at all of these 60 ground locations. The conditions of the analysis were the same as those of the case described in the previous section, where analyses were performed in two ground locations. Figure 2 shows the obtained  $G/G_0 - \delta$  and  $h - \delta$  relationship. From this data, it can be seen that there is an overall tendency that stiffness was falling and damping was increases according to the increase of the displacement. However, the degree of displacement differs greatly depending on the ground conditions. Consequently, it is difficult to create a simple model from these results only.

On the other hands, It is generally understood that if the results of element tests on the ground material are normalized and arranged using a strain value (reference strain  $\gamma_r$ ) such that  $G/G_0=0.5$ , the results can be expressed using the same parameters irrespective of the ground material, and there is a possibility that this method can be applied to the overall ground system. Accordingly, the relationships  $G/G_0 - \delta$ , and  $h - \delta$  were normalized by a reference displacement  $\delta_r$  such that  $G/G_0=0.5$ , as shown in Fig. 3 (a) and (b). Likewise, Fig. 3 (c) shows normalized participation function ( $PF$ ), also using a reference displacement  $\delta_r$ . First, from Fig. 3 (a) and (b) it can be seen that by normalizing the results of the static non-linear analysis of multiple ground locations by means of reference displacement  $\delta_r$ , it is possible to express the change in stiffness and damping using roughly the same relationships, irrespective of the natural period, the stratum thickness, and the stratum composition of the ground. Also, from Fig. 3 (c), it can be seen the overall trend of participation function  $PF$  is roughly the same regardless of the ground condition. From the above results, it can be said that if the reference displacement  $\delta_r$  is known, it is possible to predict to a certain extent the non-linear characteristics of

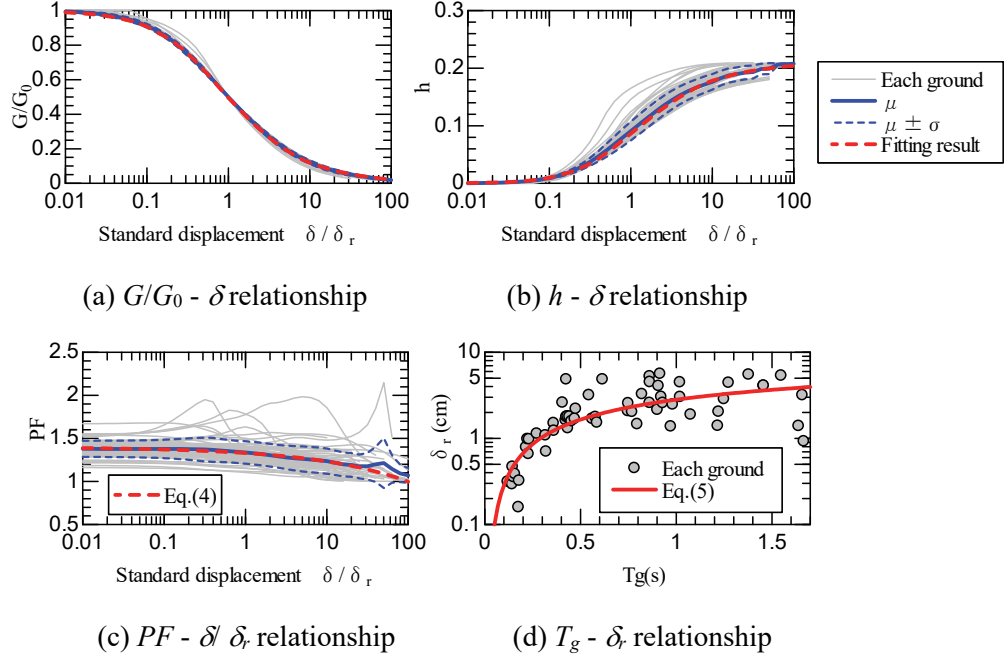


Fig. 3 Results of the static non-linear analysis after normalization using reference displacement  $\delta_r$

Table 1 Results after setting standard parameters

$C_1(0)$	$C_2(0)$	$C_1(\infty)$	$C_2(\infty)$	$C_1(1)$	$C_2(1)$	$\kappa$	$h_{\max}$
1.00	1.00	0.15	2.5	0.87	1.15	1.30	0.21

the overall ground system, regardless of the period and stratum composition of the ground.

Accordingly, the standard soil parameters that can be utilized in common regardless of the ground conditions were evaluated. First, the  $G/G_0 - \delta$  and  $h - \delta$  relationships, were expressed by means of a GHE-S model, as in the previous section, and model parameters that satisfied the average values of all of the results (blue lines in the figures) were determined by trial and error. The identified parameters are indicated in Table 1 Based on these, the  $G/G_0 - \delta$  and  $h - \delta$  relationships, expressed using the proposed method are indicated by red lines in Fig. 3 (a) and (b). It can be seen that these eight parameters have made it possible to reproduce the average value of each result with good accuracy. Next, participation function  $PF$  was also calculated as shown in Fig. 3 (c) which is expressed by Eq.(4). Note that each coefficient in Eq.(4) has been determined by the non-linear least-squares method.

$$PF = 1.4 - 0.07 \cdot \left( \frac{\delta}{\delta_r} \right)^{0.38} \quad (4)$$

Results obtained using Eq.(4) are also indicated by the red line in Fig. 3 (c). It is found that Eq.(4) expresses the overall trend of the participation factor in a good accuracy. Fig. 3 (d) shows the relationship between the reference displacement  $\delta_r$  and the natural period  $T_g$  of the ground. It can be seen that the reference displacement  $\delta_r$  increases along with the increase in the natural period  $T_g$ . Despite the reference displacement  $\delta_r$  is affected by many factors such as the natural period, the stratum thickness and stratum composition,  $\delta_r$  was likely to be evaluated only by using the natural period  $T_g$ . That is to say, the reference displacement  $\delta_r$  was estimated by a regression expression with respect to the natural period  $T_g$  of Fig. 3 (d) which is determined by a non-linear least-squares method.

$$\log \delta_r = 1.45 - \left( \frac{1}{T_g} \right)^{0.3} \quad (5)$$

The relationship between the natural period  $T_g$  and the reference displacement  $\delta_r$  expressed by Eq.(5) is also indicated by the red line in Fig. 3 (d). It is found that Eq.(5) traces the general tendency for the reference displacement to increase along with the increase in the period. By integrating these results, it is possible to construct an equivalent single-degree-of-freedom model using only the natural period  $T_g$  as a parameter.

### 3.2 Simulation of the ground response using an equivalent single-degree-of-freedom model

In order to verify the accuracy of this method, dynamic response using the equivalent single-degree-of-freedom model was compared with the reference responses obtained by employing a multi degree of freedom model. The two different ground characters were selected for the simulation, as shown in Fig. 4. The proposed equivalent single-degree-of-freedom model was constructed according to the steps described below, and the ground surface earthquake motion was calculated.

Step 1 Set the initial stiffness  $K_0$  from the natural period  $T_g$  of the ground.

Step 2 Set the GHE-S parameters and the natural period  $T_g$  from Table 1, and also set the reference displacement  $\delta_r$  (cm) from Eq. (5).

Step 3 Construct an equivalent single-degree-of-freedom model from Steps 1 and 2, and conduct a non-linear dynamic analysis.

Step 4 Calculate the participation function  $PF$  from the maximum displacement  $\delta_{max}$  obtained from Step 3 of the above and Eq.(4).

Step 5 Multiply the response waveforms of the relative acceleration, the relative velocity and the relative displacement by the participation function  $PF$ , and convert the results into the response at the ground surface position.

The comparison of the ground surface earthquake motion obtained using the proposed procedure and that from detailed non-linear ground response analysis are shown in Fig. 5 (ground A) and Fig. 6 (ground B). As seen in these figures, the proposed simple method was able to estimate the time histories and frequency characters of the surface waveform from the reference model in a good accuracy. . In addition, the same comparison was carried out at all the 60 ground locations used in this study. Figure 7 shows the comparison of the maximum displacements of the surface. As seen in this figure, both results show good agreement It consequently follows that this method is effective when carrying out a simple prediction of the ground surface in a region where the results of the ground investigation are incomplete.

## 4 CONCLUSION

This paper proposes a method for performing non-linear dynamic analysis using an equivalent single-degree-of-freedom model that can be used even for ground that has a complex stratum composition. The results obtained are shown below:

(1) A method was proposed for performing static non-linear analysis of the ground, which makes it possible to successively take into consideration the accumulation of localized strain accompanying non-linear behaviour of the ground. As a result, it was possible to evaluate the decrease of the stiffness and the increase of the damping according to the displacement.

(2) Based on the results of the static non-linear analysis of the ground, a simple single-degree-of-freedom model of the ground was proposed. It was confirmed through numerical simulations that the proposed method gave almost identical waveforms to those obtained from detail dynamic analysis with a multi-degree-of-freedom model.

(3) As a result of the application of the static non-linear analysis method to the ground, it became clear

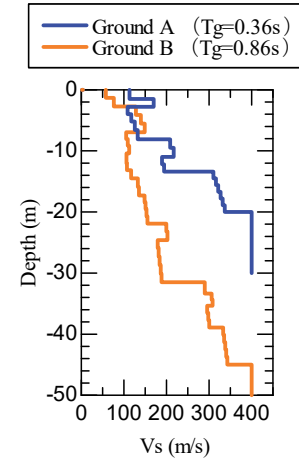
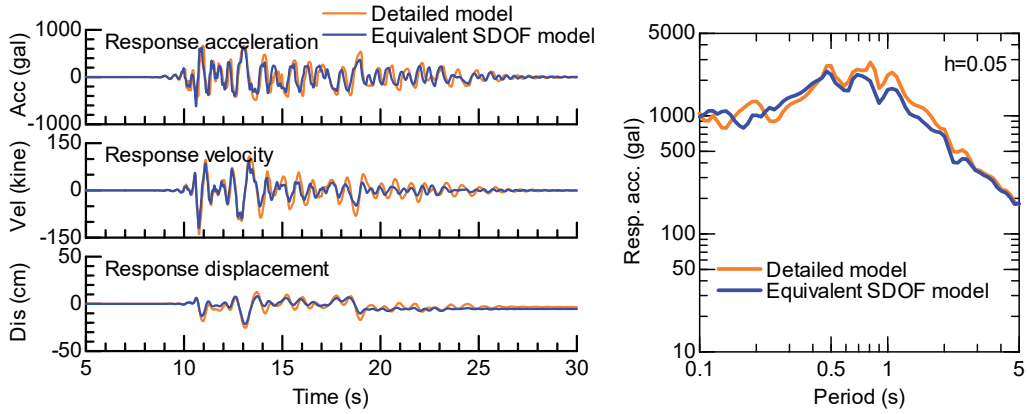


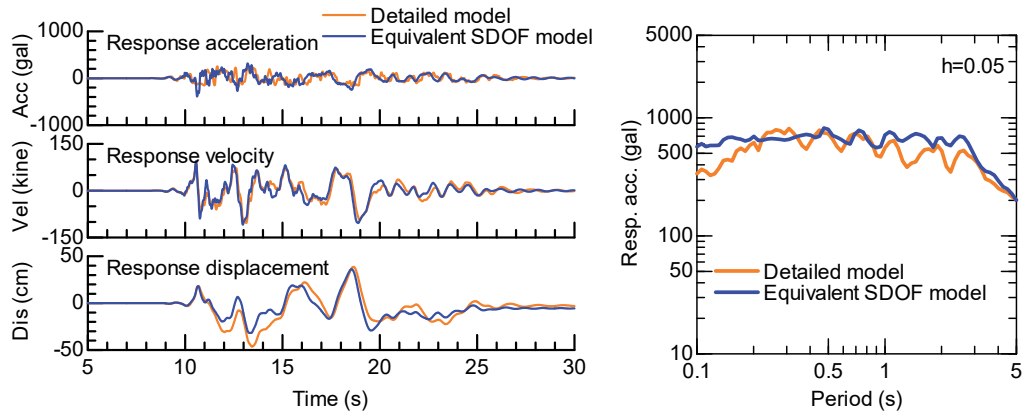
Fig. 4 Velocity construction of the ground used in the calculation



(a) Time history waveform

(b) Response spectrum

Fig. 5 Results of the prediction of the ground surface waveform by using only the natural period  $T_g$  (Ground A ( $T_g=0.36$  sec))



(a) Time history waveform

(b) Response spectrum

Fig. 6 Results of the prediction of the ground surface waveform by using only the natural period  $T_g$  (Ground B ( $T_g=0.86$  sec))

that by normalizing ground displacement with a reference displacement  $\delta_r$ , it was possible to uniformly express the tendency for stiffness to decrease and for damping to increase, irrespective of the softness or hardness of the ground, or the stratum composition. In addition, the standard parameters which express the  $G/G_0 - \delta/\delta_r$  and  $h - \delta/\delta_r$  relationships were proposed, and the reference displacement  $\delta_r$  needed for those relations was expressed by a function of the natural period of the ground  $T_g$ . As a result, it was possible to construct an equivalent single-degree-of-freedom model from the natural period  $T_g$  of the ground alone. The efficacy of the proposed method was confirmed by the numerical analysis using 60 ground conditions.

Even if only a few data with regard to soil properties property is available, particularly in case of an existing railway facility, the proposed method enables economical dense calculation of trackside surface motion. These sets of waveforms will be utilized to identify locations where

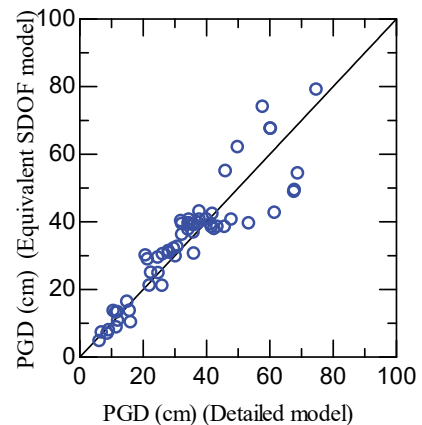


Fig. 7 Results of the evaluation the ground surface displacement using the proposed method

devastating damage is will be expected in the case of a strong earthquake.

Consequently, by using this method, it is possible to rationally extract locations which require priority attention in an earthquake, when the target of the measurement is an existing group of structures for which ground data is inadequate, and to predict ground surface earthquake motion immediately after a large scale earthquake.

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