

Evaluation of Damping Modification Factors for Seismic Response Spectra

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ABSTRACT

Seismic response spectra with structural damping ratio other than nominal 5% (of critical damping) are essential for the design and evaluation of structures in performance-based seismic engineering. Such response spectra are also essential for the design and evaluation of structures with seismic isolation and energy dissipation systems. A number of formulations for damping modification factors (DMF) have been proposed in the literature for scaling the 5% damped response spectra. Dependence of the DMF on several ground motion parameters has also been identified. Few seismic design codes have already incorporated simplified DMF based on these studies. This paper critically reviews the available formulations for DMF for seismic response spectra. Analytical investigations on the ground motion response spectra at soil sites, based on a wide range of simulated ground motion records, have been carried out. It has been observed that the DMF for ground motion response spectra at soil sites is significantly dependent on site period, which has not been identified in previous studies. The influences of earthquake shaking level, earthquake source-site distance (near-field and far-field events), soil plasticity index, and the rigidity of bedrock have also been investigated.

Keywords: Seismic design, design code, response spectra, damping, modification factor

1. Introduction

Seismic design and assessment of structures are generally based on response spectrum analyses in which response spectra representing earthquake ground motions for a specified return period with nominal 5% of critical damping are used. Also, in most seismic design codes, response spectra represent design earthquake ground motions with 5% of critical damping. However, in reality, structural and non-structural systems may have damping ratios other than 5% of the critical damping. Damping ratio (ζ) as a percentage of critical damping represents energy dissipation by the structure. In the seismic design and assessment of structures, two types of damping are usually considered: viscous damping and hysteretic damping. Energy dissipation in a structure in the elastic range (viscous damping) occurs due to various mechanisms, including cracking, interactions with non-structural elements, and soil-structure interactions. For mathematical convenience, these damping mechanisms altogether are represented as viscous damping. The concept of equivalent viscous damping for the seismic design and analysis of the structure has been used to incorporate both viscous and hysteretic damping (Blandon and Priestley 2005).

In recent years, research on the seismic design and assessment of structures is directed towards the development of direct displacement based procedures. In the direct displacement based procedures, a multi-degree-of-freedom (MDOF) structure is replaced by an equivalent single-degree-of-freedom (SDOF) structure (substitute structure) characterised by the secant stiffness to maximum displacement response and equivalent viscous damping (elastic and hysteretic damping) (Priestley et al. 2007). The equivalent viscous damping of the substitute structure is significantly higher than 5% of critical damping.

Energy dissipation devices have been increasingly used to enhance the seismic performance of important structures. Energy dissipation through frictional sliding, yielding of metal, phase transformation in metals, deformation of viscoelastic solids or fluids, and fluid orificing provides the capability of as much as 40% of critical damping in the first mode response of the structural system. Although energy dissipation characteristics of various supplemental damping devices may not be ideally viscous; they can, however, be related to an equivalent damping ratio (Lee et al. 2004).

Response spectra for damping higher than the notional 5% of critical damping can be obtained by developing response spectrum prediction equations that can directly estimate spectral ordinate at various levels of damping. They can also be obtained by developing response spectrum damping modification factors to translate existing prediction equations or code-based response spectra with 5% of critical damping to response spectra for other damping ratios. Significant research effort is required to develop ground motion prediction equations for various levels of damping which may possess similar shortcomings as the second approach (Stafford et al. 2008). However, the second approach has distinctive advantage as it is applicable for modifying both the ground motion prediction equations (5% of critical damping) and the code-based response spectra. This paper adopts the second approach to develop damping modification factor (DMF) for scaling the response spectra of 5% critical damping to higher damping levels (up to 40% of critical).

2. Damping Modification Factor (DMF) in Design Codes

The adoption of damping modification factor (DMF) in design codes was mainly inspired by the pioneer work of Newmark and Hall (1973 and 1982). Newmark and Hall (1973) proposed DMFs (Equation 1) for constant velocity, constant acceleration and constant displacement regions. The DMF were derived from median estimate of maximum displacement response of SDOF system with $\zeta < 20\%$. The relationship proposed by Newmark and Hall (1973) has been adopted in ATC-40 (1996) and FEMA 273 (1997). UBC (1997) and ASCE7-05 (2006) also incorporated the relationship adopted in FEMA 273 for constant velocity region.

$$DMF = \begin{cases} 1.514 - 0.321 \cdot \ln(\zeta) & \text{(constant velocity region)} \\ 1.400 - 0.248 \cdot \ln(\zeta) & \text{(constant acceleration region)} \\ 1.309 - 0.194 \cdot \ln(\zeta) & \text{(constant displacement region)} \end{cases} \quad (1)$$

The DMF in EC8 (2004) (Equation 2) adopted the equation derived by Bommer et al. (2000), which replaced the earlier formulation (Equation 3) of the pre-norm version of the code (EC8, 1994).

$$DMF = \sqrt{\frac{10}{5 + \zeta}} \quad (2)$$

$$DMF = \sqrt{\frac{7}{2 + \zeta}} \quad (3)$$

Priestley et al. (2007) suggested for revising the DMF in EC8 to Equation (4), especially for sites where forward directivity velocity pulse characteristics might be expected.

$$DMF = \left(\frac{7}{2 + \zeta} \right)^{0.25} \quad (4)$$

The study of Kawashima and Aizawa (1986) was adopted in Caltrans Seismic Design Criteria (2010) (Equation 5). However, Equation (5) is applicable for damping ratio $5\% \leq \zeta \leq 10\%$.

$$DMF = \frac{1.5}{40 \times (\zeta/100) + 1} + 0.5 \quad (5)$$

The study of Ramirez et al. (2002) has been adopted in NEHRP (2003) which is similar to the DMF adopted in UBC (1997) for up to $\zeta = 20\%$, beyond which the DMF is lower in NEHRP (2003) (Figure 1).

In the Chinese code for seismic design of buildings (GB50011 2010), the seismic design response spectrum is adjusted for different damping ratios according to Equation (6).

$$DMF = 1 + \frac{0.05 - (\zeta/100)}{0.06 + 1.4} \quad (6)$$

In the Japanese Seismic Design Code (Otani and Kanai 2002), the effect of damping ratios is taken into account in the form of response reduction factor according to Equation (7).

$$DMF = \frac{1.5}{1 + 10 \times (\zeta/100)} \quad (7)$$

Figure 1 shows significant differences amongst the code-based period independent DMF. The lowest values of DMFs have been suggested in the Japanese seismic design code. The most conservative DMF have been suggested by Priestley (2007). It is noted that Priestley et al. (2007) suggested the revision of EC8 (1994) DMF for near field earthquake ground motion. The great difference in the specified DMF in the design codes signifies the need for in-depth study on DMF.

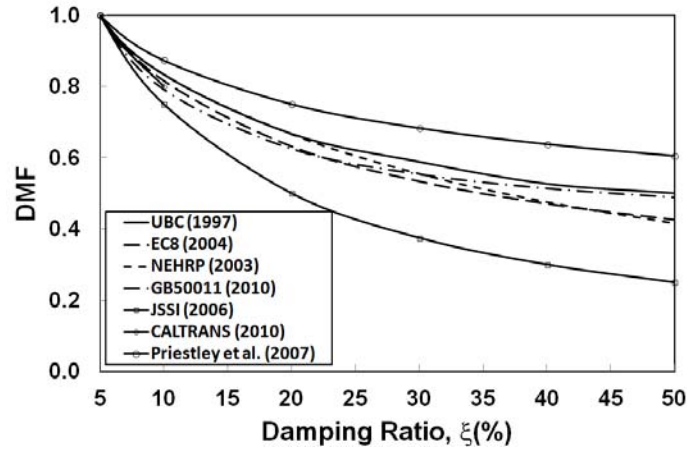


Figure 1: Damping modification factor (DMF) in seismic design codes

3. Description of the Model and Input Seismic Ground Motion

3.1 Definition of Damping Modification Factor (DMF)

For a linear SDOF system with viscous damping subjected to earthquake ground acceleration, the equation of motion can be written as

$$m\ddot{u}(t) + c\dot{u}(t) + ku = -m\ddot{u}_g(t) \quad (8)$$

where m , c , and k are mass, damping and stiffness of the system; $u(t)$, $\dot{u}(t)$, $\ddot{u}(t)$, are relative displacement, relative velocity, relative acceleration of the system; and $\ddot{u}_g(t)$ is the ground acceleration. Displacement response spectra of the system can be defined as $S_d \equiv |u(t)|_{\max}$.

DMF with respect to displacement response of the system can be defined as

$$DMF = \frac{|u(t)|_{\max, \xi}}{|u(t)|_{\max, \xi=5\%}} = \frac{RSD(T, \xi)}{RSD(T, \xi=5\%)} \quad (9)$$

where ξ is the damping ratio, T is the vibration period, RSD is the response spectral displacement.

The DMFs derived from displacement response of the system are identical to the factors derived from either pseudo acceleration or pseudo relative velocity response of the system, as they are related by the natural vibration frequency or period of the SDOF system (Chopra 2007) according to Equations (10) and (11).

$$DMF = \frac{RSD(T, \xi)}{RSD(T, \xi=5\%)} = \frac{\omega^2 \cdot RSD(T, \xi)}{\omega^2 \cdot RSD(T, \xi=5\%)} = \frac{PRSA(T, \xi)}{PRSA(T, \xi=5\%)} \quad (10)$$

$$DMF = \frac{RSD(T, \xi)}{RSD(T, \xi = 5\%)} = \frac{\omega \cdot RSD(T, \xi)}{\omega \cdot RSD(T, \xi = 5\%)} = \frac{PRSV(T, \xi)}{PRSV(T, \xi = 5\%)} \quad (11)$$

where $PRSA$ is the pseudo acceleration response, $PRSV$ is the pseudo relative velocity response and $\omega (=2\pi/T)$ is the natural vibration frequency of the SDOF system. The DMF derived in this paper is based on $PRSV$. It is noted that in the seismic analyses of structures relative velocity and absolute acceleration are approximated by the corresponding pseudo relative velocity and pseudo absolute acceleration, respectively. This approximation is suitable for small damping ratios but may show considerable differences especially for highly damped absolute acceleration and absolute velocity response spectra (Song et al. 2007). However, the proposed DMF model is primarily developed for displacement based seismic design and assessment of structure where damping modification is mainly applied to the displacement response spectra.

3.2 Earthquake Ground Motion and Site Soil conditions

In order to cover a wide spectrum of ground shaking levels, synthetic earthquake accelerograms, with maximum response spectral velocity (RSV_{max}) of around 20, 100 and 300 mm/s at soil-bedrock interface were generated by stochastic simulations of the seismological model using computer program GENQKE (Lam et al. 2000). For each level of ground shaking, two sets of time histories were generated: one represents near-field (NF) (source-site distance, $R=50$ km) ground motions, which are rich in high frequency seismic waves, and the other represent far-field (FF) (source-site distance, $R=100$ km) ground motions, which are comparatively rich in low frequency seismic waves. Each set contains six simulated acceleration time histories.

Five soil columns with weighted average shear wave velocities (V_s) = 100, 150, 200, 300, and 500 m/s and four soil plasticity indices (PI=0%, 15%, 30% and 50%) have been included in the study. There are altogether 20 soil columns of different thicknesses (H) and with a wide range of initial site period, T_i , from 0.12 to 2.4 s. This range of site period covered sandy soil sites (0.14–0.95 s) and soft soil sites (1.97–2.3 s) as considered in the study by Henderson et al. (1990) and Heidebrecht et al. (1990). The nonlinear characteristics of the soil layers were captured by two strain-compatible material parameters, namely, secant shear modulus G and damping ratio ξ . The dynamic properties of soil adopted in this study were obtained by Lam and Wilson (1999). Responses of the soil sites have been calculated using computer program SHAKE (Schnabel et al. 1972). The responses of the soil sites have been calculated considering bedrock shear wave velocities (shear rigidity of bedrock) of 1000 m/s, 2000 m/s and 3000 m/sec and also for rigid (non-transmitting) bedrock conditions. Response spectra are generated for 0.01-8.0 second (50 data points) with $\xi = 5-40\%$.

4. Results and Discussions

4.1 The influence of vibration period

The influence of vibration period (T) on the DMF was investigated in several previous studies (Ashour 1987; Wu and Hanson 1989; Ramirez et al. 2002; Naeim and Kircher 2001; Lin and Chang 2003 and 2004; Atkinson and Pierre 2004; Lin et al. 2005; Boomer and Mendis 2005; Cameron and Green 2007; Lin 2007, Stafford et al. 2008; Cardone et al. 2009; Hatzigeorgiou 2010; and Hao et al. 2011). Significant discrepancies can be observed in the

proposed period dependent DMFs (Hubbard and Mavroeidis 2011). According to the fundamental concepts of structural dynamics (Chopra 2007), ground motion at very short period and very long period are not significantly affected by damping. This essentially means that the DMF at very short period and very long period will converge to unity.

Figure 2 presents the DMF for five soil sites (PI=0%) (with different site natural period) analysed in this study. It can be observed that DMF reaches unity at $T=0.01$ s. The tendency of DMF towards unity can also be observed at long periods, although in this study response spectra have been calculated for up to $T=8.0$ s. It is evident from Figure 2 that the lowest values of DMF for different soil sites do not occur at the same vibration period. Further in-depth analyses reveal that the lowest values of DMF occur at shifted site period, T_s . It is noted that shifted site period T_s is associated with large shear strains that the soil sites experience during earthquake ground shaking and is different from the initial site natural period T_i (Tsang et al. 2006 and Tsang et al. 2012). Hence, it would be meaningful to investigate the DMF functions by normalising the period values by the shifted site natural period T_s .

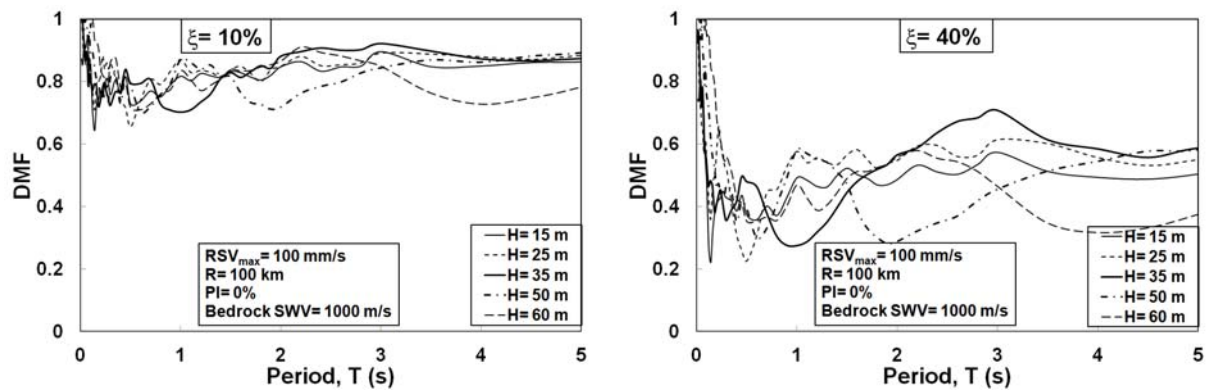


Figure 2: Influence of vibration period (T) on DMF

Figure 3 reproduces the DMF in terms of period ratio, PR (T/T_s). The influence of PR on DMF is more pronounced for $\zeta=40\%$. This is a significant finding which has not been investigated in earlier studies. It is noted that earlier studies are based on statistical analyses of recorded earthquake ground motion records on wide range of sites (typically categorised into site classes based on weighted average shear wave velocities in the top 30 m of soil layers) in high seismic regions. The effect of T_s has been masked by the averaging in the statistical processing of recorded ground motion records.

It can be observed from Figure 3 that five distinct PR (PR=0.01, 0.25, 0.5, 1.0 and 2.0) control the general behaviour of DMF for the five soil sites considered in this study. For small PR=0.01, the DMF factor is 1.0. For PR<1.0, the DMF increases from PR=1.0 to PR=0.5 and then decreases from PR=0.5 to PR=0.25. Afterward, the DMF increases to unity from RP=0.25 to PR=0.01. It is noted that such a trend may not be evident for soil column $H=15$ m where site period (T_s) is typically less than 0.2 s. However, the vibration period $T < 0.1$ s (PR= 0.5 for $H=15$ m) is considered not important for the seismic design and analyses of structures. For PR>1.0, the DMF generally increases with the increase in the PR, which is consistent with the fundamental concept of structural dynamics.

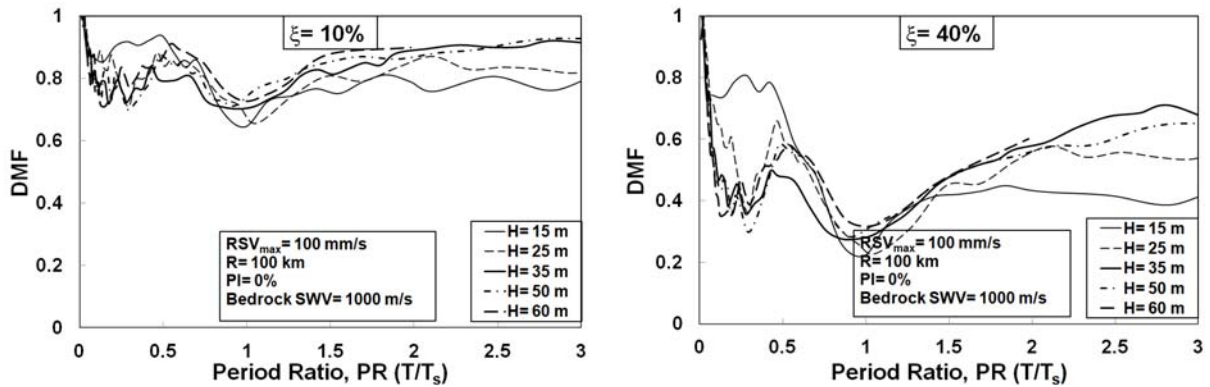


Figure 3: Dependence of DMF on Period Ratio, PR (T/T_s)

4.2 Influence of Damping Ratio

Figure 4 shows the influence of damping ratio, ξ (as percentage of critical damping) on DMF for $H=35$ m soil column subjected to far field ($R=100$ km) earthquake ground motion with $RSV_{max}=100$ mm/s. The logarithmic decrement of DMF with the increment in the damping ratio (ξ) is evident in Figure 4. Newmark and Hall (1973) also proposed such logarithmic decrement of DMF with the increase in the damping ratio (Equation 1). It can be observed from Figure 4 that DMF for $PR=0.25$ and $PR=3.0$ represent the upper and lower boundaries of DMF suggested in the reviewed design codes (Refer Figure 1). This further signifies the masking effect of site period in the statistical analyses for the DMF in the earlier studies.

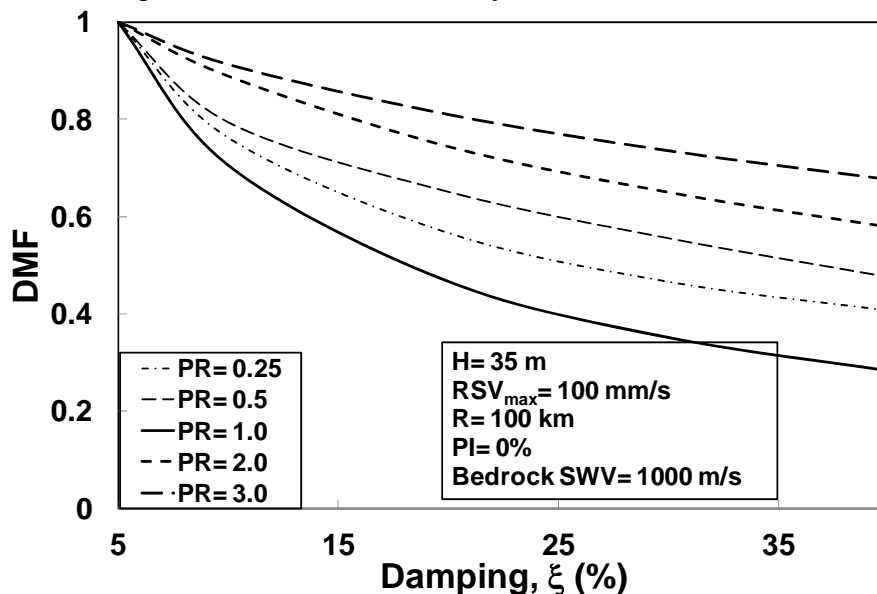


Figure 4: Influence of damping ratio (ξ) on DMF

4.3 Influence of earthquake shaking levels

The influence of earthquake shaking level (characterised by RSV_{max}) has been shown in Figure 5. It can be observed that at resonance period ($PR=1.0$), higher shaking level produces higher DMF, although the difference is not noteworthy for ground motion with $RSV_{max}>100$ m/s. The DMF decreases with the increase in the shaking level for all other PR. Boomer and Mendis (2005) have also observed that DMF decreases with increasing moment magnitude of earthquake events; whereas, Cauzzi and Faccioli (2008) observed that DMF moderately depends on the magnitude of earthquake events. On the other hand, Hao et al. (2011)

observed that earthquake moment magnitude has significant influence on DMF except for soft soil sites. It is noted that none of these studies paid adequate attention to the site period.

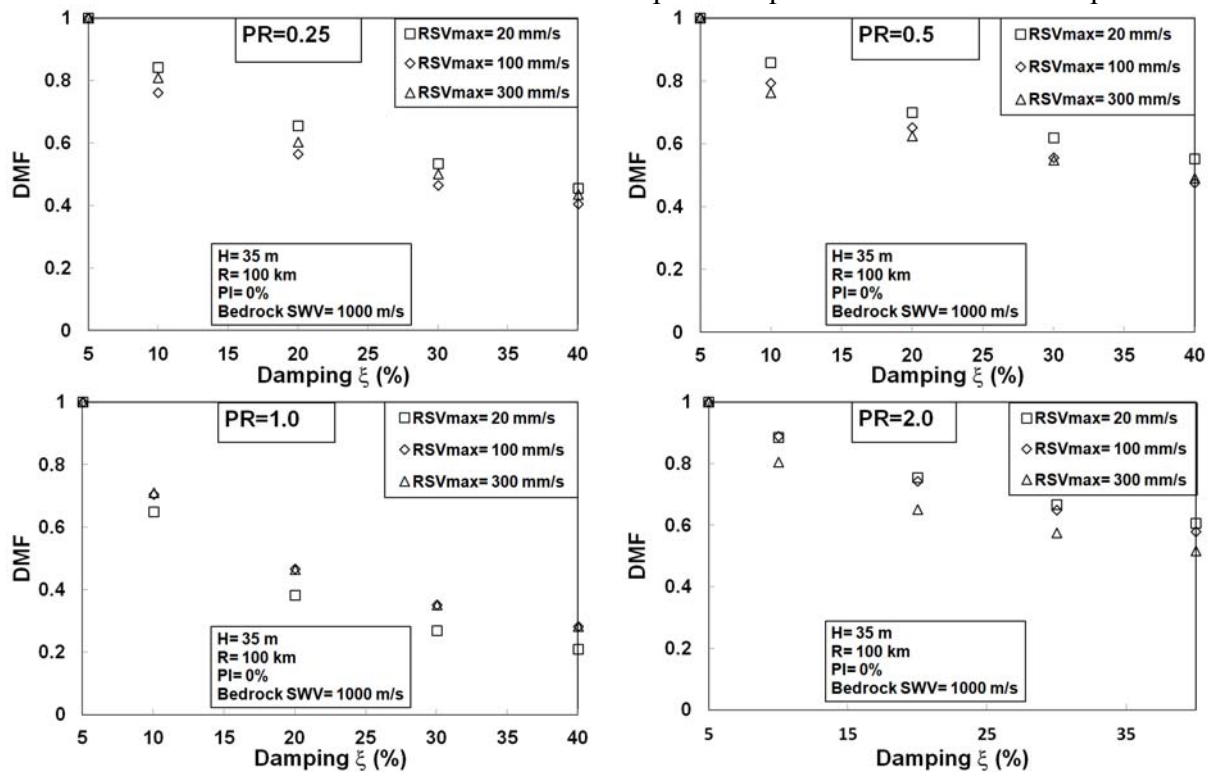


Figure 5: Influence of earthquake shaking level on DMF

4.4 Influence of source-site distance

The influence of source-site distance has been shown in Figure 6. Earthquake ground motions used in the analyses were generated considering two source-site distances: R= 50 km (Near Field event, NF) and R=100 km (Far Field event, FF). It is noted that RSV_{max} remains constant for both events in order to investigate the influence of site-source distance alone. It can be observed from Figure 6 that, at resonance period (PR=1.0) the DMF is slightly greater in FF events. However, this trend is reversed for other PR. Bommer and Mendis (2005) observed that DMFs decrease with increase in the source-site distances. However, the effect of source-site distance was found to be negligible in Hao et al. (2011), especially for site-source distance closer than 100 km.

4.5 Influence of geotechnical properties of soil sites

Plasticity Index (PI) of the soil is the controlling parameter for dynamic properties of the soil, especially when subjected to earthquake ground motion. With the increase in the PI, the rate of shear modulus degradation and damping of soils decrease (Vucetic and Dobry 1991), which has significant influence on the seismic response of the site. However, the influence of soil PI on DMF has not been observed to be significant especially for PR>1.0 (Figure 7). For PR<1.0, the general tendency is that the DMF increase with the increase in the PI of the soil sites. It is noted that in order to depict the key factors contributing to the DMF, the soil sediments has been modelled as homogeneous materials overlying the bedrock. In reality, properties of soil sediments could vary with depth. However, it is believed that the effect of non-homogeneous soil sediment on DMF may not be significant as the effect might be reflected in the calculation of site period (T_s).

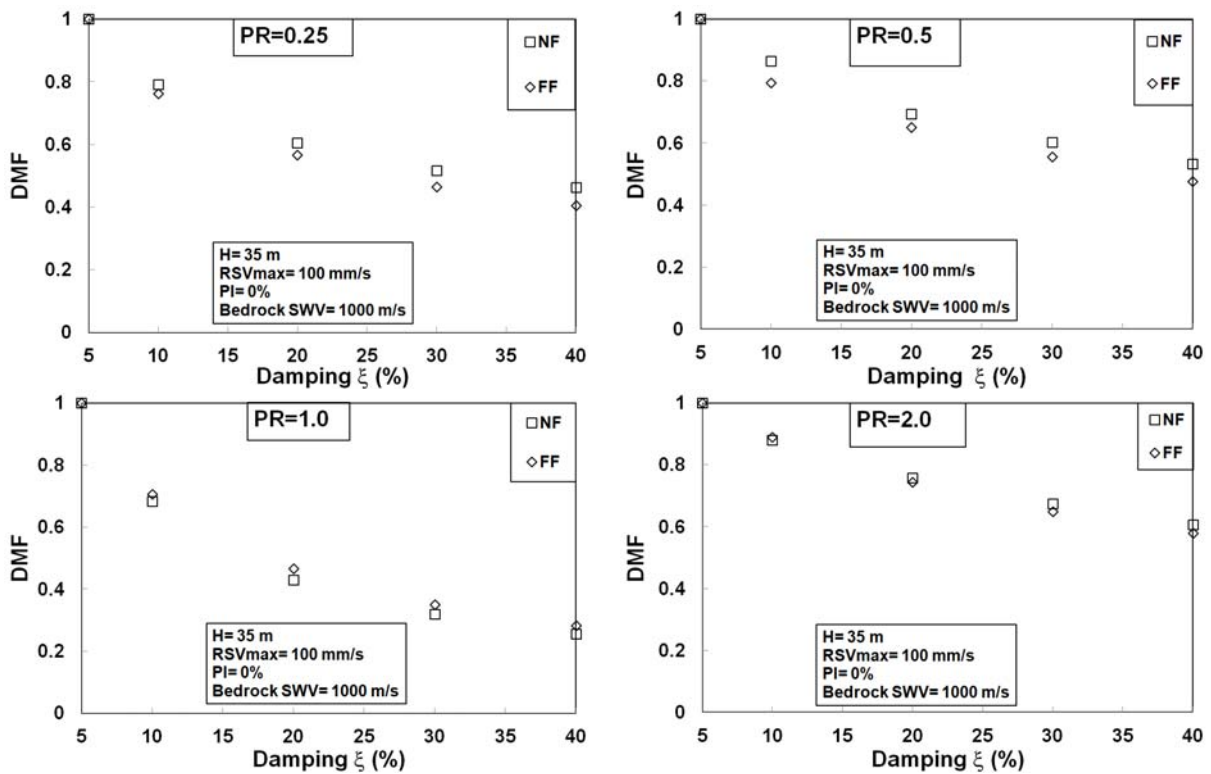


Figure 6: Influence of earthquake source-site distance (NF and FF) on DMF

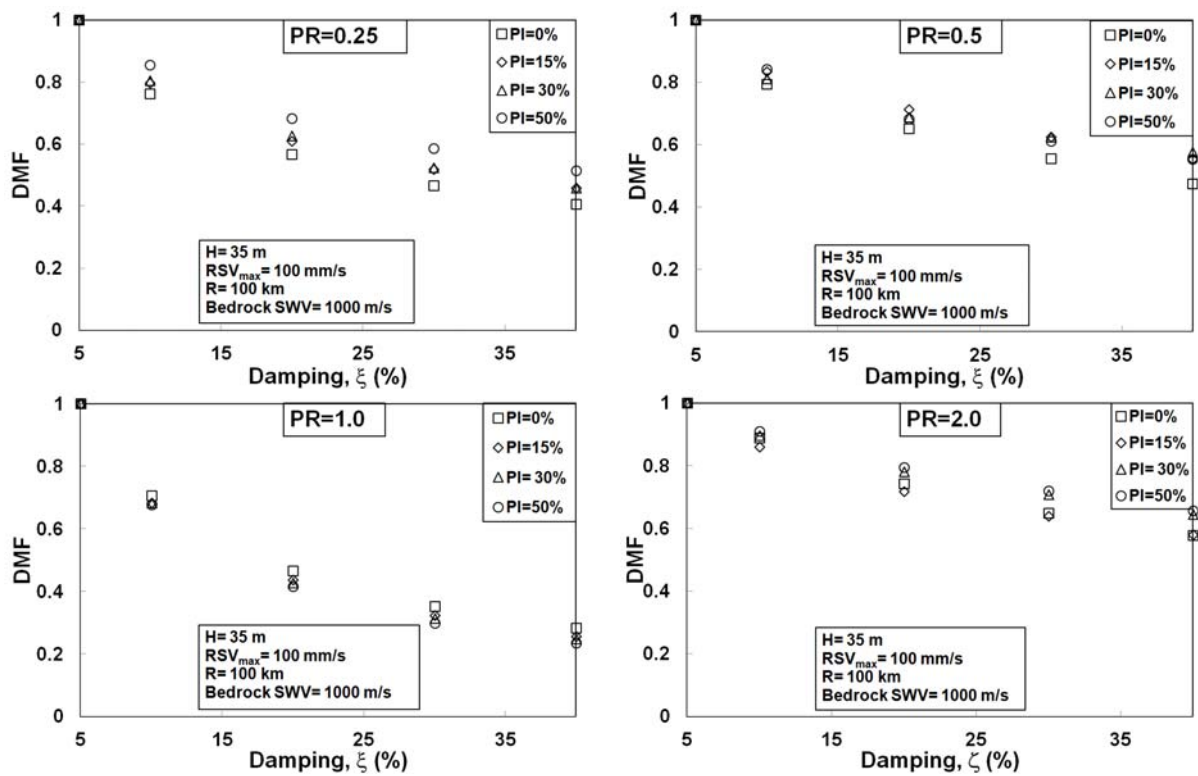


Figure 7: Influence of soil PI on DMF

4.6 Influence of bedrock rigidity

The shear rigidity of the bedrock plays an important role in the seismic site response analyses, as the amount of seismic waves reflected back from the soil-bedrock interface considerably depends on the impedance contrast between rock and soil (Tsang et al. 2012). Site modifications due to earthquake ground shaking are associated with filtering mechanisms and superposition of reflected waves within soil layers overlying bedrock. Hence, the response of the soil and the superstructure can be sensitive to the shear rigidity of the bedrock materials. However, the effect of bedrock rigidity on the DMF has not been found significant for the range of vibration period considered herein (Figure 8), although non-transmitting bedrock condition (rigid bedrock overlying soil sediments) shows slightly increased DMF for PR other than 1.0.

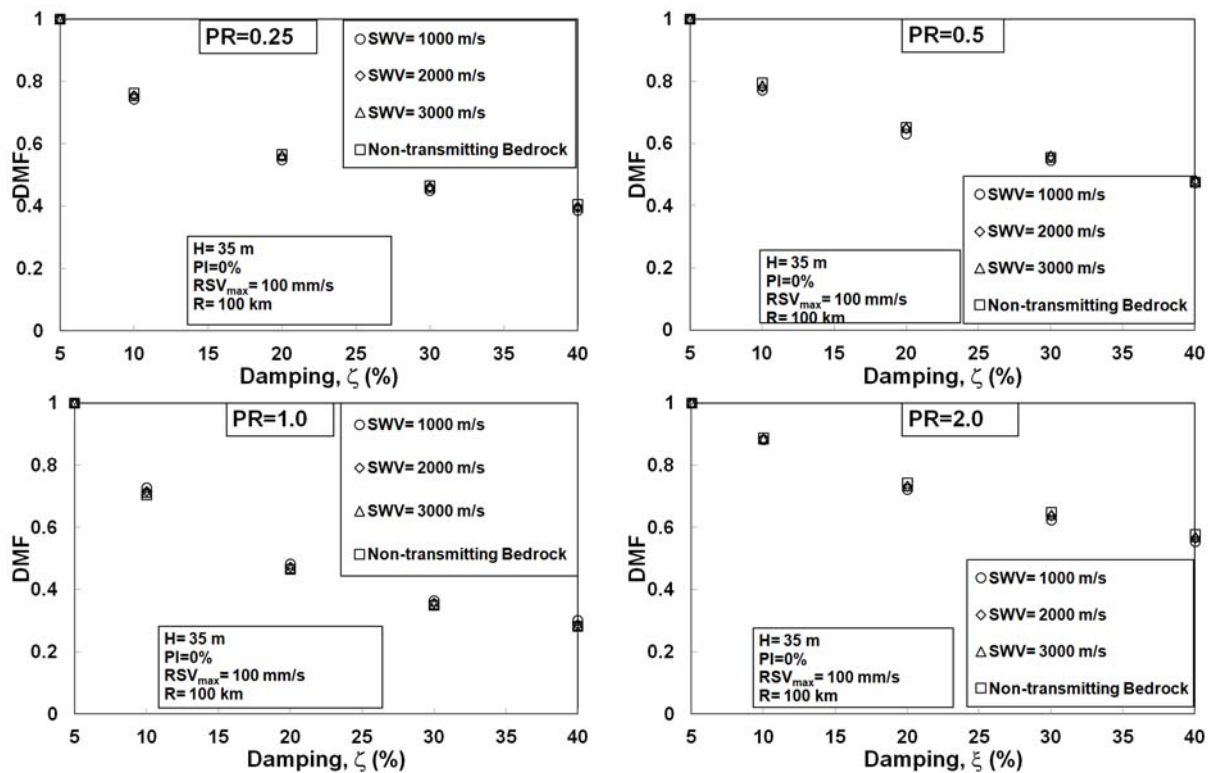


Figure 7: Influence of bedrock rigidity on DMF

5. Conclusions

Response spectra with structural damping ratio higher than nominal 5% of critical damping are required for displacement-based seismic design and analyses of structures and also for the design of structures with supplemental damping systems. Yet, the discrepancies in the damping modification factor (DMF) for scaling the response spectra for higher than 5% of critical damping in the design codes is very high. This might be due to inherent biasness of the recorded earthquake ground motion used in the statistical analyses performed in obtaining response spectral DMF.

A large number of ground response analyses using simulated earthquake records covering wide range of soil sites and shaking levels have been carried out in this study to systematically study the influence of a number of parameters that may influence the DMF.

The DMF has been found to be dependent on the vibration period, unlike code-based period independent DMF. The influence of vibration period on the DMF has also been pointed out in a number of previous studies. However, the significant outcome of this study is the observation that the DMF is highly dependent on the Period Ratio, PR (T/T_s). The decrement of DMF with the increment in the damping ratio (ζ) at different PR is logarithmic, similar to the observation in Newmark and Hall (1973).

DMF slightly increases with the increase in the shaking level for PR=1.0; however, the DMF slightly decreases with increase in the shaking level for other PR. On the other hand, the DMF is larger to some extent in the NF earthquake events at PR=1. At other PR, the DMF is little larger in FF earthquake events. The influence of soil PI and the bedrock rigidity on the DMF has not been found significant.

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