# Comparison of pushover methods for simple building systems with flexible diaphragms

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## Abstract

Nonlinear static (pushover) methods have become increasingly popular in seismic analysis as they allow the explicitly consideration of nonlinear structural behaviour, while retaining the simplicity of the static analysis. Even though significant advances of pushover methods have occurred in recent years, the applicability of such methods for unreinforced masonry buildings with flexible diaphragms has remained relatively unexplored.
This paper investigates the accuracies of various pushover methods in predicting the peak seismic response of buildings with flexible diaphragms. The peak displacement demands of simple two-degrees-of-freedom systems, considered to represent the basic characteristics of structures with flexible diaphragms, are estimated using the N2, modal pushover analysis and adaptive capacity spectrum method, and compared against time-history analyses.
The results show that pushover methods generally become less reliable when the diaphragm is made flexible. However, the modal pushover analysis appears to give better results and to be the most promising for the development of a pushover method suitable for masonry buildings with flexible diaphragms.

**Keywords:** Nonlinear static method, pushover analysis, flexible diaphragm

## 1. INTRODUCTION

The nonlinear static methods (also referred to as pushover methods) have become popular in practice as they allow the explicit consideration of the nonlinear structural behaviour without the need to define the often complex hysteretic characteristics of the structural elements. In addition, the method retains the simplicity of the static analysis, and the code-defined hazard spectra may be used as the “loading”. The initial development of the nonlinear static methods focused on low-rise regular (or planar) buildings, in which the inelastic response is approximated by an equivalent single-degree-of-freedom (SDOF) system as obtained from static pushover analysis. Subsequent developments saw the inclusion of higher-mode effects for tall structures (Sasaki et al., 1998; Chopra and Goel, 2002, Sucuoğlu and Günay, 2010), adaptive pushover methods to account for the effects of member yielding (Bracci et al., 1997; Antoniou and Pinho, 2004) and the inclusion of torsional response (Chopra and Goel, 2004; Fajfar et al., 2005; Kaatsız and Sucuoğlu 2014). Importantly, all methods were developed for buildings in which the floor and roof diaphragms remained rigid in their own plane. In recent years, they were also adapted to the seismic analysis of bridges in the transverse direction, in which the deck (or diaphragm) is relatively flexible (Casarotti and Pinho, 2007; Isaković et al., 2008).

Despite the aforementioned improvements, in common practice the pushover method for unreinforced masonry buildings is the conventional procedure in which the SDOF response is essentially assumed. Problems arise when one uses the conventional method in assessing existing heritage buildings, which often contain flexible timber floor and roof diaphragms. Due to the in-plane flexibility of the diaphragms, the structural response can contain multiple dominant modes, which invalidates the SDOF behaviour assumed in the conventional method. Therefore, there is a need to investigate whether more advanced pushover methods can be used to accurately estimate the peak seismic demands in buildings with flexible diaphragms. This issue is explored using simple two-degrees-of-freedom systems, by comparing the displacement ductility demands predicted by various pushover methods against the "actual" responses obtained from nonlinear time-history analyses.

## 2. OVERVIEW OF METHODS

Three different pushover methods are evaluated. The first is the N2 method (Fajfar, 2000) contained in Eurocode 8 (herein referred to as the conventional method). The second is the Modal Pushover Analysis (MPA) developed by Chopra and Goel (2002), and the last is a modified version of the Adaptive Capacity Spectrum Method (ACSM) proposed by Casarotti and Pinho (2007) in association with a displacement-based adaptive pushover procedure (Antoniou and Pinho, 2004). A brief discussion of these methods is given below. Readers are referred to the cited works for the development and the theoretical background of the methods.

### 2.1. N2 method

The N2 method consists of five main steps:

1. Develop a nonlinear model of the structure and define the earthquake input in terms of a response spectrum.
2. Conduct a pushover analysis of the model, with an invariant lateral force distribution proportional to $\boldsymbol{m\phi}$, where $\boldsymbol{m}$ is the mass matrix and $\boldsymbol{\phi}$ is the assumed displacement shape. Plot the base shear – control node displacement (usually at the centre of mass of the roof) to obtain the pushover curve.
3. Convert the pushover curve to an equivalent SDOF system. This conversion is done by dividing both the control node displacement and the base shear by $\Gamma = \boldsymbol{\phi}^T\boldsymbol{m1}/\boldsymbol{\phi}^T\boldsymbol{m\phi}$, provided $\boldsymbol{\phi}$ is normalised to the control node displacement. This factor is often referred to as the modal participation factor. The base shear is further divided by the mass of the equivalent SDOF system, $m^* = \boldsymbol{\phi}^T\boldsymbol{m1}$, to obtain pseudo-acceleration.
4. Based on the properties of the equivalent SDOF system from Step 3, and the input spectrum, estimate the peak inelastic displacement of the SDOF system.
5. Convert the displacement of the SDOF to the control node displacement of the structure. The result of the pushover analysis at that control node displacement is

considered to represent the peak demands under the considered earthquake.

Several assumptions are used in the N2 method. In step 2, the use of an invariant lateral force distribution implies that the higher mode effects are considered negligible and the inertial force distribution to remain constant for the entire duration of loading. In addition, the diaphragms are considered rigid, so that a physically meaningful location of the control node, typically at the centre of mass of the roof, can be identified. In step 3, the conversion to an equivalent SDOF system is conducted assuming a single mode response, with the inertial force distribution that is identical with the displacement shape. Finally in step 4, the peak inelastic displacement demand is typically estimated using an empirical formula, for instance Vidic et al. (1994), with its own assumptions.

### 2.2. Modal Pushover Analysis

Extensions of the conventional method in including the higher mode effects have been investigated, predominantly for tall buildings. In this study, Modal Pushover Analysis (MPA) of Chopra and Goel (2002) is considered as the representative of the methods that include higher mode effects. In comparison to the N2 method, the MPA requires multiple pushover analyses, with an invariant pushover force in each mode, proportional to the mass multiplied by the elastic mode shape. The responses (e.g. displacement, drift, plastic hinge rotation) resulting from the modal pushover analyses are combined together to obtain the total responses. The MPA contains two main assumptions. Firstly, it assumes that the coupling between the (elastic) modal coordinates resulting from yielding of the system to be negligible. This is reflected in the independent pushover analyses using invariant force distributions proportional to the elastic mode shapes. Secondly, the peak modal responses are combined using mode combination rules, which is strictly applicable for linear systems only. In addition, the MPA also considers the floor diaphragms to be rigid, so that a unique control node can be established, usually at the centre of mass of the roof.

### 2.3. Adaptive Capacity Spectrum Method

The adaptive procedures attempt to address the invariant force distribution used in the previous methods. They aim to capture the progression of damage by continually modifying the pushover force distribution based on the instantaneous state of the structure at each pushover step. In this study, one such adaptive pushover algorithm by Antoniou and Pinho (2004) is examined in combination with the Adaptive Capacity Spectrum Method (ACSM) by Casarotti and Pinho (2007). The ACSM defines the equivalent SDOF system at each pushover step, using a substitute-structure approach of the Direct Displacement-Based Design method (Calvi and Kingsley, 1997). In contrast to the N2 or the MPA method, the ACSM exhibits two stages of adaptive processes. Firstly, the pushover analysis is conducted adaptively, based on the mode shapes calculated from the tangent stiffness of the structure at each step. Secondly, the determination of the equivalent SDOF system is also adaptive as it is based on the deformed shape of the structure at each pushover step. Perhaps the most attractive aspect of the ACSM for structures with flexible diaphragms is that the definition of the equivalent SDOF does not require a physical location of the control node. Instead, the equivalent SDOF system is expressed in terms of the “system” displacement, mass and acceleration such that the SDOF system retains the same amount of work done on the multi-degree-of-freedom (MDOF) structure. However, the treatment of the higher-mode response is not as clear as the MPA. The effects of multiple modes can be included in the adaptive pushover algorithm, however, the use of the equivalent SDOF system at each pushover step essentially results in the response to be described by a single mode (i.e. the current deformed shape).

## 3. DEFINITION OF SIMPLE SYSTEM

Simple two-degrees-of-freedom systems as shown in Figure 1 are used this study. This model is considered to be the simplest idealised system able to express the salient response characteristics of structures with flexible diaphragms. The model consists of two oscillators, wall 1 and wall 2, representing the parallel in-plane walls with their attribute masses ($m_1$, $m_2$), stiffnesses ($k_1$, $k_2$), and damping properties ($c_1$, $c_2$). The two oscillators are coupled by a shear spring ($k_d$) representing the stiffness of the flexible diaphragm. The out-of-plane walls are neglected for simplicity, and the distributed mass of the flexible diaphragms is assumed to be negligible as far as the modes of vibration are concerned (the floor mass is "rigidly" lumped on the walls). The nonlinear behaviour of the in-plane walls are defined by a modified Takeda rule, while the diaphragm is considered to remain elastic.

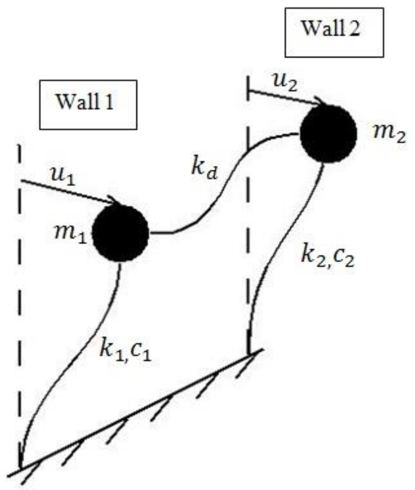



Figure 1. Simple 2-DOF system

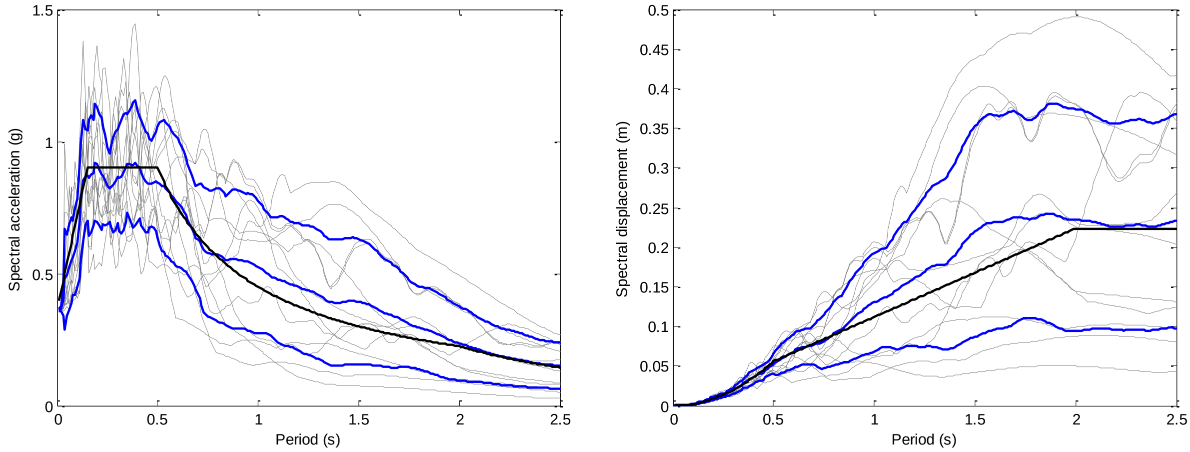
The behaviour of the system is governed by the following parameters:

1. Mass ratio, $R_m = \frac{m_2}{m_1}$. Note that mass ratio has been set to 1 in this study.
2. Period ratio of the uncoupled walls $R_T = \frac{T_2}{T_1}$. The uncoupled period is defined by $T_1 = 2\pi\sqrt{\frac{m_1}{k_1}}$ for wall 1. Similar definition applies to wall 2, and is denoted by $T_2$. Without loss of generality, we let $T_2 \leq T_1$ such that $R_T$ is bound between zero and one. Therefore, wall 1 may be referred to as the "flexible wall", and wall 2 as the "stiff wall". This condition is applied throughout this study.
3. The relative stiffness of the diaphragm to the total lateral stiffness of the system, $k_d/k_{tot}$, where $k_{tot} = k_1 + k_2$.
4. The reference period of the system, taken to be the period of the rigid diaphragm system, $T_{rig} = 2\pi\sqrt{\frac{m_1+m_2}{k_{tot}}}$.
5. Damping of the system, assigned to be 5% of critical damping for each mode at any value of $k_d/k_{tot}$.
6. Yield displacements of wall 1 and wall 2. These are specified using the parameters

$R_{y1}$ and $\alpha_1$. Specifically for wall 1, the yield displacement is given by $u_{y1} = \frac{u_{el1}}{R_{y1}}$ and for wall 2, $u_{y2} = \frac{u_{el2}}{1+\alpha(R_{y1}-1)}$, where $u_{el1}$ and $u_{el2}$ are the peak elastic displacement of uncoupled wall 1 and wall 2 respectively. $R_{y1}$ is the yield force reduction factor specified for the uncoupled wall 1, and $\alpha_1$ signifies the strength ratio of the two walls, defined as $\alpha_1 = \frac{R_{y2}-1}{R_{y1}-1}$.

## 4. GROUND MOTIONS

Twelve natural accelerograms were obtained from the European Strong-motion Database and Selected Input Motions for displacement-Based Assessment and Design (SIMBAD) using the REXEL software (Iervolino et al., 2009). The records were scaled to be compatible with Type 1 spectrum of Eurocode 8 on soil type B. Figure 2 shows the comparisons of the mean, mean plus and minus standard deviation spectra of the records with the target spectrum.




(a) Pseudo-acceleration spectrum (b) Displacement spectrum

Figure 2. Mean, mean plus and mean minus standard deviation spectra of accelerograms

## 5. METHODOLOGY

The analysed cases are summarised in Table 2. For the conventional N2 method, the first-mode proportional and the uniform pushover force distributions were investigated. For structures with flexible diaphragms, the location of control node is no longer unique. Hence two different possible control node locations were investigated, either on the flexible wall or the stiff wall. For the MPA, the control node of mode 1 was placed on the flexible wall, while that of mode 2 was placed on the stiff wall, in order to capture the dominant mode of each wall. The square-root-of-sum-of-squares (SRSS) modal combination rule was used for simplicity. For the adaptive pushover algorithm, the SRSS modal combination rule was used for the “spectral scaling” to include the higher mode effects in the adaptive pushover force variation. It is possible that the complete-quadratic-combination (CQC) rule could have provided more accurate predictions for these methods, when the periods of the two walls were close to each other (i.e. $R_T \approx 1$). Nevertheless, the results obtained using SRSS (Section 6) indicate the comparable accuracies for the case in which the initial wall periods are similar and the case in which they are very different. It is the authors’ opinion that the choice of the modal combination rule had little influence on the outcome of this study.

Table 2: Analysis variables for the comparison of nonlinear static methods

| Nonlinear static method | Pushover analysis | Location of control node | Comments |
|---|---|---|---|
| N2 | Invariant, first mode-proportional | Flexible wall | - |
| N2 | Invariant, first mode-proportional | Stiff wall | - |
| N2 | Invariant, mass-proportional | Flexible wall | - |
| N2 | Invariant, mass-proportional | Stiff wall | - |
| MPA | Invariant, first and second modes | Mode 1 - flexible wall<br>Mode 2 – stiff wall | SRSS combination |
| ACSM | Displacement-based adaptive | - | SRSS combination for spectral amplification |

In practice, the inelastic displacement demand (Step 4 of Section 2.1) is typically calculated from empirical expressions in conjunction with a code-defined elastic spectrum. In comparing the nonlinear static methods against time-history results, however, it is desirable to eliminate the errors associated with the use of a code-defined spectrum. Hence in this study, the inelastic displacement spectra were instead directly developed for each record. In deriving the inelastic displacement spectra, the modified Takeda hysteresis was used, and the viscous damping was set to 5% of critical.

Even though the inelastic displacement spectra were directly used in the calculation of the peak displacements, linear interpolation was used to obtain data points lying between the calculated period values. This resulted in certain errors in the estimation of the peak displacements of the equivalent SDOF systems. Assuming a normal distribution of the mean errors, the upper and lower bounds of the baseline errors were defined by the 95% confidence interval, and are indicated in the results.

## 5. RESULTS

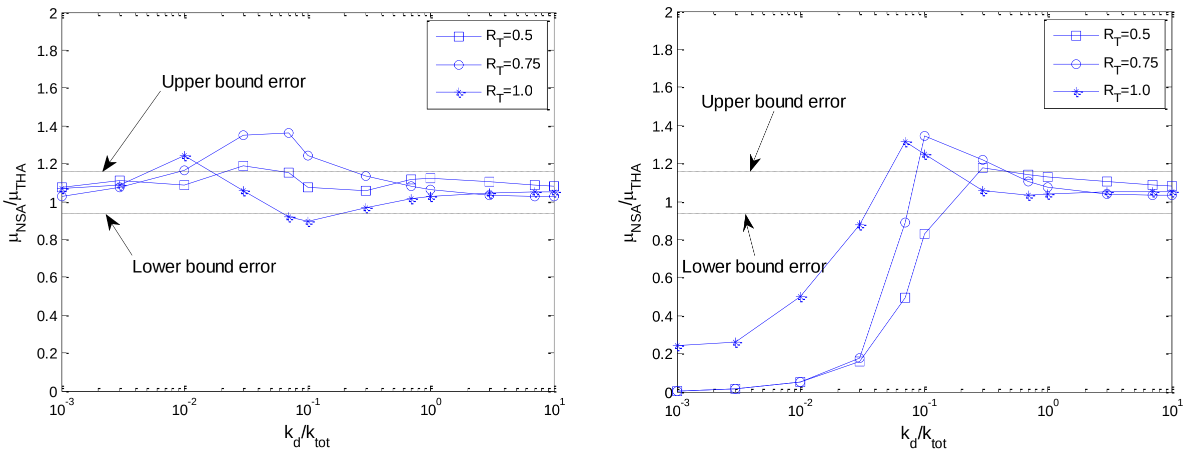
### 5.1. N2 method with first-mode proportional pushover force

Typical results of the N2 method with the first-mode proportional pushover force are shown in Figure 3 and Figure 4. The estimated median ductility values ($\mu_{NSA}$) are shown normalised to the median results of the time-history analysis ($\mu_{THA}$), and plotted against the diaphragm stiffness ratio, $k_d/k_{tot}$. Small values of $k_d/k_{tot}$ indicate flexible diaphragms. In Figure 3, the control node is placed on the flexible wall. Figure 4 shows the results when the control node is placed on the stiff wall instead. The results correspond to $T_{rig}$ of 0.27 s, $R_{y1}$ of 3 and $\alpha_1$ of 0.5. The dotted lines indicate the confidence interval on the mean error associated with the calculation of the peak displacement demand of the equivalent SDOF system, as described in Section 4.

The ductility demands of the walls are accurately predicted when the diaphragm stiffness is large. Furthermore, when the control node is placed on the flexible wall, the ductility demand of the flexible wall is rather accurately captured for all values of diaphragm stiffnesses. On the other hand, the method significantly underestimates the ductility demand of the stiff wall, for $k_d/k_{tot}$ approximately less than 0.1. The sudden reduction in the predicted ductility of the stiff wall highlights the limitation of the first-mode proportional pushover force. Since the response of the stiff wall becomes governed by the higher mode, while the response of the flexible wall remains in the fundamental mode as the diaphragm becomes flexible

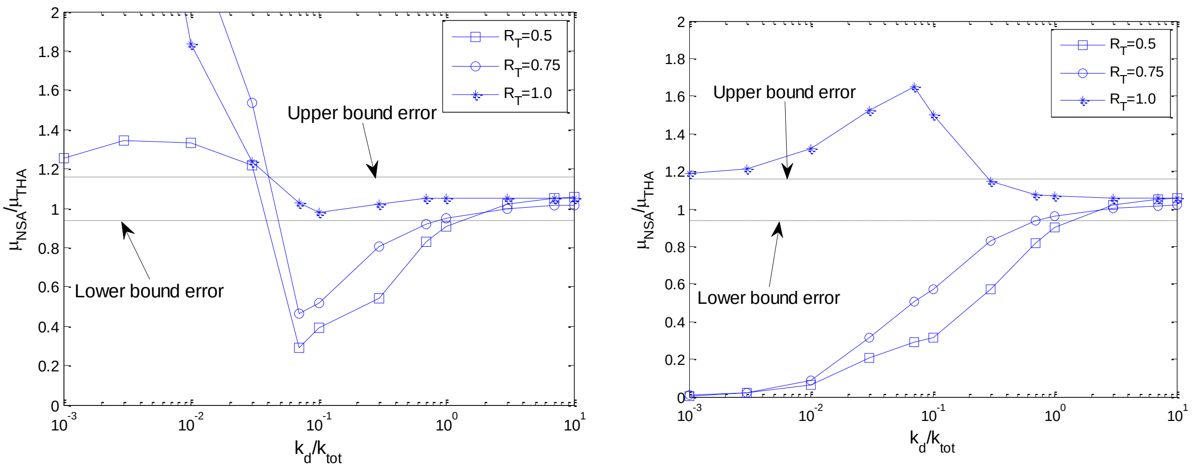
(Nakamura, 2014), the first-mode pushover force can significantly underestimate the demand imposed on the stiff wall.

When the control node is placed on the stiff wall, the method also gives accurate prediction when the diaphragm is relatively rigid, for $k_d/k_{tot}$ larger than approximately 1. However, the accuracy again reduces as the diaphragm becomes flexible. In contrast to the case when the control node is placed on the flexible wall, the prediction becomes erroneous for both walls.




(a) Wall 1 (flexible wall) (b) Wall 2 (stiff wall)

Figure 3. N2 method with first-mode proportional pushover and control node on flexible wall ($T_{rig}$ = 0.27 s, $R_{y1}$ = 3, $\alpha_1$ = 0.5)

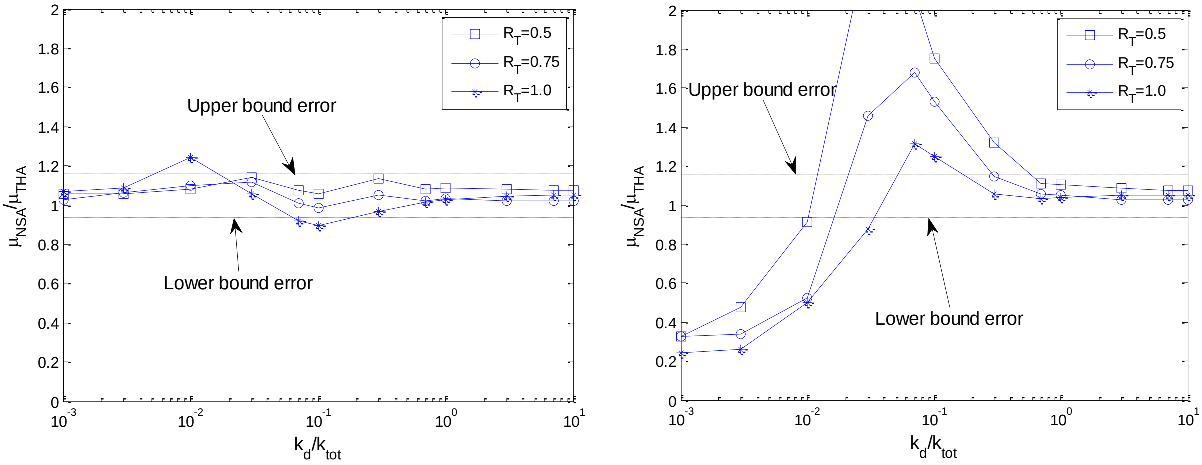



(a) Wall 1 (flexible wall) (b) Wall 2 (stiff wall)

Figure 4. N2 method with first-mode proportional pushover and control node on stiff wall ($T_{rig}$ = 0.27 s, $R_{y1}$ = 3, $\alpha_1$ = 0.5)

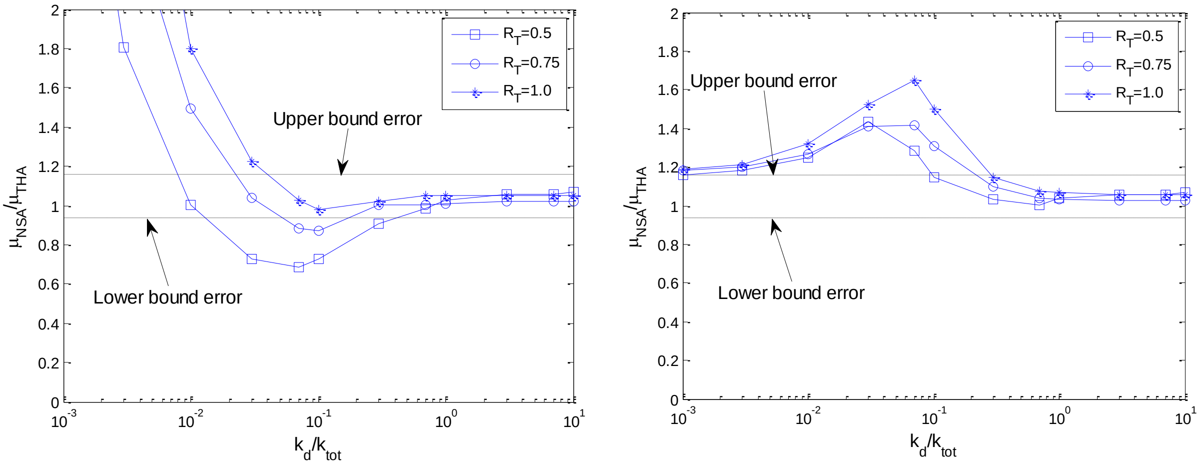
## 5.2. N2 method with uniform pushover force

Typical results of the N2 method with the uniform pushover force are shown in Figure 5 and Figure 6 when the control node is placed on the flexible wall and stiff wall respectively. The Figures correspond to $T_{rig}$ of 0.27 s, $R_{y1}$ of 3 and $\alpha_1$ of 0.5. The plots show that the response of the rigid diaphragm system can be estimated accurately. In addition, the importance of the location of the control node can be observed. As the diaphragm becomes flexible, the prediction is seen to diverge for the wall on which the control node is *not* placed. For the wall on which the control node *is* placed the prediction is rather accurate for a wide range of diaphragm stiffnesses, with some local deviations occurring in the range of $k_d/k_{tot}$ between 0.01 and 1.




(a) Wall 1 (flexible wall) (b) Wall 2 (stiff wall)

Figure 5. N2 method with uniform pushover and control node on flexible wall ($T_{rig}$ = 0.27 s, $R_{y1}$ = 3, $\alpha_1$ = 0.5)



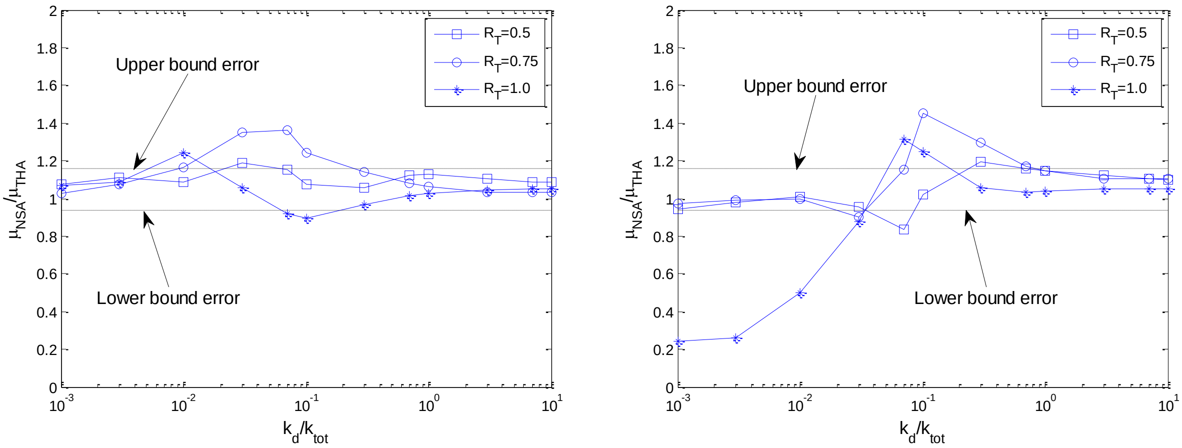

(a) Wall 1 (flexible wall) (b) Wall 2 (stiff wall)

Figure 6. N2 method with uniform pushover and control node on stiff wall ($T_{rig}$ = 0.27 s, $R_{y1}$ = 3, $\alpha_1$ = 0.5)

## 5.3. Modal Pushover Analysis

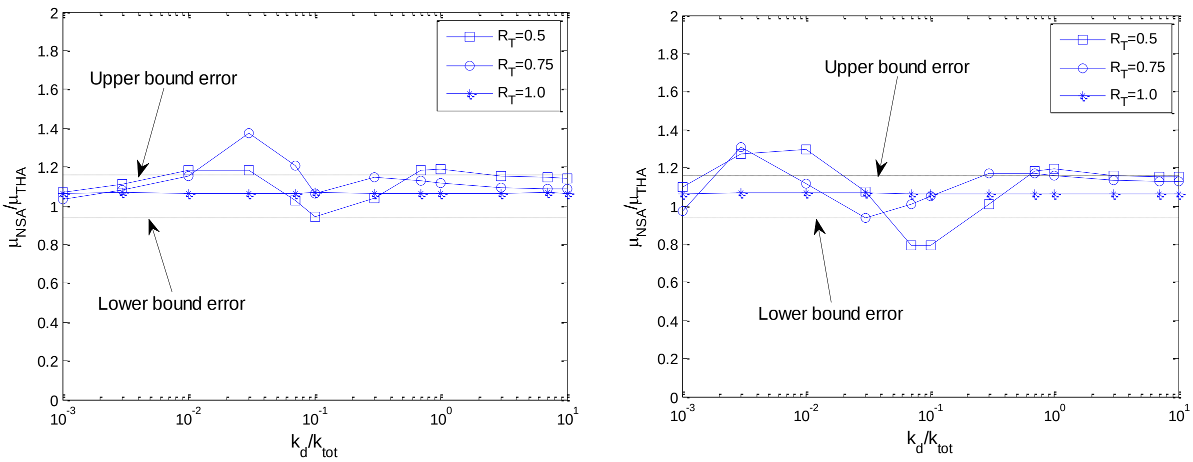
Figure 7 shows the median peak ductility of MPA normalised to the time-history results for $T_{rig}$ of 0.27 s, $R_{y1}$ of 3 and $\alpha_1$ of 0.5. Figure 8 shows the results corresponding to the same system parameters except $\alpha_1$ is set to 1. Both plots show more consistent predictions of the ductility demands for both walls, as compared to the conventional method. The predictions are also accurate for both the rigid diaphragm and completely flexible diaphragm conditions. For the $k_d/k_{tot}$ values approximately between 0.01 and 1.0, local deviations in the predictions are observed.

Even though the predictions are good in most cases, large errors can result when $R_T = 1$ and $\alpha_1 \neq 1$. The error arises in MPA in these cases because of its assumption that the uncoupled elastic modes can be used to estimate the inelastic "modal" responses. When $R_T = 1$ (with $R_m = 1$, as assumed throughout this study), the structure becomes symmetrical in the elastic range. The higher mode then comprises of the purely asymmetric mode, which does not participate in the elastic responses. In fact for a purely torsional mode, the pushover results in a zero base shear. However, such a mode can be important in the inelastic responses when there is a difference in the strengths of the wall, that is, when $\alpha_1 \neq 1.0$.




(a) Wall 1 (flexible wall) (b) Wall 2 (stiff wall)

Figure 7. Modal Pushover Analysis ($T_{rig}$ = 0.27s, $R_{y1}$ = 3, $\alpha_1$ = 0.5)

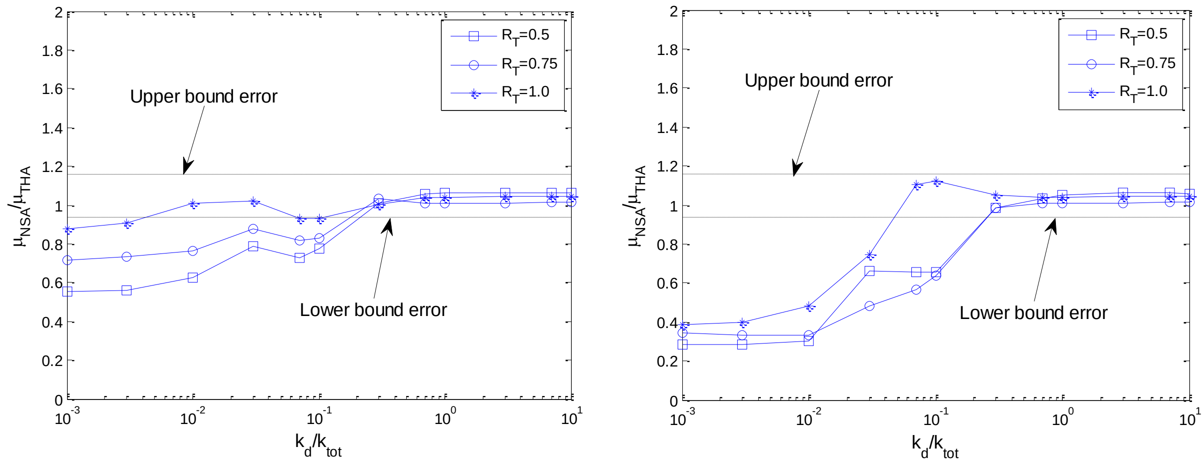



(a) Wall 1 (flexible wall) (b) Wall 2 (stiff wall)

Figure 8. Modal Pushover Analysis ($T_{rig}$ = 0.27 s, $R_{y1}$ = 3, $\alpha_1$ = 1)

## 5.4. Adaptive Capacity Spectrum Method

Figure 9 shows the median peak ductility calculated from the ACSM and normalised to the time-history results for $T_{rig}$ of 0.27 s, $R_{y1}$ of 3 and $\alpha_1$ of 0.5. Similar to the MPA, the ACSM gives more consistent results for both walls when compared to the conventional method with invariant pushover forces. The prediction, however, gradually becomes inaccurate as the diaphragm flexibility increases. Although the higher mode effects are included in the pushover force distribution, the method essentially considers a single-mode response at each pushover step. Hence when the diaphragm becomes flexible, the multi-mode nature of the response is not captured as well as in the MPA.




(a) Wall 1 (flexible wall) (b) Wall 2 (stiff wall)

Figure 9. Adaptive Capacity Spectrum Method ($T_{rig}$ = 0.27s, $R_{y1}$ = 3, $\alpha_1$ = 0.5)

## 6. SUMMARY AND CONCLUSIONS

The applicability of nonlinear static methods for structures with flexible diaphragms has been investigated using simple systems. The ductility demands of inelastic 2-DOF were estimated by the N2, the MPA and the ACSM procedures and compared with the results of the time-history analysis.

When the diaphragm stiffness was large, all methods estimated the peak ductility demands well. However, their accuracies reduced as the diaphragm became flexible.

The N2 method with the first-mode proportional pushover force resulted in severe underestimations of the stiff wall's ductility as the diaphragm flexibility increased. This was due to the separation of the stiff wall's response from the assumed first-mode response.

The N2 method with the uniform pushover force resulted in good correlations with the time-history results for the wall on which the control node was placed. Large errors were obtained, however, for the wall that did not have the control node.

The MPA provided a more consistent level of predictions for both walls as compared to the N2 method. In general, the predictions were accurate for both the rigid diaphragm and the completely flexible diaphragm conditions. However, local deviations in the ductility predictions were observed for $k_d/k_{tot}$ between 0.01 and 0.1. The assumption of the uncoupled modal coordinates lead to inaccurate results when the structure was symmetric in the elastic range ($R_T = 1$) but had strength irregularity ($\alpha_1 \neq 1$).

The ACSM also provided a consistent level of predictions for both walls. However, since the method essentially assumes a single-mode response at each pushover step, the error in the ductility estimation increased as the diaphragm flexibility increased. The method had the tendency to underestimate the ductility demands.

From the foregoing discussions, it appears that MPA provides the most promising framework on which to develop a pushover method suitable for unreinforced masonry buildings with flexible diaphragms.

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