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Analysis of brick veneer walls on steel frame subjected to seismic loads

Vidal P. Paton-Cole[1], Emad F. Gad[2] and Nelson T. K. Lam[3]

1. Corresponding Author. PhD student, Department of Infrastructure Engineering, The University of Melbourne, Parkville, VIC, 3010, Australia. Email: v.paton-cole@student.unimelb.edu.au

2. Professor, Faculty of Engineering and Industrial Sciences, Swinburne University of Technology, VIC, 3122, Australia. Email: egad@swin.edu.au

3. Associate Professor and Reader, Department of Infrastructure Engineering, The University of Melbourne, Parkville, VIC, 3010, Australia. Email: ntkl@unimelb.edu.au

Abstract

A brick-veneer wall consists of a single leaf non-structural masonry which is attached by metal ties to a steel or timber frame behind the wall. This form of construction features a diverse assemblage of materials with significant disparity in stiffness and ductility properties. In earthquake conditions, interactions between the veneer wall and the structural frame in the out-of-plane direction is a cause for concern in view of the risk of brick wall collapsing. This paper examines the complex interaction between the wall and frame based on full-scale shaking table tests and computer analysis. A full-scale one-room test house has been shown to perform well on the shaking table which was subjected to varying levels of ground shaking based on the 1940 *El Centro* ground motion. An analytical model representing the brick veneer wall assemblage has been subsequently developed on ANSYS and verified by comparison of the simulated time-histories with those recorded from the shaking table experiments. Results obtained from simulations by the verified finite element model are presented in this paper.

Keywords: structural frame, brick veneer, out-of-plane, earthquake loading, finite element model.

1.0 Introduction

In regions of low seismicity like Australia, damage to brick veneer wall in the out-of-plane direction have been observed in recent years resulting from strong wind events and moderate earthquake ground shaking (Melchers, 1990; Page, 1991). Most related wall damages have reportedly been associated with wall ties that were widely spaced, too flexible or poor workmanship with inadequate anchorage to the brick veneer or structural backing. Some failures were associated with corroded or missing ties. The vulnerability of this category of buildings was clearly exhibited following the poor performance of these structures during past earthquake events in New Zealand (Thurston and Beattie, 2008b; Priestley et al., 1979; Pender and Robertson, 1987). The 1987 Edgecumbe earthquake in New Zealand saw numerous failures related to ties and warranted the need to understand the seismic performance of brick veneer-tie interaction (Jacks and Beattie, 1990). Brick veneer wall failures have also been associated with wall ties in a nationwide survey conducted in Canada (Valsangkar, 1991). Following the 1994 Northridge earthquake, it was reported that many wood frame residential constructions with veneers suffered considerable damage (Bruneau, 1995). Most of the failures were generally attributed to poor connection and flexibility mismatch between the wood frame work and masonry, typically resulted in severe damage to the non-structural masonry. These reportedly poor performances highlight the significant role of the tie in the overall performance of a brick veneer wall system. However, despite its importance, design and detailing guidelines for brick veneer ties in regions of low seismicity are limited and are lacking in areas of moderate to high seismicity. Thus, there exists a gap in quantitative information on the seismic performance of such a method of construction. Failure of such non-structural components (brick veneer in this context) might be life threatening and in addition the economic losses and disruption to amenities can be great. Thus, the safety of such components in residential houses is not just important at a personal level but of benefit to the community.

Over the last quarter of a century, during which time brick veneer attached to steel studs have gained increased popularity, few studies have been conducted to better understand the behaviour of the wall system (Gad, 1997; Valsangkar, 1991; Drysdale and Breton, 1991; Yi et al., 2003; Arumala, 1991). While these studies try to establish the wall behaviour, a clear understanding of its interaction with the structural frame has not been thoroughly reported. Moreover, despite these reported studies are representative of brick veneer walls on steel stud system, they do not represent the characteristic properties and geometrical construction details that are representative of the high strength (G550) thin walled steel studs uniquely used in New Zealand and Australia. This study reported herein is therefore focussed on understanding the out-of-plane performance of brick veneer walls attached to steel stud frames with G550 characteristic properties (AS1397, 2001), designed in accordance with provisions in the design guide for residential steel-framed housing (NASH, 2005).

Results obtained from the extensive seismic test program (Paton-Cole et al., 2009) have been used as the basis to develop analytical models representative of brick veneer on steel stud wall systems. This paper is focussed on analytical studies to investigate the elastic and inelastic out-of-plane behaviour of brick veneer walls. Two dimensional (2D) finite element (FE) models have been developed to represent the full scale test house. The models were implemented in the finite element program ANSYS. Experimental results were used to calibrate the FE models and studies were undertaken to examine the out-of-plane behaviour of the walls subjected to design level earthquakes compliant with the New Zealand Earthquake Loading Standard (NZS1170.5, 2004). This study forms the basis for simulating the behaviour of a wide range of veneer walls and developing simple design procedures for suggested provisions to be incorporated in design codes of practices.

2.0 Summary of results from experimental study

Comprehensive details of the test setup, loading protocols and experimental results have been reported elsewhere (Paton-Cole et al., 2009; Paton-Cole et al., 2012). In the experimental study, a representative full scale structure was constructed in conjunction with standard New Zealand masonry construction (NASH, 2005; NZS4210, 2001). The primary objective was to examine the out-of-plane performance of the full scale brick veneer walls which were not connected at the four corners of the test house. A total of nine high level earthquake tests (up to 2.7 times El-Centro (EC)) were undertaken and observations made during the tests have been reported (Paton-Cole et al., 2009; Paton-Cole et al., 2011). Based on the experimental observations (Paton-Cole et al., 2009), it was concluded that the test house reported in this research exhibited extremely good performance.

Similar excellent performance of brick veneer steel-framed houses (constructed with same full scale components as those used in this study) cladded with 70 mm thick bricks and laterally supported by the steel frame via brick ties were reported in a reconnaissance earthquake survey report following the 4th September 2010 Darfield earthquake in New Zealand (Bruneau et al., 2010). In a more damaging event that struck Christchurch on 22nd February 2011, numerous out-of-plane collapse of brick veneer in cavity wall structures and in timber frame houses were reportedly observed (Buchanan et al., 2011). Brick veneer on light steel-frame was reported to generally perform exceptionally well (Clifton and MacRae, 2013; Clifton et al., 2011; Buchanan et al., 2011) with minimal hairline cracks to the internal linings and exterior cladding.

3.0 Analytical Finite Element (FE) modelling

Advanced FE modelling software ANSYS (ANSYS, 2009) was used to develop the FE models. The software has an extensive library of elements and material models capable of modelling the physical responses of various structural analyses. Most significantly, the program offers extensive dynamic and nonlinear analysis options that are deemed vital for simulating the dynamic behaviour of brick veneer walls that are supported by a steel stud frame. Since the out-of-plane walls that were of interest in the experimental test setup were disconnected at the corners, the possibility of a two-way bending action on the veneer wall was eliminated. The one-way bending action of the veneer wall was clearly demonstrated in the relative displacement obtained from the experimental results (Paton-Cole et al., 2009). On this basis, the three-dimensional (3D) test house was simplified into a two-dimensional (2D) FE model as shown in Figure 1. Two dimensional (2D) elastic beam elements with tension, compression, and bending capabilities were used to model the structural frame members and assumed to be linearly elastic. The structural frame members with “N” members were lumped together into a single beam and the sectional properties of the lumped section provided in the FE model. The elastic modulus for the steel studs was 200x10^3 MPa and Poisson’s ratio of 0.3. Being that the primary focus was on the out-of-plane behaviour of the veneer wall, the plasterboard was not explicitly modelled since its contribution in the out-of-plane direction is considered insignificant. A spring-damper element was used to represent the equivalent lateral stiffness offered by the in-plane walls. The higher roof mass was used as a basis for estimating the equivalent stiffness of the whole test house based on single-degree-of-freedom system modelling.

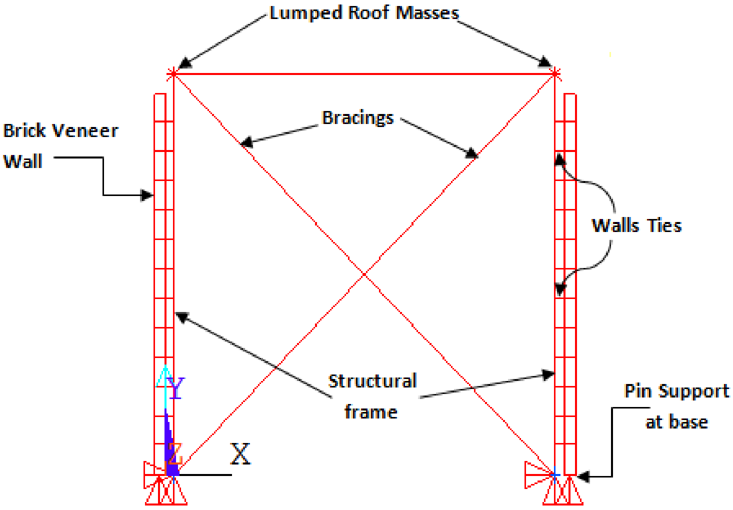
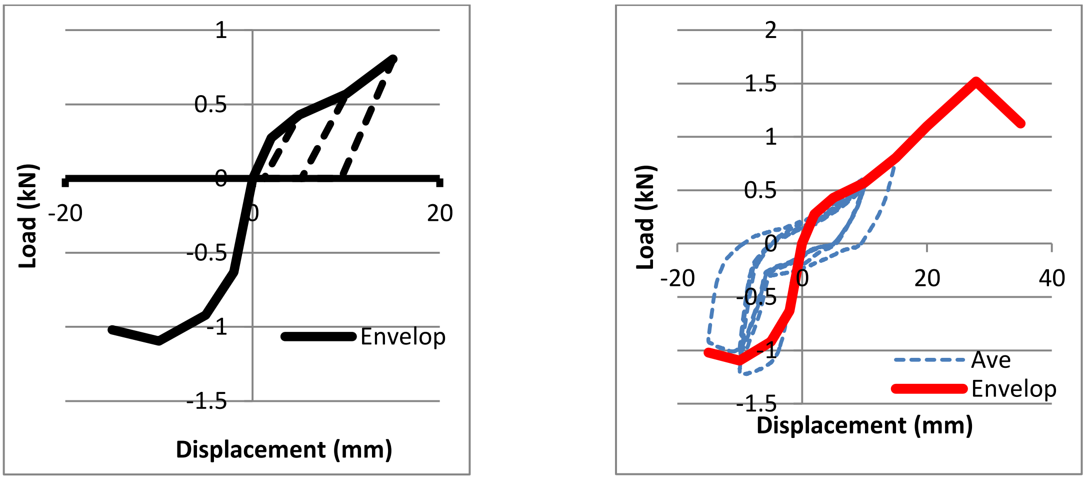


Figure 1: Full scale Finite Element model

A structural mass element was simplified into two lumped masses on top of the model to simulate the roof mass of 1500kg. Based on experimental shaking table tests, the viscous damping ratios of 3.5% and 5% were adopted and used in the numerical models. The 70mm thick veneer walls were modelled with beam elements adequate enough to capture the flexural behaviour of the brick veneer walls under out-of-plane loading. The relative displacements profiles from the experiment provide noticeable flexural deformation at mid-height (Paton-Cole et al., 2009). Therefore, the full thickness of the brick veneer wall modelled and represented with the equivalent section properties of the full scale walls was assumed to be linearly elastic. The weight of the wall was taken as 145kg/m^2 to comply with requirements for Masonry Construction practice in New Zealand (NZS4210, 2001). The elastic modulus for the brick veneer was taken to be 13.75 GPa based on prism test results (Heath, 2012). The mortar joints in the veneer walls were not explicitly modelled as the rigid body behaviour of the wall was of primary interest in the inelastic stage. The brick ties were modelled with spring elements which have nonlinear generalized force-deflection capability. The behaviour of the brick ties connections as shown in Figure 2 were typically representative of in-service conditions evaluated from a series of laboratory tests of component tie-stud connections.



(a) (b)

Figure 2: Idealised tie behaviour: (a) Displacement cycles up to ±15mm
(b) Ultimate tensile load behaviour

The studs and veneer walls were assumed pinned at the bottom. The base of the veneer was constructed without damp proof course which means that sliding along the base course was prevented. Observation during the experimental test shows no sign of failure of the mortar joint at the shaking table interface.

3.1 Evaluation of tie connections

The exceptional performance of the full scale brick veneer walls in the experiment was associated with the flexibility of the tie-stud connections (Paton-Cole et al., 2012). From the experimental investigation conducted based on the methodology prescribed by the tie standard (AS/NZS2699.1, 2000), the elastic deformation response of the tie connection was observed to be taken up by the flexing of the flange. This is consistent with experimental observations made during testing of the full scale experimental tests (Paton-Cole et al., 2009). The ultimate failure mechanism of the connection was straightening of ties prior to pullout of the screw from the stud flange. Laboratory measurements indicate straightening of the ties measured to a maximum of 18 mm while the stud flange deform approximately 8-10 mm. Due to the high deformation accommodated by the tie and its connections to the flange, it is therefore concluded that the flexibility of the connection of the ties to the flange of the studs assisted the out-of-plane veneer to survive the severe earthquakes to which they were subjected (Paton-Cole et al., 2009). The flexibility of the tie connections may have reduced the forces imposed on the veneer and yet the ties provided sufficient displacement capacity.

Estimate of the overall tie stiffness gave an average characteristic axial stiffness of 0.181 kN/mm. The ties can be rated as *"Medium Duty"* as per AS/NZS2699.1 (2000). It is evident that ties attached on to timber studs are much stiffer than ties on the "C" section studs. This reaffirms claims by the authors (Paton-Cole et al., 2009) that the tie evaluation procedure based on characteristic axial stiffness as per AS/NZS 2699.1 for ties attached to steel studs is overly conservative for it does not represent in-service condition of the tie connections.

4.0 FE analysis results

4.1 Vibration characteristics

Modal analysis was performed on the FE model in order to evaluate the dynamic modal parameters of the 2D representation of the test house. The primary parameters investigated were natural frequencies and mode shapes. The dynamic properties of the full scale test house were evaluated using the Ambient Response Testing and Modal Identification Software (ARTeMIS) as reported in Paton-Cole et al. (2011). These reported properties form the basis of calibrating and validating the analytical model. Based on modal analyses, four distinct modes of vibration were obtained as summarised in Table 1. The first mode was a lateral sway (racking) of the entire test house where the frame and the veneer walls moved back and forth together. The natural frequency of the first mode was 5.8Hz. The second mode was a sway mode of the out-of-plane walls only and had a natural frequency of 11.9Hz. In this mode, the brick veneer walls were simultaneously pulling and pushing the frame in opposite directions. This mode could be referred to as a *"clapping"* mode of the veneer. The third mode was also a sway mode of the out-of-plane walls but with the veneer walls moving together in the same direction. This mode had a natural frequency of 13.6Hz. The fourth mode was a flexural response of the brick veneer walls and had a natural frequency of 25.5Hz. The experimental and analytical dynamic characteristics shows close correlation and a clear indication of how well the FE model would be capable of predicting the behaviour of the test house and ultimately the out-of-plane behaviour of the brick veneer walls.

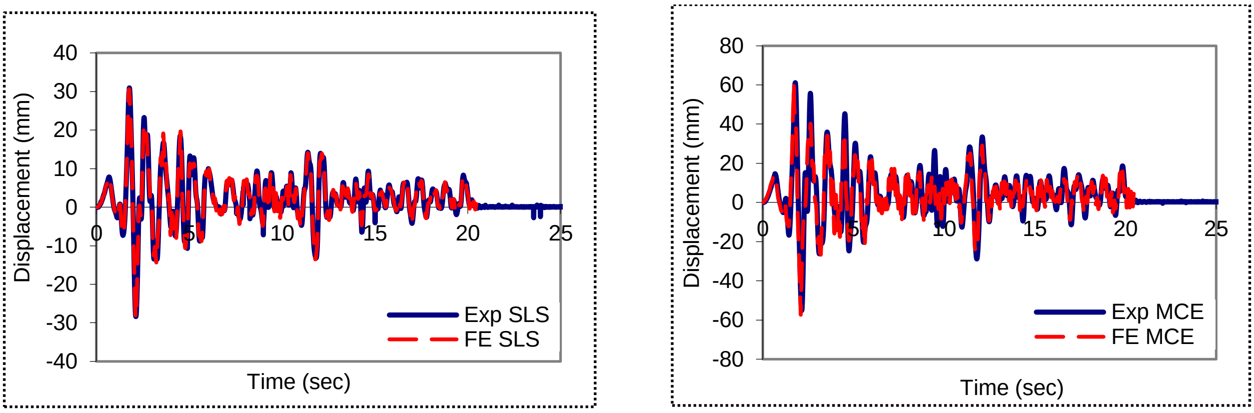
Table 1: Summary of modal frequencies

	Direction of testing	Mode of vibration	Natural Frequency (Hz)	
			ARTeMIS (Experiment)	ANSYS (FE model)
Full scale two-dimensionally analytical model	N-S [a]	First	5.8	5.8
		Second	-	11.9
		Third	13.5	13.6
		Fourth	26.8	25.5

[a] *North-South*

4.2 Response displacement

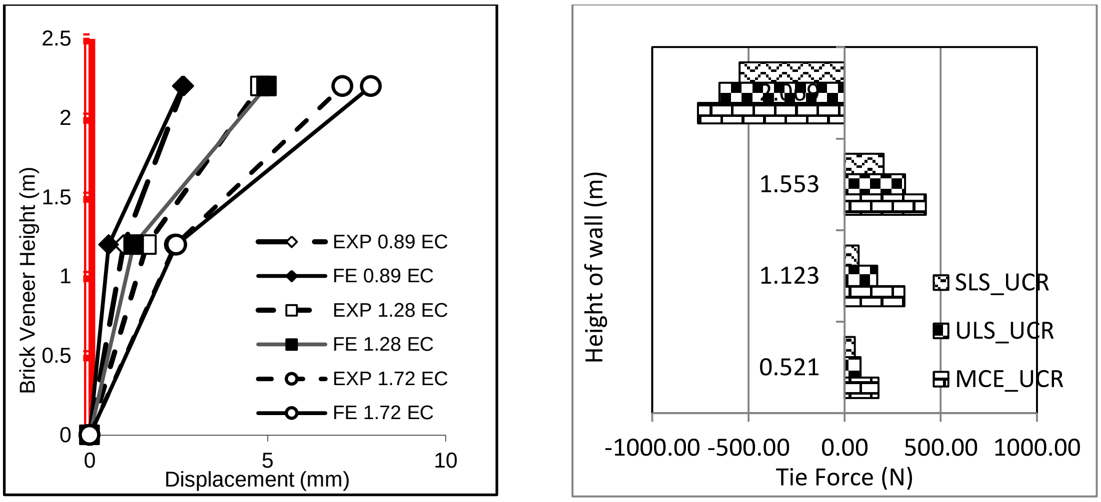
Upon calibration, the FE model was verified against the response displacement obtained from the shaking table experiments in order to validate the behavioural trends and response of the model under earthquake loads. The model was analysed with a sequence of ground motion (NZS1170.5, 2004) representative of the design level earthquakes (Severability Limit State SLS, Ultimate Limit State ULS and Maximum Considered Earthquake MCE) for New Zealand. To adequately represent the actual base excitation, the displacement time history recorded at the shaking table during the experiment were used as input to the base of the FE model. Evaluation of the peak response displacements (experimental and analytical) at the top of the test house shows a close correlation as shown in Figure 3. This justifies the accuracy of the FE model and its capability in predicting the response behaviour of the test house.



(a) (b)

Figure 3: Comparison of peak response displacement: (a) SLS (b) MCE

Given that only one way bending was considered, the behaviour of the veneer walls relative to the studs was influenced by the stiffness of the brick ties, the relative stiffness between the frame and veneer and the condition at the base support. To further understand how the veneer wall interacts with the structural frame under seismic load, the relative displacements and distribution of tie forces along the veneer wall were evaluated and compared against experimental results as shown in Figure 4. Both sets of results show very close match as indicated in Figure 4a. The maximum relative displacement between the frame and out-of-plane veneer wall at MCE was about 8mm which the ties and their connections could comfortably accommodate without any sign of distress. Experimental observations show there were no visible signs of damage to the out-of-plane veneer walls at this level of relative displacement. This explains how exceptional the tie and its connection performed whilst maintaining connectivity with the frame and veneer. As the veneer wall was restrained by the wall ties, the displacement demand in the lower wall ties were quite small compared to the top row of ties. As shown in Figure 4a, the relative displacement indicates that with increase in base excitation, the wall is likely to crack at mid-height. This is supported with preliminary analyses conducted in this study in which a wind load of 1 kPa applied to the veneer wall indicates maximum bending moment at around mid-height position. It implies that cracking will eventually occur at mid-height. The behaviour of the cracked veneer has also been investigated and discussed in Section 4.3.



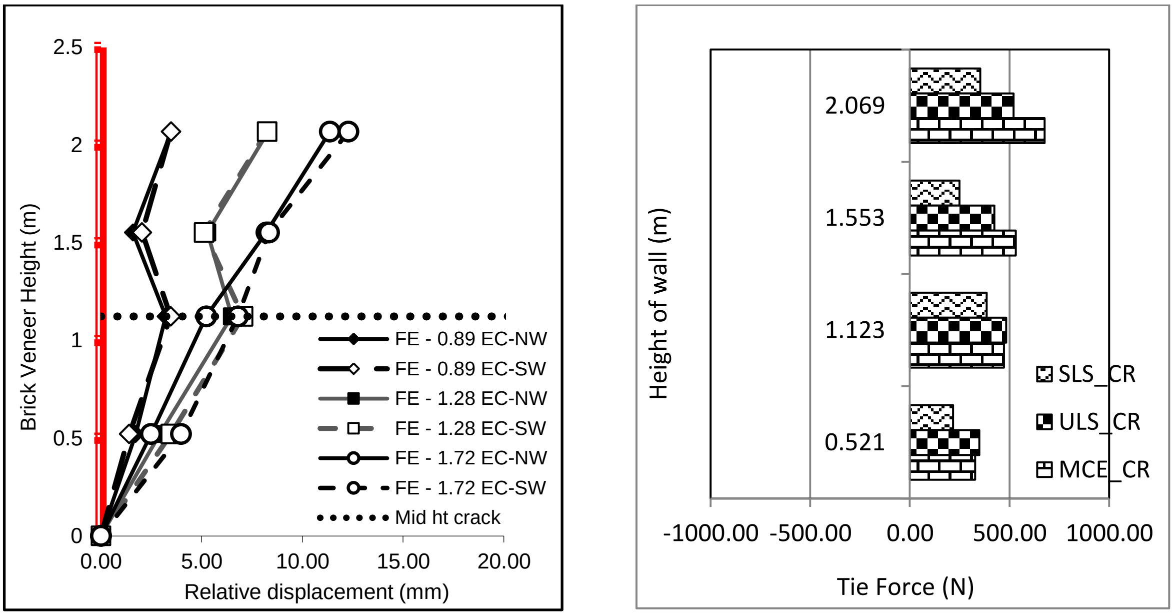
(a) (b)

Figure 4: Behaviour of brick veneer wall: (a) peak relative displacement envelope (b) tie forces from FE model

As indicated earlier, the brick ties connecting the veneer to the structural frame are the key elements in the brick veneer wall system. Premature failure of the ties would trigger an early out-of-plane failure. As shown in Figure 4b, the higher tie forces are induced in the top row of ties. The high displacement demand at the top of the walls (refer Figure 4a) accounts for the maximum forces in the top row of ties. Despite the smaller displacement in compression (veneer wall pushing against the stud) at the top, the compressive forces were actually higher because of the high stiffness of the tie connection (refer Figure 2). At the MCE level of shaking intensity (when a relative displacement of about 8 mm was obtained), the maximum tie forces are much smaller than the minimum characteristic residual strength specified by the tie standard (AS/NZS 2699.1) for heavy duty rated ties. This provides a substantive conclusion that the most of the tie forces are absorbed by the flange flexibility which further provides adequate support to the brick veneer wall. Thus, the tie standard (AS/NZS2699.1, 2000) is not appropriate for assessing ties connected to steel studs.

4.3 Behaviour of cracked brick veneer wall

When brick veneer walls are loaded in the out-of-plane direction, it can be conceded that the onset of cracking will commence in a horizontal bed joint. Once the wall is cracked, it is expected that the top half of the wall will rotate about the crack as a rigid body. The rigid body behaviour is obviously seen from the relative displacement profiles in Figure 4a. Based on this hypothesis, a discrete crack was introduced along the horizontal bed joint at mid-height in the veneer. In the past, similar approach has been employed in related studies conducted to evaluate brick veneer wall systems (Drysdale and Chidiac, 1989; Page et al., 1996; Arumala, 1988). In this study, the crack was modelled using *“Node-to-Node”* contact elements available in the ANSYS library. Figure 5a shows the displacement response profiles for the cracked brick walls. The numerical model reveals that once the veneer is cracked, the rigid body motion increases the displacement demand at mid-height. Most importantly, the significant observation from the numerical studies is the alteration of the tie forces along the cracked veneer wall as given in Figure 5b. The forces in the top row of ties were smaller for the low level intensity of excitation. With increase in earthquake intensity, it was observed that the rigid body mechanism ultimate reverses the higher displacement demand to the top as the collapse probability of the wall increases which would likely happen from the top end of the wall.



(a) (b)

Figure 5: Behaviour of cracked brick veneer wall: (a) peak relative displacement envelope (b) tie forces

Despite the wall is cracked, the corresponding relative displacement demand at mid-height of the wall under the MCE level of shaking intensity is approximately 7 mm. While noting that the increase in displacement demand at mid height will significantly increase the force in the structural backup frame, most of the displacement would be accommodated by the flexible flange of the stud. The assistance offered by the flexible tie-stud connection enables the wall to maintain its integrity without collapsing despite the high level earthquake. Such flexible behaviour will not occur if the tie-stud connection is stiffer. Thus, excessive displacement under rocking will have possible serviceability implications and will ultimately lead to a premature pullout failure of the ties either at the stud or veneer end. From the analysis presented in this study, it is clear that the cracked veneer wall is critical as it represents the worst case scenario of the brick veneer wall and should therefore be considered in engineering design of such residential structures.

5.0 Conclusions

A two-dimensional Finite Element model has been developed to simulate the behaviour of a single storey steel framed brick veneer structure that was experimentally investigated under out-of-plane seismic loads. The FE model consists of line elements with the structural frame and brick walls represented with linear elastic material properties. The brick ties whose idealised behaviour was obtained from small scale component tests were represented with nonlinear inelastic material models. Under out-of-plane earthquake loads, experimental and analytical analyses show that the flexible wall tie connections provides sufficient relative displacement capacity which assist the wall to accommodate higher deformation while maintaining its integrity without collapsing.

Analyses have shown that the top row of ties carry most of the load when the wall is uncracked. Once the wall is cracked, the mid-height ties support greater forces than top ties. It is therefore suggested that the cracked condition which represents the worst case scenario should be considered in the design of light framed residential structures.

Laboratory and analytical investigation have indicated that the stiffness requirements for earthquake rated ties in AS/NZS 2699.1 may be conservative. Ties attached to timber studs would exhibit much stiffer behaviour and would result in higher tie loads. This implies that the stiffness representation of ties on the timber studs is not applicable to ties on the steel studs. It is therefore suggested that the tie standard be revised for brick ties on the steel studs.

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