Analysis of influences of an irregular site with uncertain soil properties on spatial seismic ground motion coherency

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Abstract

Analysis of recorded ground motions showed that spatial coherency at a site with irregular subsurface topography is different from that at a flat-lying site. Furthermore, the properties of soil cannot be realistically considered as deterministic, as there always exist certain uncertainties in engineering practice. This paper investigates the effect of irregular topography and random soil properties on coherency loss of spatial ground motions on surface of a layered site. The random soil properties considered in the analysis are shear modulus, soil density and damping ratio of each layer, which are modelled by the independent one-dimensional random fields in the vertical direction, and all are assumed to follow normal distributions. The coherency loss function of the surface ground motion is derived in two steps: firstly, the ground motion time histories are generated based on the ground motion power spectral density functions derived from one-dimensional wave propagation with random site properties. Then, the coherency loss function on the generated surface ground motions is statistically derived by using Monte-Carlo simulation method. A numerical example is presented to illustrate the method proposed in the present paper. Numerical results show that coherency loss function of the spatial ground motions directly relates to the site amplification spectra ratio of the two local sites, the influences of irregular topography and random soil properties on the coherency loss function cannot be neglected.

Keywords: Coherency loss function; Irregular topography; Uncertain soil properties; Monte-Carlo simulation

1. Introduction

For some lifeline structures, such as long span bridges, pipelines, damns, communication transmission systems, their supports undergo different motions during an earthquake, which is known as the ground motion spatial variations. It has been recognized that the effect of spatial variation of seismic ground motions on responses of long-span structures cannot be neglected and in cases might even govern the structural responses [1]. Therefore it is important to reliably model earthquake ground motion spatial variations for structural response analysis. There are three main reasons give rise to the spatial variability of seismic ground motions [2]: (1) wave passage effect due to different arrival times of waves at different locations; (2) incoherence effect of seismic waves due to scattering in the heterogeneous medium of the ground; (3) local site effect owing to different local soil properties underneath each site. The wave passage effect and incoherence effect have been extensively studied by many researchers, and many empirical coherency models have been proposed especially after the installation of the SMART-1 array in Lotung, Taiwan. Zerva and Zervas [3] overviewed these models.

Most of these empirical coherency models were based on the data recorded by strong motion arrays located on approximately flat-lying alluvial site(e.g. SMART-1 array). In contrast, Somerville et al [4] analysed the coherency loss function on a site located on folded sedimentary rocks, and found that the spatial incoherence does not show a strong dependence on station separation and frequency. Moreover, there always exist uncertainties in defining the properties of soils. This results from the natural heterogeneity or variability of soils, the limited availability of information about internal conditions and sometimes the measurement errors. These uncertainties associated with system parameters are also likely to have influence on the responses. Researches of uncertain soil properties on the ground motion coherency are relatively less. Zerva and Harada [5] simplified horizontal stochastic layers of a site as a 1-DOF system with random characteristics to study the effect of soil stochasticity on the coherency function. They pointed out that the effect of soil layer's stochasticity should also be incorporated in spatial variation model because the variability in the soil characteristic will reduce the coherency function at the stochastic layer predominant frequency. It should be noted herein, that a 1-DOF system cannot realistically represent local site owing to the fact that multiple predominate frequencies exist corresponding to different modes of the site. Liao and Li [6] developed an analytical stochastic methodology to evaluate the seismic coherency function. In which, a numerical approach to compute coherency function is developed firstly by combining the pseudo-excitation method with wave motion finite element simulation techniques, then the orthogonal expansion method is introduced to study the effect of uncertain soil properties on the coherency function. The results also demonstrate that the coherency values tend to decrease in the vicinity of the resonant frequencies of the site. However, it is difficult and sometimes a little bit arbitrary to select the absorbing boundary conditions in this method, and it is difficult to explain why the coherency function varies significantly in a relatively short distance.

In a recent study, Bi and Hao [7] proposed a methodology to simulate the ground motions on a canyon site with multiple soil layers based on one-dimensional wave propagation theory and spectral representation method. This method also provides a feasible way to study the effect of irregular topography and random soil properties on the coherency loss functions. In this paper, the spatially varying bedrock motions are assumed to consist of out-of-plane *SH* wave or in-plane combined *P* and *SV* waves and propagate into a random layered soil site with an assumed incident angle. Uncertain soil properties considered are shear modulus, density and damping ratio of each layer, which are modelled as a one dimensional random field [8], and all follow normal distributions in the vertical direction. For each realization of the random fields, the soil properties are deterministic, hence, the ground motion time histories can be generated based on the method proposed in Ref. [7]. The coherency function of the motions on the ground surface is then statistically estimated based on the Monte-Carlo simulation method. A numerical example is presented to illustrate the effect of irregular site and uncertain soil properties on the coherency loss function.

2. Theoretical basis

2.1 Spatial Ground motion simulation at a site with deterministic soil properties

Consider horizontally extended multiple soil layers resting on an elastic half-space (bedrock), the spatially varying bedrock motions are assumed to consist of out-of-plane *SH* wave or in-plane combined *P* and *SV* waves and propagate into the deterministic layered soil site with an assumed incident angle. The bedrock motions at different locations are assumed to have the same power spectral density, and are modelled by a filtered Tajimi-Kanai [9] power spectral density function or other stochastic ground motion attenuation models. The bedrock spatial variation is modelled by an empirical coherency loss function. The cross power spectral density function of surface motions at *n* locations of the layered site can be written as:

$$S(i\omega) = \begin{bmatrix} S_{11}(\omega) & S_{12}(i\omega) & \cdots & S_{1n}(i\omega) \\ S_{21}(i\omega) & S_{22}(\omega) & \cdots & S_{2n}(i\omega) \\ \cdots & \cdots & \cdots & \cdots \\ S_{n1}(i\omega) & S_{n2}(i\omega) & \cdots & S_{nn}(\omega) \end{bmatrix} \tag{1}$$

where

$$S_{ii}(\omega) = \left|H_i(i\omega)\right|^2 S_g(\omega) \qquad i = 1,2,...,n$$

$$S_{ij}(i\omega) = H_i(i\omega)H_j^*(i\omega)S_g(\omega)\gamma_{i'j'}(d_{i'j'}, i\omega) \qquad i, j = 1,2,...,n \tag{2}$$

are the auto- and cross-power spectral density function respectively. $S_g(\omega)$ is the ground motion power spectral density on the bedrock; $\gamma_{i'j'}(d_{i'j'}, i\omega)$ is the coherency loss function between location i' and j' on the bedrock; $H_i(i\omega)$, $H_j(i\omega)$ are the site transfer function at support *i* and *j* on the ground surface, which can be formulated based on the one-dimensional wave propagation theory [10]. According to this theory, the out-of-plane motion is independent of the in-plane motion. Hence, the transfer functions for the out-of-plane motion and in-plane motion can be formulated independently.

Decomposing the Hermitian, positive definite matrix $S(i\omega)$ into the multiplication of a complex lower triangular matrix $L(i\omega)$ and its Hermitian $L^H(i\omega)$

$$S(i\omega) = L(i\omega)L^H(i\omega) \tag{3}$$

the stationary time series $u_i(t), i = 1,2,...,n$, can be simulated in the time domain directly

$$u_i(t) = \sum_{m=1}^{i}\sum_{n=1}^{N} A_{im}(\omega_n)\cos[\omega_n t + \beta_{im}(\omega_n) + \varphi_{mn}(\omega_n)] \tag{4}$$

where

$$A_{im}(\omega) = \sqrt{4\Delta\omega}\left|L_{im}(i\omega)\right|, \qquad 0 \le \omega \le \omega_N$$

$$\beta_{im}(\omega) = \tan^{-1}\left(\frac{\mathrm{Im}[L_{im}(i\omega)]}{\mathrm{Re}[L_{im}(i\omega)]}\right), \qquad 0 \le \omega \le \omega_N \tag{5}$$

are the amplitudes and phase angles of the simulated time histories which ensure the spectrum of the simulated time histories compatible with those given in Eq. (1); $\varphi_{mn}(\omega_n)$ are the random phase angles uniformly distributed over the range of $[0, 2\pi]$,

φ_{mn} and φ_{rs} are statistically independent unless $m = r$ and $n = s$; ω_N represents an upper cut-off frequency beyond which the elements of the cross power spectral density matrix given in Eq. (1) is assumed to be zero; $\Delta\omega$ is the resolution in the frequency domain, and $\omega_n = n\Delta\omega$ is the *n*th discrete frequency.

2.2 Random field theory

The properties of each soil layer are deterministic in the method proposed in Ref. [7]. However, in engineering practice there are always some uncertainties in the soil properties. In order to describe the variability of soil properties, a random field theory [8] is widely used. In this theory the random soil property $u(z)$ is characterized by the mean value $\bar{u}$, standard deviation σ_u and the correlation distance δ_u. σ_u measures the intensity of fluctuation or degree to which the actual value of $u(z)$ may deviate from. δ_u measures the correlation level or persistence of the property from one point to another in a site, small values of δ_u suggest rapid fluctuation about the average, while large values of δ_u imply a slowly varying component is superimposed on the average value of $\bar{u}$.

Consider a one dimensional random field $u(z)$ with mean value $\bar{u}(z)$ and standard deviation σ_u, its local average process $u_Z(z)$ of $u(z)$ over the interval Z centered at *z* is defined as:

$$u_Z(z) = \frac{1}{Z}\int_{z-Z/2}^{z+Z/2} u(z')dz' \tag{9}$$

It can be seen that the local average $u_Z(z)$ will vary depending on the specific location of the interval *z* within the statistically homogeneous soil layer. The mean and variance of $u_Z(z)$ are

$$\begin{gathered} E[u_Z(z)] = E[u(z)] = \bar{u}(z) \\ Var[u_Z(z)] = \sigma_u^2 \lambda(Z) \end{gathered} \tag{10}$$

where $\lambda(Z)$ is variance reduction function of $u(z)$, which measures the reduction of point variance σ_u^2 under local average. The variance function $\lambda(Z)$ can be derived from auto-correlation function $\rho_u(\Delta z)$ in the following form

$$\lambda(Z) = \frac{2}{Z}\int_0^Z (1 - \frac{\Delta z}{Z})\rho_u(\Delta z)d(\Delta z) \tag{11}$$

By using the exponential auto-correlation function [11]

$$\rho_u(\Delta z) = \exp(-2|\Delta z| / \delta_u) \tag{12}$$

the variance reduction function can be derived as

$$\lambda(Z) = \frac{1}{2(Z/\delta_u)^2}\left[2(Z/\delta_u) + e^{-2(Z/\delta_u)} - 1\right] \tag{13}$$

2.3 Monte-Carlo simulations

Monte-Carlo simulations have been widely used in many scientific fields including random parameters. It was found that for the range of variability usually present in soil properties, Monte-Carlo based method, though computationally intensive, is the simplest and most direct method. Other methods, which are basically expansion based, do not provide accurate results when the coefficient of variation of soil properties are large [12].

In this study, the shear modulus, density and damping ratio of each soil layer of the site are regarded as random fields, and are assumed to follow normal distributions. These random fields can be modelled by introducing the mean value, standard deviation and correlation distance of each parameter as mentioned in section 2.2. Take shear modulus for example, $G=\overline{G}+\sigma_G\sqrt{\lambda(Z)}\phi=\overline{G}(1+COV\times\sqrt{\lambda(Z)}\phi)$, where $\overline{G}$ and σ_G is the mean value and standard deviation of shear modulus, $\lambda(Z)$ is the variance reduction function and ϕ is a normal distributed random process with zero mean and unity variance. $COV=\sigma_G/\overline{G}$ is the coefficient of variation. For each realization of the random fields, the parameters are deterministic, so the ground motion simulation methodology in section 2.1 can be incorporated to generate ground motions at different locations of the site.

The lagged coherency function, $\gamma_{ij}(\omega)$, between ground motions i and j is given by

$$\gamma_{ij}(\omega)=\frac{\left|S_{ij}(i\omega)\right|}{\sqrt{S_{ii}(\omega)S_{jj}(\omega)}} \tag{14}$$

in which $S_{ij}(i\omega)$ is the smoothed cross-power spectral density function of the ground motions between station i and j, $S_{ii}(\omega)$ and $S_{jj}(\omega)$ are the corresponding smoothed auto-power spectral density functions. An 11-point Hamming window is used to smooth the spectra. Eq.(14) can be used to estimate the lagged coherency loss function of any two ground motions. Taking random soil parameters into consideration, ground motions at locations i and j according to different soil properties are firstly generated and the mean lagged coherency function can be derived based on the Monte-Carlo simulation method.

3. Numerical example

A canyon site with multiple soil layers resting on an elastic half space is selected as an example (Fig. 1), in which h is the layer depth, G shear modulus, ρ density, ξ damping ratio and υ Poisson's ratio. The mean values of these parameters for each soil layer are indicated in the figure. Without losing generality, shear modulus, soil density and damping ratio are assumed random in all soil layers, and all follow a normal distribution. According to a more specific review and summary [13], in most common field measurements, the COV for the cohesion and undrained strength of clay and sand are among 10% to 60%. The statistical variation of the soil density is, however, relatively small as compared with other soil parameters. In the present study, the COV of shear modulus and damping ratio of all soil layers are assumed to be 20%, 40% or 60%, while the COV of soil density is assumed to be 5%. Moreover, based on the limited data, for soil properties in most field measurements, the vertical correlation distances are between 1-5m, the correlation distance is assumed to be 4m in this paper. For comparison, coherency loss functions on the ground surface with deterministic properties and on the base rock are also presented.

The motion on the bedrock is assumed to have the same intensities and frequency contents and is modelled by the filtered Tajimi-Kanai power spectral density function

$$S_g(\omega) = \frac{\omega^4}{(\omega_f^2 - \omega^2)^2 + (2\omega_f \omega \xi_f)^2} \frac{1 + 4\xi_g^2 \omega_g^2 \omega^2}{(\omega_g^2 - \omega^2)^2 + 4\xi_g^2 \omega_g^2 \omega^2} \Gamma \tag{15}$$

where $\omega_g = 10\pi$ rad/s, $\xi_g = 0.6$, $\omega_f = 0.5\pi$, $\xi_f = 0.6$ and $\Gamma = 0.0034$ m^2/s^3.

The Sobczyk model [14] is selected to describe the coherency loss between the ground motions at points i' and j' ($i \neq j$) on the bedrock:

$$\gamma_{i'j'}(i\omega) = \left|\gamma_{i'j'}(i\omega)\right| \exp(-i\omega d_{i'j'} \cos\alpha / v_{app}) = \exp(-\beta \omega d_{i'j'}^2 / v_{app}) \cdot \exp(-i\omega d_{i'j'} \cos\alpha / v_{app}) \tag{16}$$

where β is a coefficient which reflects the level of coherency loss, $\beta = 0.0005$ is used in the present paper, which represents highly correlated motions; $d_{i'j'}$ is the distance between the points i' and j', and $d_{i'j'} = 100$ m is assumed; α is the incident angle of the incoming wave to the site, and is assumed to be 60°; v_{app} is the apparent wave velocity in the bedrock, which is 1768 m/s according to the bedrock property and the specified incident angle.

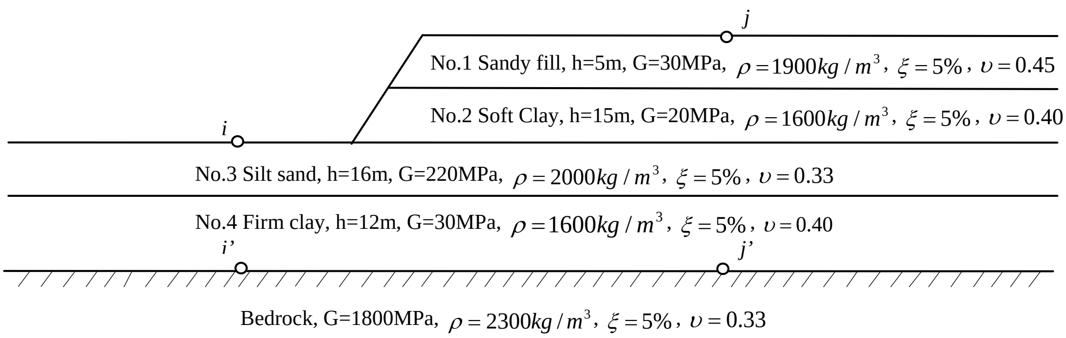

Figure 1. A canyon site with multiple soil layers

Fig.2 shows the coherency loss functions of spatial ground motions in the three directions on the ground surface. Compared to the coherency loss function on the base rock, the coherency loss unction on the ground surface of a canyon site is totally different. The values on the ground surface are smaller than that on the bedrock at every frequency. This is, however, as expected, the lagged coherency measures the similarity of the motions at two different locations, if site amplifies the ground motions at the same extent, the coherency loss is caused by incoherence effect and wave passage effect only, local soil site has no effect on the lagged coherencies. However, if site amplification spectra are different at different locations, the local site effect decreases the similarity of the motions on the ground surface compared with that on the bedrock, so the values are smaller. As also shown in Fig.2, many peaks and troughs appear when the canyon site is considered. For the out-of-plane motion, four obvious troughs can be obtained around frequencies 0.78, 1.84, 4.20 and 7.10 Hz, this is because the amplification spectra ratio between locations j and i ($\left|\frac{H_j(i\omega)}{H_i(i\omega)}\right|$) vary significantly at the vicinity of these frequencies as shown in Fig.3a, the evident mean amplification spectra differences cause the obvious variations of the lagged coherency between motions on the ground surface and at the bedrock. Similar results can be observed for the in-plane horizontal and vertical motions (Fig.2b, Fig.2c, Fig.3b and Fig.3c). It is interesting to note that larger coefficients of variation of soil properties in general lead to smaller lagged coherencies between the motions on the ground surface, but could result in larger coherency values at certain frequencies corresponding to smaller mean amplification spectra ratios shown in Fig.3. This is expected because smaller amplification spectra ratios indicate less extent of differences between two considered local sites at the corresponding frequencies.

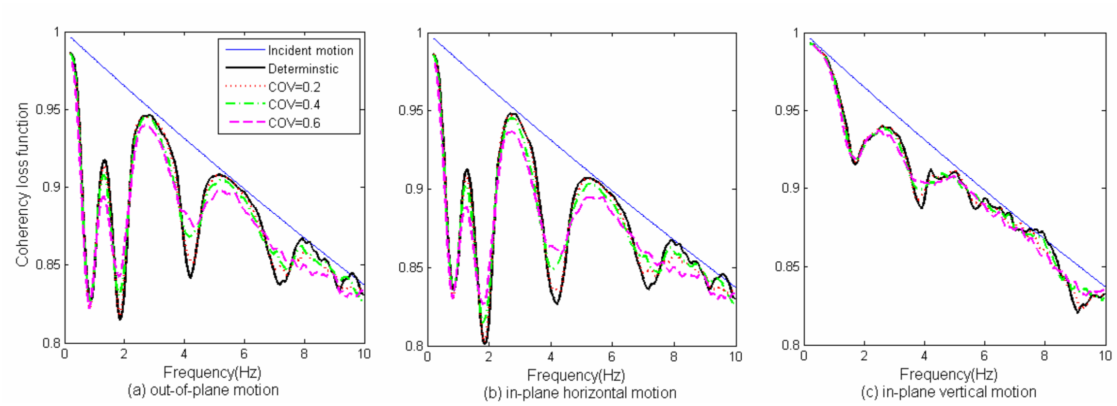

Figure 2. Effect of random soil properties on coherency loss function of a canyon site

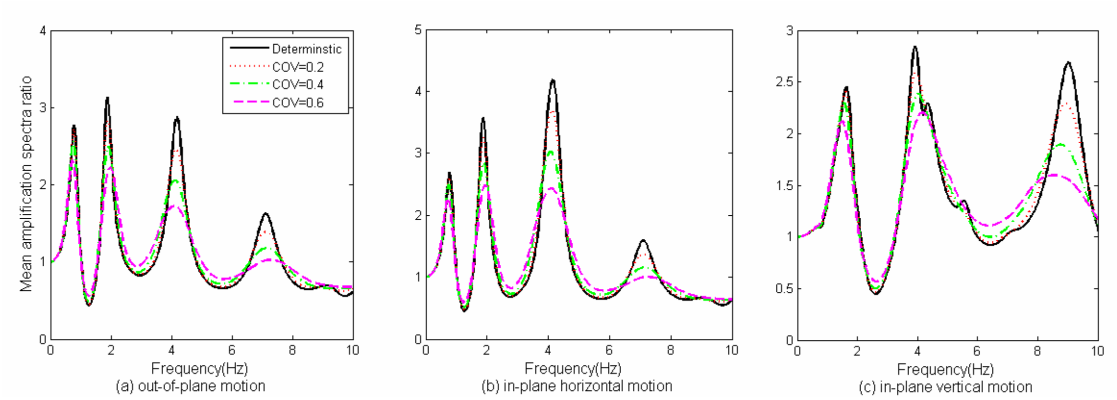

Figure 3. Effect of random soil properties on spectra ratio of a canyon site

4. Conclusions

This paper investigates the effects of irregular topography and random soil properties on the coherency loss function between spatial ground motions on surface of a layered soil site. The shear modulus, density and damping ratio of each soil layer are considered as random variables in the analysis, are assumed to follow normal distribution, and modelled by the one-dimensional random fields in the vertical direction. It is found that the coherency loss function between spatial motions on ground surface directly relates to the site amplification spectra ratio of the two considered locations. Obvious decrease in coherency loss function can be observed in the vicinity of frequencies where the amplification spectra ratio of the two sites varies significantly. Larger coefficient of variation of the soil properties in general result in smaller coherency loss values, but may cause slightly larger coherency loss values at frequencies where the amplification spectra ratio of the two sites is small. The results presented in the paper demonstrate that irregular topography and random soil properties of the local site have significant effect on the coherency loss functions of spatial ground motions.

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