Response of Tall Building Structures using Panel Elements with In-Plane Rotational Stiffness

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Abstract

Generally, strain based finite elements are useful to assess the response of tall buildings. It should be noted that use of many lower order finite elements may reveal some analytical problems in analysis of tall buildings. The absence of an appropriate in-plane rotational stiffness in some finite elements and the existence of parasitic shear effects in the governing displacement functions of the element are of the most known effects which may appear in the finite element analysis of tall building structures. The proposed finite elements in this study, have been developed based on the strain functions which can model the general behaviour of shear wall panels and wide column elements in a tall building. Furthermore, based on using the assumption of horizontal strains which are negligible in all heights of the element, a unified lateral displacement will be obtained for the proposed panel elements. Hence, the rigid body motion of the floor slabs is simply employed to the mathematical formulation of the proposed elements. Yet, two rotational degrees of freedom which denote in-plane rotations are also defined at both chords of the element. The proposed panel elements can be simply used in modelling and analysis of tall buildings. Results of some analysed structures are given to denote the accuracy and efficiency of the proposed panel elements.

Keywords: high-rise building, stain function, strain based finite element, drilling degree of freedom, shear-flexure interaction, coupling effect.

1. INTRODUCTION

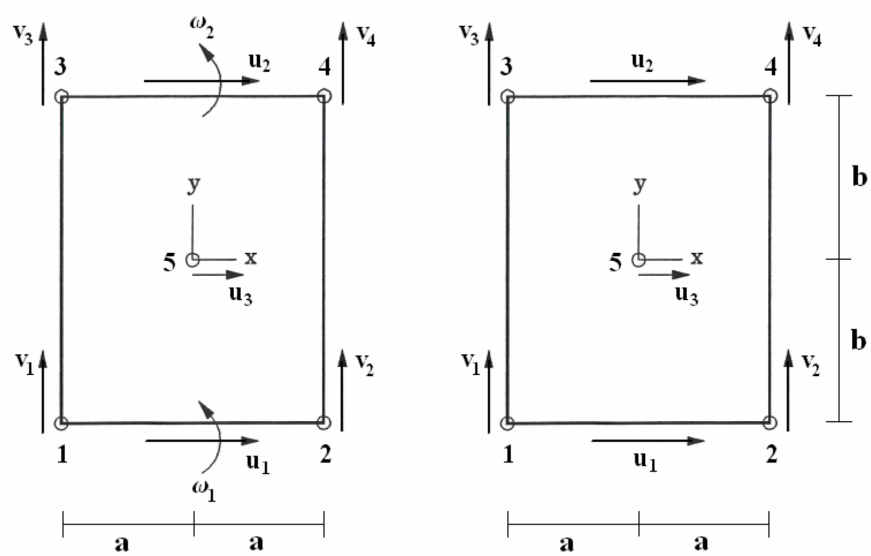
The advantage and benefit of application of coupled shear walls as well as shear cores in symmetric and asymmetric high-rise buildings to resist against lateral forces have already been recognized. It is noteworthy to mention that every structural asymmetry with plane in tall buildings may lead to large floor and storey eccentricities. The existence of these conditions causes considerable torsional effects in overall behaviour of high-rise structures (Tso *et al* 1990 & 1986, Makarios 2005, Georgosis 2006). There are many computational methods which have been developed in order to analyse the structural behaviour of symmetric and asymmetric high-rise buildings. These analytical procedures are known as the continuum method with basis of closed form solution, the frame method which is classified in the wide column and the solid wall analogies, the finite element and the finite strip methods (Coull *et al* 1991, Macleod *et al* 1977, Kim *et al* 2005, Cheung *et al* 1998).

Based on researches, it can be concluded that application of the finite element method to analyze tall building structures is generally categorized in using displacement-based and strain-based finite elements. As a general conclusion, it is important to note that application of all developed finite elements is not straightforward in the analysis of high-rise buildings. Some major problems are involved. The probable existence of parasitic shear effects in lower order plane stress elements, the exact definition of in-plane rotational freedom at each node as well as the inefficiency of using lower order finite elements to model the coupling beams in the structural system of tall buildings are some of the most famous problems in this respect (Macleod 1969, Kwan *et al* 1994, Oztorun 2006, Paknahad *et al* 2007).

Two developed strain based finite elements are presented in this paper. The presented finite elements were developed based on the governing displacement functions of a beam element and its corresponding strain functions. Both of these strain-based finite elements can show the internal shear–flexure interaction of shear wall panels. The degrees of freedom which were defined to both finite elements include external and internal horizontal translations as well as four vertical translations. The analytical concept which was used for the definition of drilling degrees of freedom is the rotation of vertical fibbers at connecting nodes. The presented finite elements can be simply used to model and analyse tall building systems.

2. FORMULATION OF THE PRESENTED FINITE ELEMENTS

The strain based elements P**S9** and P**S7** which are presented in this paper, have been developed based on the governing displacement function of a simple beam element. These elements are shown in Figure 1. The nodal degrees of freedom associated with the element P**S9** include one horizontal and two vertical translations as well as one in-plane rotation which are defined at both chords of this element. There is one translational degree of freedom u_3 which is defined at the central node of the element. The finite element P**S7** has seven degrees of freedom similar to the element P**S9** with the exception of two in-plane rotational freedoms ω_1, ω_2. The definition of the degree of freedom u_3 is based on developing two higher-order finite elements with respect to previous researches by Ha *et al* (1989) and Kwan (1992). As shown in Figure 1a, the degrees of freedom $u_1, \omega_1 = -\partial u_1/\partial y, v_1, v_2$ and $u_2, \omega_2 = -\partial u_2/\partial y$, v_3, v_4 are defined at lower and upper chords of the panel element P**S9**, respectively. Furthermore, two drilling degrees of freedom ω_1 and ω_2 are defined as $\omega = -\partial u/\partial y$, which are either of the vertical fiber rotation at the connecting nodes between beam elements and shear wall panels or wide column elements in tall building structures. As shown in Figure 1b, no in-plane rotational degrees of freedom are defined for panel element PS7. Therefore it is needed to use transition elements to produce rotational coupling effects and rigid body motion of floor slabs in analyzing the behavior of tall buildings (Kwan 1991). The degrees of freedoms which have been defined to element P**S7** are $u_1, v_1, v_2, u_2, v_3, v_4, u_3$.



(a) Panel Element PS9 (b) Panel Element PS7

Figure 1 The presented strain based panel elements PS9 and PS7

It is needed that governing displacement functions u(y) and v(x,y) which denote lateral and vertical displacement of the panel elements PS9 and PS7**,** should satisfy three principal analytical conditions according to the finite element method. Both functions u(y) and v(x,y) must be formulated in such a way which should be able to denote strain-free rigid body motion and compatibility within the finite element. In addition to these conditions, it is needed that the governing displacement functions could be able to show the strain distribution corresponding to pure bending behaviour. This is an essential improved characteristic for both finite elements PS9 and PS7 which denotes their behaviours are free of parasitic shear effects (Meshkat-Dini *et al* 2002 & 2007). It is important to note that satisfaction of the third condition causes the tall building analyses should be done without using fine meshes. Equating the lateral, vertical and shear strain functions i.e. $\varepsilon_x, \varepsilon_y, \gamma_{xy}$ to zero and integrating of them will lead to the displacement functions associated with the rigid-body motions of the finite element. These displacement functions are defined by equations (1a) and (1b).

$$u(y) = \alpha_1 - \alpha_3 y \tag{1a}$$

$$v(x,y) = \alpha_2 + \alpha_3 x \tag{1b}$$

As the displacement functions associated with the panel element PS7 must be formulated based on the seven desired degrees of freedom, hence it is needed to seven independent constants α_i corresponding to $u_1, v_1, v_2, u_2, v_3, v_4, u_3$ as shown in Figure 1b. According to the formulation of a simple beam element, the other four constant coefficients α_i are apportioned among the governing strain functions $\varepsilon_x, \varepsilon_y, \gamma_{xy}$ as given in equations (2a) to (2c).

$$(\varepsilon_x)_{PS7} = 0 \tag{2a}$$

$$(\varepsilon_y)_{PS7} = \alpha_4 + \alpha_5 x \tag{2b}$$

$$(\gamma_{xy})_{PS7} = \alpha_6 + \alpha_7 y \tag{2c}$$

It is worth mentioning that according to use similar procedure as mentioned above for the panel element PS9, there are also six constant coefficients α_i which are needed to distribute among the three strain functions $\varepsilon_x, \varepsilon_y, \gamma_{xy}$. These coefficients were distributed as follows;

$$(\varepsilon_x)_{PS9} = 0 \tag{3a}$$

$$(\varepsilon_y)_{PS9} = \alpha_4 + \alpha_5 x + \alpha_6 xy \tag{3b}$$

$$(\gamma_{xy})_{PS9} = \alpha_7 + \alpha_8 y + \alpha_9 y^3 \tag{3c}$$

It is obvious that both constant coefficients α_7 and α_9 are corresponded to degree of freedom u_3 in the panel elements PS7 and PS9, respectively. For both presented elements ε_x is set to zero because of using the assumption of horizontal strains are approximately negligible in all heights of the panel element. This is an applicable simplification based on the assessment of overall behaviour of several analysed tall buildings (Taranath 1998). Furthermore, equations (2a) and (3a) represent the rigid floor diaphragm assumption which must be used in tall building modelling. The coefficient α_4 in equations (2b) and (3b) denotes a constant strain distribution along the centroidal axis of both panel elements in Figure 1. The second statement in equation(2b) indicates a linear variation of ε_y with respect to y-direction, meanwhile the term $\alpha_5 x + \alpha_6 xy$ in equation (3b) represents a linear variation of bending strains in both x and y directions.

Based on γ_{xy} functions as given in equations (2c) and (3c), an independent linear shear strain function corresponding to ***Euler-Bernoulli*** beam element as well as a simple higher-order function for γ_{xy} based on the ***Timoshenko*** beam theory have been defined, respectively. Obviously, it is found that two constant coefficients α_6 and α_7 are remained to describe the changes of shear strains within the element PS7. It should be also noted that three constant coefficients α_7 , α_8 and α_9 have been used to determine function γ_{xy} base on four internal rotational deformations θ_1, θ_2 (i.e. $\partial v/\partial x$)

and ω_1, ω_2 (i.e. $\omega = -\partial u/\partial y$) which describe horizontal and vertical fiber in-plane rotations at the upper and lower chords of the panel element PS9, respectively. The horizontal and vertical fiber in-plane rotations which were mentioned above are described based on formulation of higher order beam elements. It is worth mentioning that the beam elements with or without internal degrees of freedoms can be generally used in the formulation of both wide column and solid wall panel elements in the frame method (Smith et al 1984 & Kwan 1993& Prathap 1982 & Reddy 1997).

The governing strain fields of both panel elements PS7 and PS9 satisfy the internal compatibility of strains within the element. It should be noted that these two strain variations as defined in equations (2) and (3) describe a pure bending state. It is essentially based on the general beam behaviour. Both bending and shear strain functions are analytically independent. Therefore, it can be predicted that based on using the elements PS7 and PS9, there will be no spurious shear-flexure interaction mode effects in the analysis of tall buildings. The existence of this characteristic causes that the proposed panel elements can be used in coarse meshes and the numerical procedure converges fast.

3. DETERMINATION OF GOVERNING DISPLACENMENT FUNCTIONS

The constant coefficients α_1, α_2 and α_3 in equations (1a) and (1b) represent the rigid body motion associated with both panel elements PS7 and PS9. By equating the lateral and bending strain expressions of equation (2) to the strain-displacement relationships of $\varepsilon_x = \partial u/\partial x, \varepsilon_y = \partial v/\partial y$ and integrating, then the displacement functions u(y), v(x,y) are obtained.

$$[u(y)]_{PS7} = \alpha_1 - \alpha_3 y + f_1(y) \tag{4a}$$

$$[v(x,y)]_{PS7} = \alpha_2 + \alpha_3 x + \int \varepsilon_y dy + f_2(x) \tag{4b}$$

Two complementary functions $f_1(y)$ and $f_2(x)$ are obtained according to the shear function γ_{xy} as given in equation (5).

$$\left(\frac{\partial f_1(y)}{\partial y}\right)_{PS7} + \left(\frac{\partial f_2(x)}{\partial x}\right)_{PS7} = -\alpha_5 y + \alpha_6 + \alpha_7 y \tag{5}$$

It is also seen that both $f_1(y)$ and $f_2(x)$ are an exact function of y and x respectively. Hence, it is concluded from equation (5) the following statements.

$$f_1(y) = -\frac{1}{2}\alpha_5 y^2 + \alpha_6 y + \frac{1}{2}\alpha_7 y^2 \tag{6a}$$

$$f_2(x) = 0 \tag{6b}$$

The full displacement functions u and v are given in equations (7).

$$[u(y)]_{PS7} = \alpha_1 - \alpha_3 y - \frac{1}{2}\alpha_5 y^2 + \alpha_6 y + \frac{1}{2}\alpha_7 y^2 \tag{7a}$$

$$[v(x,y)]_{PS7} = \alpha_2 + \alpha_3 x + \alpha_4 y + \alpha_5 xy \tag{7b}$$

The complementary functions $f_1(y)$ and $f_2(x)$ associated with the panel element PS9 are obtained based on using similar procedure as mentioned above. The resulting statement for $f_1(y)$ is a higher order polynomial of variable y and $f_2(x)$ is also equal to zero.

$$f_1(y) = -\frac{1}{2}\alpha_5 y^2 - \frac{1}{6}\alpha_6 y^3 + \alpha_7 y + \frac{1}{2}\alpha_8 y^2 + \frac{1}{4}\alpha_9 y^4 \tag{8}$$

All constant coefficients α_i in two governing displacement functions u(y), v(x,y) associated with both panel elements PS7 and PS9, are determined by equating the nodal translations and rotations to the defined degrees of freedom for each of finite elements and solving the system of equations thus obtained. Following this approach as well as using equations (4a) and (4b) will lead to the strain-displacement matrix [B] associated with the elements PS7 and PS9. The strain-displacement matrix [B] indicates an analytical relationship between the strain vector $\{\varepsilon\}$ and the displacement vector {D} of the finite element. This analytical relationship is symbolically noted in equation (9).

$$[\varepsilon_x \quad \varepsilon_y \quad \gamma_{xy}]^T = [B]\{D\} \tag{9}$$

Considering the above equation, the displacement vector {D} corresponding to both elements PS7 and PS9 are as follows;

$$\{D\}_{PS7} = \{u_1 \quad v_1 \quad v_2 \quad u_2 \quad v_3 \quad v_4 \quad u_3\}^T \tag{10a}$$

$$\{D\}_{PS9} = \{u_1 \quad \omega_1 \quad v_1 \quad v_2 \quad u_2 \quad \omega_2 \quad v_3 \quad v_4 \quad u_3\}^T \tag{10b}$$

It is noteworthy to indicate that based on using the finite element method, the lateral displacement function u(y) associated with the panel element PS7 is quadratic in the y direction. The function u(y) associated with the panel element PS9 is also of order of four with respect to y direction. Furthermore, there is also no common coefficient α_i to link between the bending and shear strain functions which are given in equations (2) and (3). Hence, it can be concluded that both elements PS7 and PS9 are free of parasitic shear effects. Generally, the absence of common constant coefficients α_i between ε_y and γ_{xy} removes undesirable stiffening effects which may occur in the numerical procedure of the finite element method. (Cook 1975). The stiffness matrixes of the elements PS7 and PS9 are evaluated by the standard expression as noted in equation (11).

$$[K] = t\int_A [B]^T.diag[E \quad E \quad G].[B]dA \tag{11}$$

$$[D_m] = diag[E \quad E \quad G] \tag{12}$$

It should be noted that because of using the assumption $\varepsilon_x = 0$, the material matrix $[D_m]$ will convert to a diagonal matrix of rank three as noted in equation (12). The parameter t is the thickness of the panel element shown in Figure 1. The strain-displacement matrix [B] is also obtained according to the standard finite element method.

4. NUMERICAL EXAMPLES

4.1 NATURAL FREQUENCIES OF COUPLED SHEAR WALL STRUCRUE

Both panel elements PS7 and PS9 have been used to assess the natural frequencies of a uniform coupled shear wall as shown in Figure 2. The total height of the example structure is 95.00m, storey height is 3.80m, both wall widths are 6.00m and the free span of connecting beams is also 2.00m. The cross section of all connecting beams is 30x30cm and the wall thickness is also 30cm respectively. The dynamic characteristics of this coupled shear wall have been already analysed by Kuang *et al* (1998), Savassi (2002). A number of analyses were performed using computer program TALBA (Meshkat-Dini 2006) based on some analytical finite element models. Table 1 shows a number of first natural frequencies of vibrations associated with the example structure. It is important to note that only one layer of elements is used per each story except for the first storey which modelled by a mesh includes two layers of elements. It is worth mentioning that Kuang's study is based on the discrete-continuum methodology and Savassi's research has been also accomplished based on the one dimensional finite element analysis. Therefore, it is acceptable to see differences among the results from various researches.

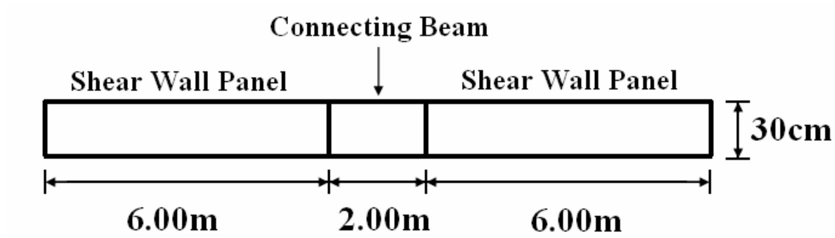


Figure 2: Dimensions of 25 storey coupled shear wall structure

Table 1: Natural frequencies (Hz) of the example coupled shear wall structure

	Vibration Mode Number				
	1	2	3	4	5
The Panel Element PS7	0.75	3.45	8.33	14.41	23.58
The Panel Element PS9	0.71	3.15	8.01	14.05	22.63
Kuang *et al* 1998 (The Discrete-Continuum Method)	0.67	2.93	7.16	13.28	21.44
Savassi 2002 (The Finite Element Method)	0.66	2.91	7.11	13.18	21.29

4.2 SHEAR CORE SUPPORTED BUILDING

The fifteen storey shear core supported building as shown in Figure 3 is considered in this study. It is important to note that because of the structural asymmetry which is introduced by the access opening, there are considerable torsional and warping stresses under the effects of lateral loads. This asymmetric structure has been already analyzed by Smith *et al* (1972), Macleod *et al* (1977), Ha *et al* (1989) and Pekau *et al* (1996). These researches have been done based on using various analytical methodologies. The structural dimensions and loading are shown in Figure 3. The story height is 3.81 m, lintel beam depth is 0.457m, Young's modulus E = 2.76×10^4 MPa It should be noted that just one layer of the elements PS7 and PS9 is used per each story of the structure.

Table 2 shows the structural rotation ϕ_{Top} with respect to the shear center of central core as well as the vertical stresses $\sigma_A, \sigma_B, \sigma_C$ which are noted at the foundation level. The agreement between the results from the panel elements PS7 and PS9 and those of other researches is almost exactly. Nevertheless, it can be seen small differences among the results for corresponding parameters shown in Table 2 with the exception of σ_B. The main source of the differences among the results can be explained based on the conceptual principles and analytical assumptions which have been considered in each of the methodologies.

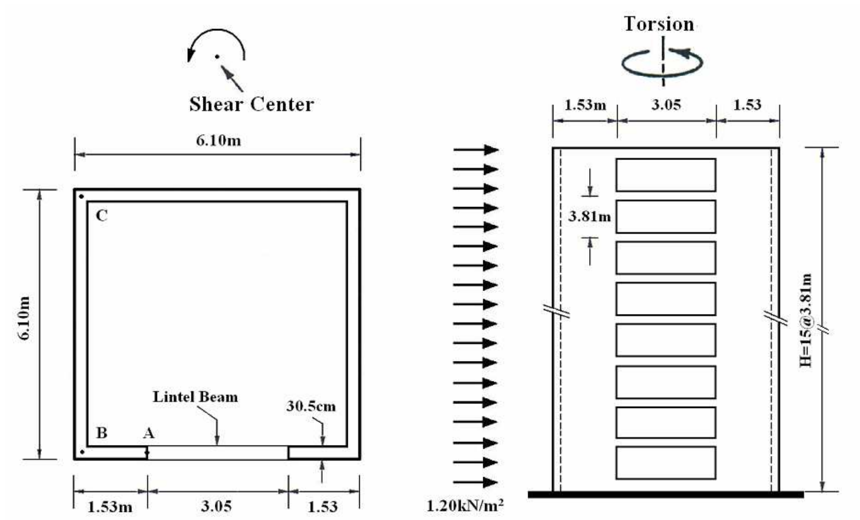


Figure 3: Shear core supported building

Table 2: Analytical results for the shear core supported building

	ϕ_{Tot} (10^{-3}rad)	σ_A (kg/cm^2)	σ_B (kg/cm^2)	σ_C (kg/cm^2)
The Panel Element PS7	**3.41**	**-32.6**	**8.7**	**44.5**
The Panel Element PS9	**3.49**	**-37.1**	**8.4**	**45.2**
Smith *et al* 1972 (The Continuum Method) (Open section structure)	**2.95**	**-36.8**	**4.1**	**45.3**
Smith *et al* 1972 (The Continuum Method) (Closed section structure)	**3.32**	**-36.2**	**4.3**	**44.9**
Macleod *et al* 1977 (The Frame Method)	**2.67**	**-44.3**	**6.3**	**40.7**
Ha *et al* 1989 (6DOFD Finite Elements)	**2.85**	**-35.4**	**6.9**	**39.7**
Ha *et al* 1989 (6DOFS Finite Elements)	**3.38**	**-35.2**	**7.7**	**42.2**
Pekau *et al* 1996 (The Finite Storey Method)	**3.70**	**-37.3**	**5.0**	**47.4**

5. CONCLUSIONS

Application of many of lower order finite elements in tall building analysis may lead to some considerable errors in analytical results. This is because of the problem of parasitic shear which renders the lower order finite element too stiff under action of bending mode especially in coarse meshes. The existence of parasitic shear effects in a number of lower order plane stress finite elements arises from their inability to represent a bending mode deflection. The presented finite elements PS7 and PS9 are able to represent the state of pure bending and are also free of parasitic shear effects. The lateral displacement function of both presented elements is not linear with height and can show a lateral deflected shape of order of two and higher with respect to variable y.

The existence of these characteristics causes that both presented elements PS7 and PS9 should be able to denote a shear-flexure interaction mode which governs tall building behaviour. The efficiency and capability of both presented strain based elements are demonstrated by applying them to analyse shear wall supported tall buildings in coarse meshes. The results obtained from application of both presented elements are in good agreement with the corresponding results of other researches. Furthermore, it is generally needed to one layer of the finite elements PS7 and PS9 per each storey of lateral load resistant systems in tall buildings.

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