1. INTRODUCTION

The Theatre of Beneventum (now called Benevento, located at 32 miles Northeast of Naples) has been investigated in this project. The theatre was built in the early 2nd century AD during the high point of the Roman Empire. It is one of the most representative examples of the achievement of Empire theatres and is still considerably well preserved.

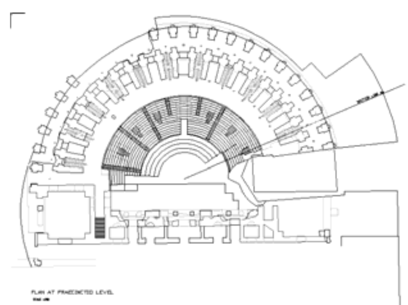
Since part of the theatre (mostly façade) is no longer present and one possible reason for this is that it collapsed in an earthquake, a seismic assessment has been carried out to investigate the performance of the structure when subjected to seismic forces. Through the analysis, the members most vulnerable to the seismic force are identified and the mechanism of the partial structural destruction has been discussed.

2. THE THEATRE of BENEVENTUM

2.1 Existing Structure

A detailed survey has been conducted on the theatre in terms of its dimension and level of damage. Several drawings have been made according to the survey. Based on the available data from the survey, a reconstruction of the initial layout of the theatre has been accomplished. Located in the west part of town, the existing theatre is approximately 93 m in diameter and 16.7 m in height. It is composed of two levels of seating (*cavea*) and the remains of one circumferential wall at the top. Then the *cavea* is further divided into wedges of seats by means of staircases. Numerous radial barrel vaults at every level support the cavea. There are also staircases providing access between different levels. Furthermore, an annular passage way is located around the building perimeter at each level. The passageways are supported by a series of cross vaults. In summary, the seating is supported on an intricate complex of annular and radial barrel vaults that reached a height of nearly 15m.

(a)

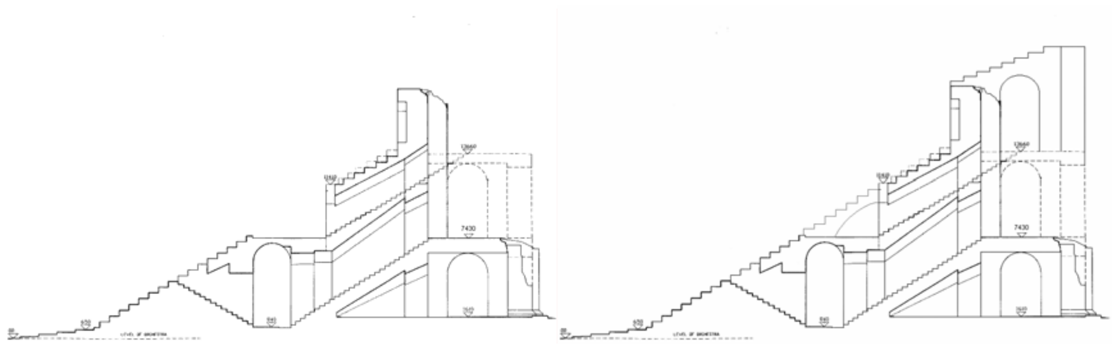


(b)

Figure 1. (a) The seating of the Theatre and b) Plan of the Theatre

2.2 Original Structure

A reconstruction of the original structure has been made on basis of the work done by an archaeologist (Sear, 2000). It is suggested that the seating at the time of the Roman Empire was usually divided into three levels, which are called *ima cavea*, *media cavea* and *summa cavea* from bottom to top respectively. Hence, there should have been another level of annular passage-way above the existing building and the seating area would extend above it, as shown in Figure 2 (b). The height of the reconstructed level has been determined by referring to other Roman Empire theatres built at the same period.



(a) (b)

Figure 2. (a) Existing elevation of the theatre and (b) Reconstructed elevation of the theatre

3. RESPONSE SPECTRUM ANALYSIS

Because of the complexity of the structure, a few assumptions have been made before the analysis is carried out. They are:

1) the concrete walls are the only structural elements;
2) the stone façade, which is much thinner than the concrete walls and has many openings, is assumed to be non-structural;
3) the façade has fallen and the reason for this might be that it was subjected to a seismic force in the transverse direction which caused out-of-plane failure, hence only loading in this direction has been considered;
4) the structures are divided into 3 levels with mass lumped at each one; the gravity load is assumed to be G + Q;
5) rigid diaphragms are assumed to exist at the 1st and 2nd levels;
6) shear force at top floor is divided into 24 equal amounts for the purpose of determining the overall in-plane behaviour of the individual structural walls.

This analysis is clearly intended to be an approximate one, and this is consistent with the lack of exact knowledge about the material properties and about the magnitude of earthquakes that have occurred at the site.

3.1 Structural Model of the Original Structure

As can be seen in Figure 1(b), the major substructures of the theatres are the massive concrete walls which are arranged radially around the building. They are three storeys high, though their layout and dimensions vary at the different levels. At the third level, there is simply one continuous curved wall. A simplified model has been developed of the original structure. The continuous curved wall at the third level has been divided into 24 parts in order to determine the distribution of shear force. The structural walls are connected horizontally by barrel vaults or stairs at the first two levels, effectively providing a rigid diaphragm at these levels. The members at the third level are assigned to be rotationally restrained at the bottom and free at the top. The complete model is illustrated in Figure 3. Each structural wall has been numbered from 1 to 24 and the most critical one under seismic force will be identified in the following section.

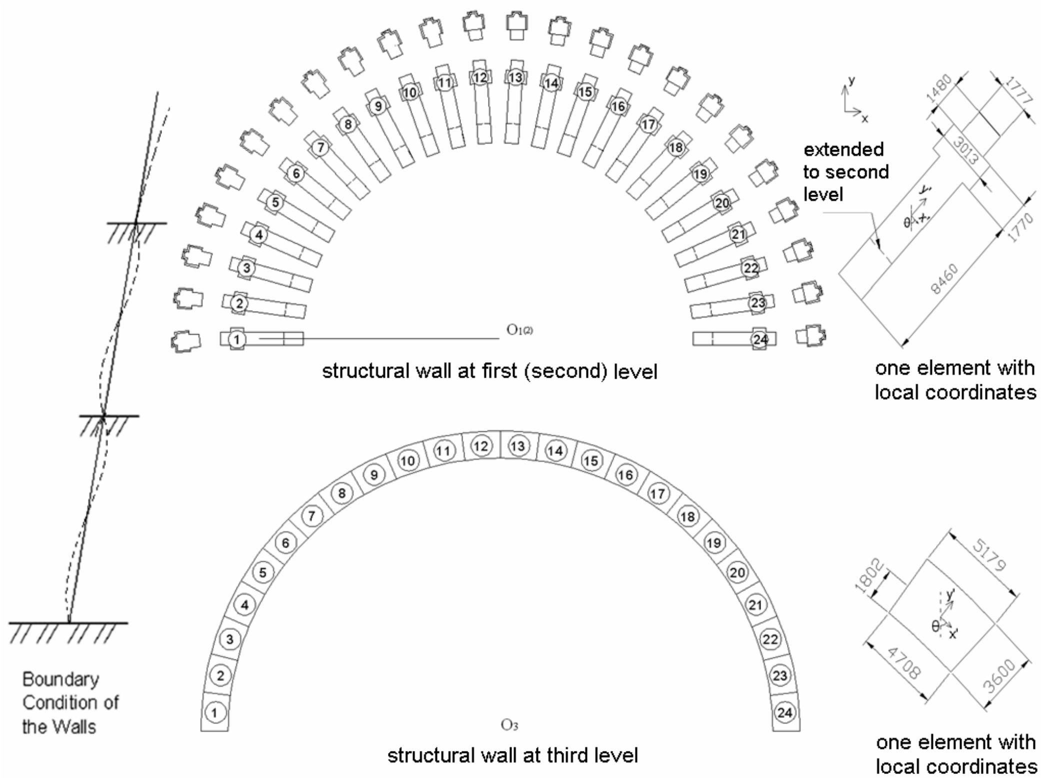
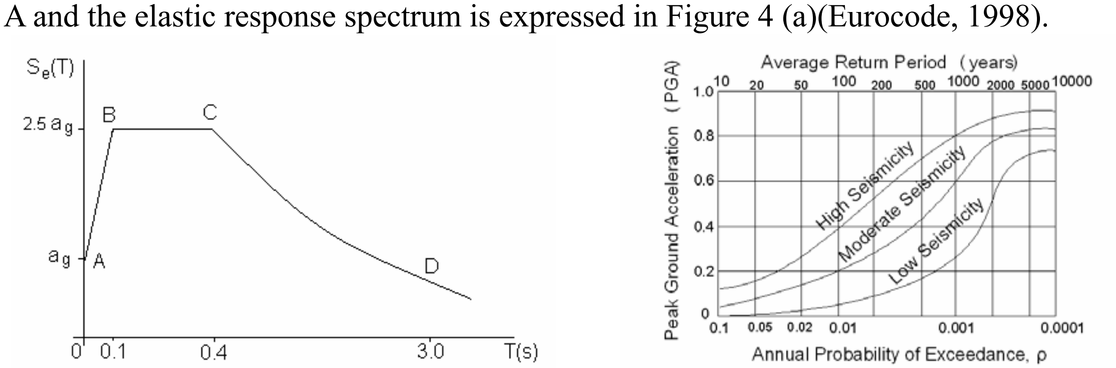


Figure 3. Model of structural wall with defined local and global coordinates

3.2 Response Spectrum

In order to define the response spectrum of the region, the seismic intensity as well as the soil condition of the region has been investigated. The seismic intensity of the region has been defined as 2.4m/s^2 in terms of its peak ground acceleration with a 500 year return period (U.S. Geological Survey, 2005). Assuming that Benevento is in an area of moderate seismicity, the approximate scaling factor for a 2500 year return period earthquake relative to a 500 year return period earthquake is assumed to be 1.6, as shown in Figure 4 (b) (Paulay and Priestley, 1990). Hence, the peak ground acceleration that the structure is likely to experience is approximately $a'_g \approx 1.6\, a_g$ = 1.6 x 2.4=3.84 m/s^2. Since the geology of Benevento is assigned as stiff soil, it should be classified as Subsoil Class A and the elastic response spectrum is expressed in Figure 4 (a)(Eurocode, 1998).



(a) (b)

Figure 4. (a) Elastic response spectrum and (b) Relationship between peak ground acceleration and annual probability of exceedence for different seismic regions

3.3 Material Properties

The material properties used in the dynamic analysis of the structure are shown in Table 1.

Table 1. Material constants

Density of Concrete	1605 kg/m^3
Compressive Strength of Concrete	6.66 MPa
Principal Tensile Strength(estimated)	0.77 MPa
Elastic Modulus (E)	6868 MPa
Shear Modulus (G)	2935 MPa

The density, compressive strength and other properties are taken from the data provided in the Romacon Project (Oleson et al., 2004). In that project, the researchers used a concrete core-drilling system to extract samples from various existing concrete *pilars* built in the sea at Roman times. The samples were subjected to a variety of material tests. The concrete used by the Romans in structures on land such as Beneventum is likely to have enhanced properties relative to those given above. The reason for this is that the ratio of pozzolanic material to lime was typically 3 to 1 for structures on land and only 2 to1 for maritime structures, such as the *pilars* (Morgan, 1914).

3.4 Analysis Procedure

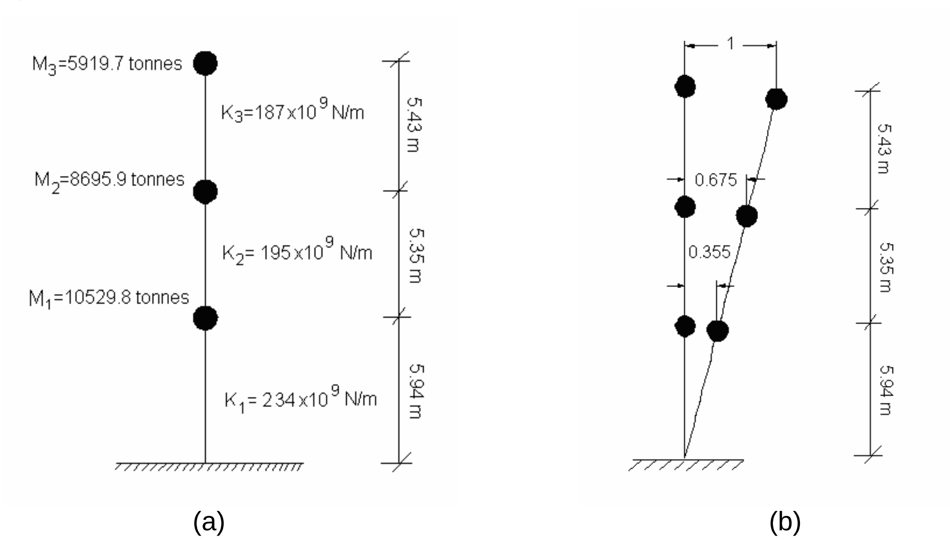


Figure 5. (a) Lumped mass model and (b)The hypothetical mode shape

The mass of the theatre at each level has been lumped (Figure 5 (a)) and the equivalent height of each storey has been determined. In order to find the stiffness at every level, only the structural walls are considered. Shear stiffness and flexural stiffness have been taken into account to determine the overall stiffness, as shown in Eq. (1):

$$k_j = \frac{1}{\frac{h^3}{3EI_g} + \frac{1.2h}{A_g G}},\ j = 1,2,3\,, \tag{1}$$

A generalised single-degree-of-freedom method has been used to determine the natural period of the structure. Only the first mode shape has been considered in determining the natural period.
Assuming displacements to increase linearly with height above the base, as shown in Figure 5 (b), the generalised properties of the system are determined by (Chopra, 2001):

$$\tilde{m} = \sum_{j=1}^{3} m_j \psi_j^2;\ \ \tilde{K}_x = \sum_{j=1}^{3} k_j (\psi_j - \psi_{j-1})^2;\ \ \ \omega_y = \sqrt{\frac{\tilde{K}_x}{\tilde{m}}};\ \ T_y = \frac{2\pi}{\omega_y}\,, \tag{2}$$

where ψ_j is the assumed shape factor at the different levels and T_y is the period corresponding to seismic force in the transverse direction.
The period obtained from applying Eq. (2) is 0.079s. Therefore, from the response spectrum in Figure 4 (a), the equivalent earthquake acceleration is A=8.65m/s^2. Shear forces at the different levels can be determined using the Single Degree of Freedom assumption shown in Figure 5 (b). Analysis of the structure based on the equivalent force method yields a displacement at the top level due to these forces of 1.85mm, a reflection of the large stiffness inherent in the structure. The seismic forces are then distributed to 24 structural walls according to their contributing stiffness by applying

$$V_{iy} = \frac{K_{ix}}{\sum K_{ix}} f_y\,, \tag{3}$$

where K_{ix} is the stiffness of each element according to the global x (longitudinal) axis and f_y is the static force at each level . As mentioned earlier, the eccentricity between centre of mass and centre of rigidity is zero in the transverse direction (global y axis shown in Figure 3), so torsional effects have been ignored.
By distributing the seismic forces onto every structural wall, the stress in the wall under seismic loading can be established. As all the structural walls are distributed radially, these distributed forces are further decomposed into two perpendicular ones which correspond to the principal axes of each structural wall. Hence, each structural wall is subjected to bending action over two principal axes. (Figure 6)

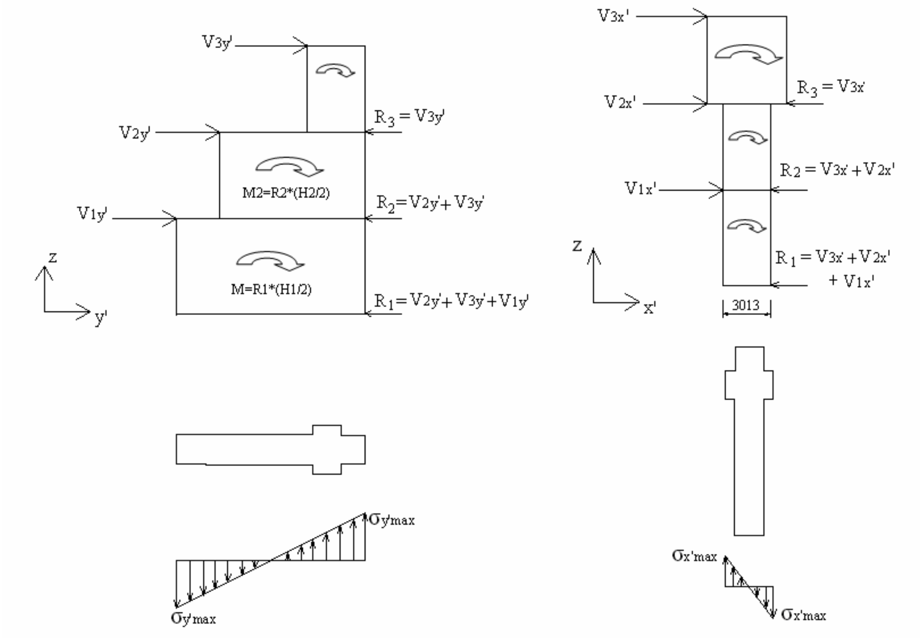


Figure 6: Structural wall under moment due to seismic forces

Besides the flexural stress due to moments, the structural walls are also subjected to the compressive stress induced by gravity, which is considered as Permanent Action and Imposed Action in this case.

The maximum tensile stress and compressive stress can be obtained by applying both the flexure stress and stress caused by gravity to the members. Therefore, the most critical structural wall can be identified.

Note that for the curved wall at the third level, the above model is not applicable because of its continuity. The whole wall has been analysed as an integral member subjected to overturning moment due to y-direction seismic forces.

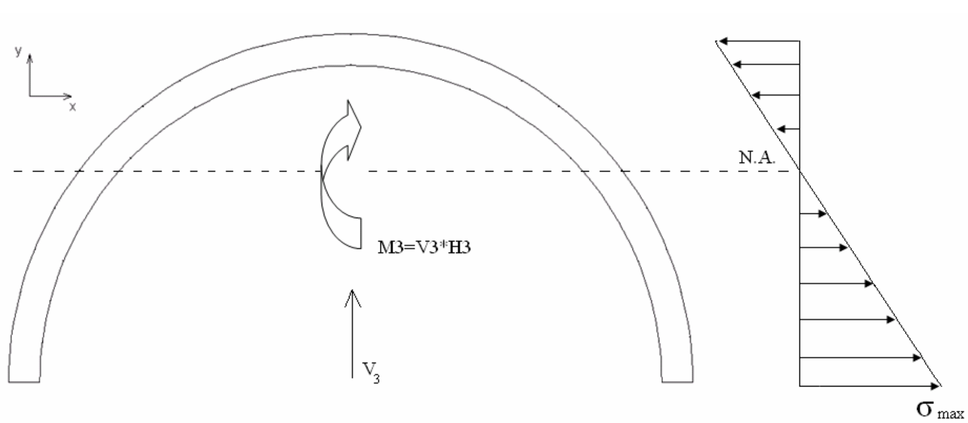


Figure 7: Flexural stress distribution at third level

Because of the massive horizontal sectional area of the structural wall, the shear stress has also been considered in this analysis.

3.5 Results from the Structural Analysis

The stresses in the walls that have been predicted by the elastic analysis under a 2500 year return period earthquake are given in Table 2. If they are compared with the concrete strength values given in Table 1, it is clear that severe damage would be expected.

Table 2. The stress at different levels

	Critical Walls	Maximum Tensile Stress (MPa)	Maximum Compressive Stress (MPa)	Maximum Shear Stress due to R_{ix}' (MPa)
Level 1	4 and 21	1.979	2.919	2.609
Level 2	4, 5, 20,21	3.209	3.819	2.135
Level 3	—	-0.074	0.386	2.900

However, no significant cracks in the main structural walls have been observed during the survey, which suggests that this level of earthquake has not been experienced.

A further analysis has been made of the non-structural facade subjected to out-of-plane displacement (N.T.K. Lam, 2001) and it has been estimated that an earthquake with a PGA of approximately 0.81 m/s^2 would have been sufficient to cause the collapse of the top level of this masonry wall, leading to the failure of the floor at this level. The maximum stresses in the structural walls induced by this level of earthquake are given in Table 3.

Table 3. The stress at different levels subjected to an earthquake with PGA of 0.81m/s^2

	Maximum Tensile Stress (MPa)	Maximum Compressive Stress (MPa)	Maximum Shear Stress due to R_{ix}' (MPa)
Level 1	0.048	0.988	0.552
Level 2	0.439	1.049	0.452
Level 3	-0.196	0.264	0.633

Clearly this level of earthquake does not exceed the strength of the main structural elements.

The masonry façade, which is located around the building perimeter, was built of travertine stone block. It has been assumed that there is a thin layer of mortar between the blocks to keep them in place. Therefore, if the façade at the third level fell, a sequential collapse might have occurred, which may have led to the further destruction of the façade at the second level and part of the first level.

4. CONCLUDING REMARKS

The results from the analysis are consistent with the observation from the survey. As both stress level and the critical parts are identified, the outcome can be applied to the preservation and maintenance of the existing structure for future earthquake events.

5. REFRENCES

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