Peak Seismic Displacement Demand and Second Corner Period

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ABSTRACT

The displacement demand on linear elastic single-degree-of-freedom systems for 5% damping would typically increase monotonically with increasing natural period up to the limit which is known as the second corner period (T_2). The maximum displacement demand (RSD_{max}) and the associated T_2 phenomena has great engineering significance in situations where the estimated RSD_{max} value is within the drift capacity of the structure as the latter can be deemed safe irrespective of its natural period properties. The earthquake magnitude dependence of T_2 is well established. The displacement response spectrum model stipulated by the current Australian Standard for Seismic Actions has the value of T_2 specified at 1.5 seconds based on an assumed earthquake magnitude of 7. Results presented in this paper offer support for this recommendation, but also reveal a possible influence on the value of T_2 from the regionally dependent stress drop associated with the earthquake rupture. This finding offers a plausible explanation for the diverse range of T_2 values that have been observed from earthquakes across the globe.

Keywords : peak displacement demand, second corner period, displacement response spectrum.

1. Introduction

The response spectral displacement (RSD) of an earthquake can be calculated as the product of $(T/2\pi)^2$ and the response spectral acceleration (RSA) value where T is the natural period of the elastic single-degree-of-freedom system. Figure 1 presents response spectra in different formats and their inter-relationships. Clearly, the modelled maximum displacement demand value (RSD_{max}) is controlled by the assumed value of the second corner period (T_2). It is noted that the seismic displacement demand estimated from this calculation might produce misleading results in the high period range depending on the nature of the response spectrum used. A traditional response spectrum in the flat-hyperbolic form that is without a second corner period (T_2) will have the value of RSD increasing indefinitely with increasing natural period. This is generally not the case with typical earthquakes except for large magnitude distant earthquakes. Realistic estimates of the seismic displacement demand can only be assured if the response spectrum features a representative second corner period (T_2) value.

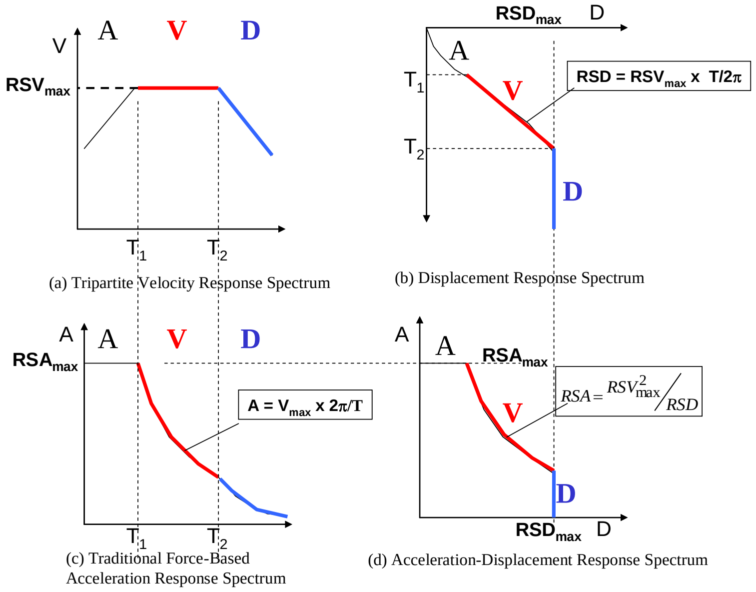


Figure 1 Response spectra presented in different formats

The determination of the value of T_2 from a (truncated) response spectrum is described in Section 2. Justifications for truncation period limit of 5 seconds is presented. Whilst the associated maximum displacement demand (RSD_{max}) value is widely considered to be relevant to the analysis of long period (flexible) structures only, it is emphasized herein that the assumed value of T_2 would also be critical to the assessment of a much wider range of structures for their risks of overturning and collapses. This concept will be explained in the light of the displacement-based seismic assessment approach as applied to vulnerable structures at the threshold of overturning or collapse.

The current Australian standard for seismic actions (AS1170.4, 2007) stipulates a constant T_2 value of 1.5 seconds. In comparison, the European Standard EC8 (EN 1998-1, 2004) stipulates a slightly higher T_2 value of 2 seconds (for Type 2 earthquakes). However, alternative models that have been developed for estimation of T_2 values have been found to be very inconsistent. Results obtained from parametric studies of some 168 records from a diverse databased of shallow earthquakes on stiff soil sites have been used to evaluate recommendations from the literature and to identify trends associated with the dependence of the value of T_2 on magnitude, distance and stress drop properties.

2. Engineering Interpretations of Second Corner Period

The second corner period (T_2) of an elastic displacement response spectrum is defined herein in accordance with the bi-linear representation of the spectrum and should be distinguished from the much higher period at which the spectrum peaks (as illustrated by the example of Figure 2a). In situations where the spectrum rises indefinitely with increasing natural period, the design maximum displacement demand value (RSD_{max}) is the highest spectral ordinate in a spectrum that has been truncated at the limiting period of 5 seconds (as illustrated by the example of Figure 2b). Justifications for this truncation limit are presented in this section.

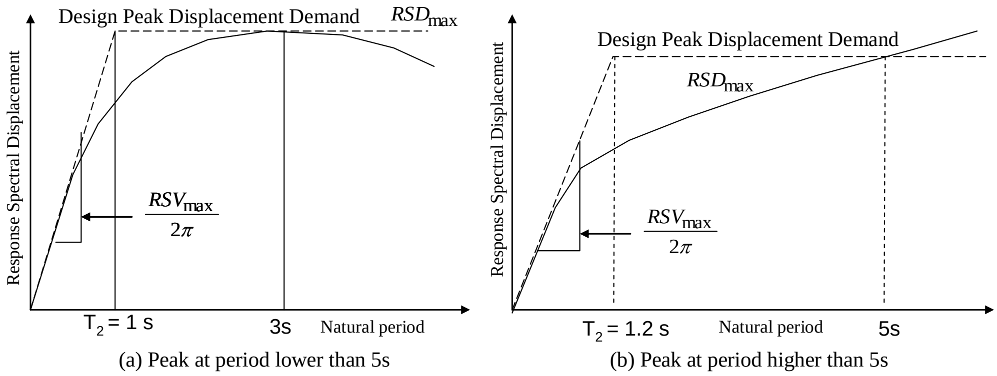
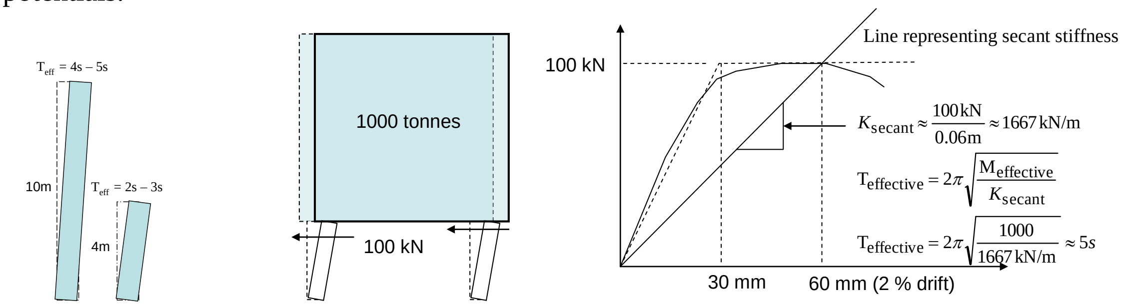


Figure 2 Definition of second corner period

The estimation of the value of RSD_{max} which is controlled by the value of T_2 is essential to the stability assessment of a wide range of structures. For example, the risks of overturning of an object in an earthquake can be evaluated by comparing the RSD_{max} value of the ground motion with the object's base dimensions. The non-linear behaviour of rocking motions can be analysed using this simple approach. Analyses of free-standing rectangular objects (and gravity structures) revealed natural period of rocking ranging between 2 - 3 seconds for 4m high objects, and 4 - 5 seconds for 10m high objects as shown in Figure 3a (Al Abadi *et al.*, 2005). Unreinforced masonry walls of twice the height are characterised by similar natural period in ultimate conditions provided that the walls are supported at the upper and lower boundaries of the wall (Lam *et al.*, 2003). Thus, the modelling of RSD_{max} for stability assessment for the majority of structures only need to consider response spectrum properties within the 5 second limit.

A similar approach of assessment can be applied to reinforced concrete structures which can accommodate a limited amount of post-yield displacement without collapsing. The risks of collapse of a building which is supported by reinforced concrete columns in the soft-storey can be evaluated by comparing the horizontal drift capacity of the columns with the estimated values of RSD_{max} (Lumantarna *et al.*, 2010). Figure 3b shows the example building module which weighs 1000 tonnes and is supported by columns of square cross-sections and with dimensions in the order of 400 mm – 500 mm. The full yield capacity of the columns can be developed with a modest horizontal drift of about 30 mm (1.0% drift) based on the stress-strain behaviour of 500 MPa steel. For a modest displacement ductility of 2, the design ultimate drift limit of the column is about 60 mm (i.e. 2% drift for 3m tall columns). It can be shown that a total horizontal resistance of 100 kN per module (i.e. 1% of the gravitational loading) can be provided by these columns even with a nominal amount of reinforcement. Consequently, the secant stiffness of the columns in such a potentially vulnerable structure is at least 1667 kN/m (100 kN divided by 0.06 m) which is translated to an effective natural period of about 5 seconds. Much lower effective natural period of the building is estimated if reinforcement exceeding the nominal amount is provided or if the phenonmenon of over-strength is taken into account. More robust buildings that are braced by shear walls have much lower effective natural period. Thus, again, the calculation of RSD_{max} values for evaluating the risks of collapses of buildings only need to consider response spectrum properties within the 5 second limit. Alternative definitions for T_2 that are associated with a higher truncation limit would only be applicable to systems with exceptionally high natural periods and hence too restrictive in their application potentials.



(a) Free-standing objects (b) Reinforced concrete soft-storey building and force-displacement relationship

Figure 3 Effective natural period of vulnerable systems

3. Recommendations for the Second Corner Period

The value of the second corner period (T_2) in the displacement spectrum is not unique and is well known to be sensitive to the moment magnitude of the earthquake. Equation (1) for prediction of the value of T_2 was developed by the authors through stochastic simulations of the seismological model (Lam *et al.*, 2000a). Predictions were based on rock conditions and within 30 km from the source of the earthquake in order that interferences by the earth crusts along the seismic wave travel path were minimized. Given that the seismological empirical source model was developed from ground motion data recorded in *Central and Eastern North America* (Atkinson, 1993), equation (1) should be most suitable for applications in stable continental regions which are characterised by high stress parameter values (often referred as “stress drop” values). The predictive expression is also supported by an early independent empirical study of ground motion records from stable continental areas in which an average T_2 value of 0.7 seconds were observed for earthquakes of magnitudes ranging between M5.5 and M6.5 (Somerville *et al.*, 1998).

$$T_2 = 0.5 + \frac{M-5}{2} \qquad (1)$$

A similar predictive expression (equation 2) was developed more recently based on observations from response spectra of earthquakes recorded in Japan, Europe and the M7.6 Chi Chi earthquake of 1999, Taiwan (Faccioli *et al.*, 2004 which was cited in Priestley *et al.* 2007). Response spectra calculated from accelerograms recorded from earthquakes of M5.4 – M6.4 within 30 km of the earthquake epicentres featured T_2 values at around 1 second which is consistent with the earlier findings by Somerville *et al.* (1998) and Lam *et al.* (2000a). Consequently, both equations (1) and (2) are roughly consistent in the low magnitude range (M < 6). However, equation (2) predicts much higher T_2 values in order to match with observations from the Chi Chi earthquake at M = 7.6.

$$T_2 = 1.0 + 2.5(M - 5.7) \qquad (2)$$

Figure 4 shows predictions for the T_2 values by the two equations along with recommendations by Somerville (2003) as cited in Faccioli *et al.* (2004) and those by FEMA 274 (NEHRP, 1997) as cited in Priestley *et al.* (2007). The codes provisions of EC8 (2004) and AS1170.4 (2007) are in general agreement with predictions by equation (1). However, EC8 provisions have been claimed to be "severely un-conservative" in view of predictions by equation (2).

As shown by the comparative plot of Figure 4, predictions from the presented models are very diverse and particularly so in the high magnitude range. It is important to note that empirical data recorded from earthquakes exceeding M6.5 in support of the presented models were very scarce. For example, no empirical data was available to constrain the models in the magnitude range of M6.5 – M7. Equation (2) was constrained in the high magnitude range by records taken from only one event: the M7.6 Chi Chi earthquake of 1999.

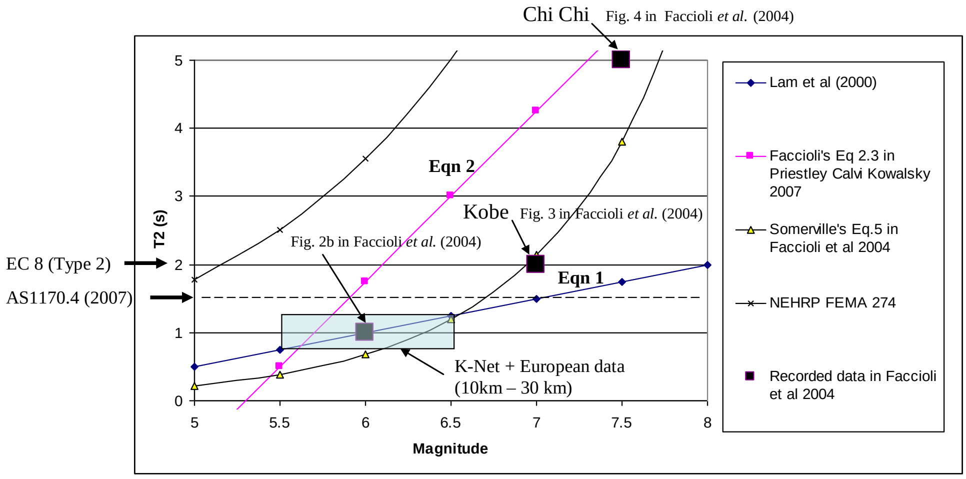


Figure 4 Recommendations for second corner periods

To resolve the notable discrepancies between equations (1) and (2), a parametric study was undertaken by the authors to study the behaviour of T_2 based on ensembles of accelerograms sourced from the PEER database (that is available for public access via the worldwide web at <http://peer.berkeley.edu/smcat/index.html>). Refer Table 1 for a summary listing of the 168 records employed in the study. Data presented herein were all recorded from Class C (stiff soil) sites which have shear wave velocities in the upper 30m varying in between 360 m/s and 760 m/s. Records from stiff soil sites were used because more records from this site class than from rock sites were available. Importantly, 22 records from the database were taken from four earthquake events which had magnitude equal to, or exceed, M6.8.

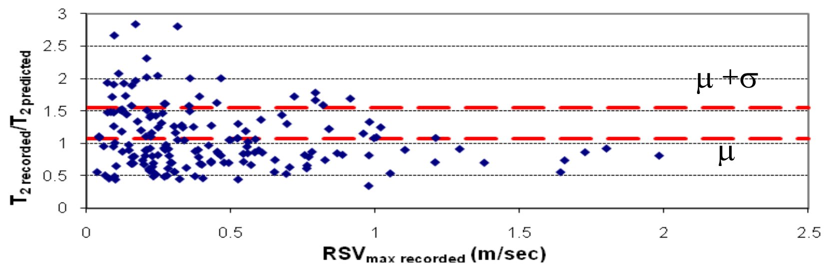
Figures (5a) and (5b) show the corner periods observed from the individual calculated response spectra that have been normalised with respect to the respective predicted corner period values as calculated from equations (1) and (2) respectively. The normalised corner periods were plotted against the recorded values of RSV_{max} obtained from the individual record. The comparison clearly shows equation (1) to be more consistent with field observations than equation (2). The model of equation (1) shows negligible overall biases (ie. $\mu \sim 1$). Current provisions by both AS1170.4 (2007) and EC8 (2004) are well supported by the evaluations presented herein. However, significant outliers are also shown in both figures indicating systematic effects on the second corner period values that have not been incorporated into the modelling. Many outliers were associated with records possessing low velocity properties. The dependence of T_2 on various seismological parameters are investigated further in the rest of the paper.

4. Dependence of Second Corner Period on magnitude, distance and stress parameters

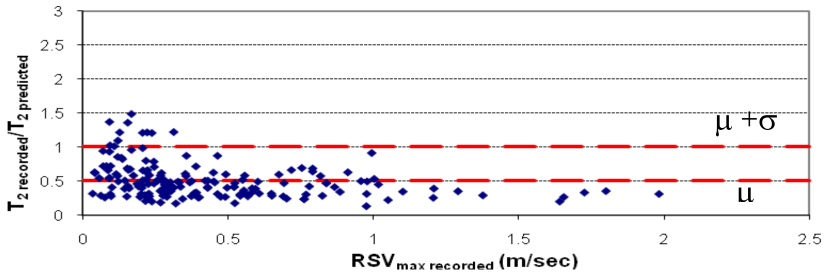
Figures 6 and 7 reveal no distinct trends on the influence of magnitude and distance on the normalised values of T_2, but again indicate that the majority of outliers were associated with records possessing low velocity properties. Other well known influential factors (other than magnitude, distance and path/site effects) are namely the stress parameters (which are often referred as "*stress drop*") and directivity effects. Directivity effects would only be significant in situations where the site was in close proximity to a major fault source and in alignment with the direction of rupture propagation. Given that *stress drop* effects have not been parameterised in most attenuation relationships, the ratio of the recorded and predicted peak ground velocities would then be indicative of the stress drop anomalies. A high recorded/predicted PGV ratio indicates higher than average stress drop behaviour. Intraplate earthquakes in stable continental regions which are typified by reverse (thrust) faulting mechanism have been observed to possess high stress drop behaviour. The influence of stress drop properties on the value T_2 is revealed in Figure 8 (the legend of the figure shows different symbols to represent records of different ranges of recorded/predicted PGV values). Interestingly, the great majority of results associated with recorded/predicted PGV values exceeding 1.5 (ie. high stress drops) have the normalised values of $T_2 < 1.0$. This suggests the dependence of T_2 behaviour on regional stress drop properties as well as the earthquake magnitude and modifications by the wave travel path. This finding offers a plausible explanation for the diverse range of T_2 values that have been observed from earthquakes across the globe.

Table 1 Catalogue of recorded accelerograms from PEER database Stiff soil (360m/sec<V_{30}<750m/sec)

Earthquake name	**Magnitude**	**Distance Range (km)**	**No. of records**
Coalinga	6.4	41 - 53	10
Coyote Lake	5.8	4 - 24	8
Friuli	6	15	2
Friuli	6.5	16	4
Imperial Valley	6.5	15	2
Irpinia	6.9	18 - 30	6
Kobe	6.9	7	2
Loma Prieta	6.9	4 - 20	10
Mammoth Lakes	6.3	5	2
Morgan Hill	6.2	3 - 31	12
N. Palm Springs	6	23 - 52	8
Nahanni	6.8	5 - 10	4
Northridge	6.7	7 - 50	52
Parkfield	6.2	15 - 18	6
San Fernando	6.6	23 - 40	8
Santa Barbara	6	12 - 28	4
Superstitn Hills	6.6	6	2
Westmorland	5.9	20	2
Whittier Narrows	6	15 - 49	24



(a) Comparison with predictions by Lam *et al.* (2000a)
Stiff soil (360m/sec<V_{30}<750m/sec)



(b) Comparison with predictions by Faccioli's expression in Priestley *et al.* (2007)
Stiff soil (360m/sec<V_{30}<750m/sec)

Figure 5 Comparison of recommended second corner period values

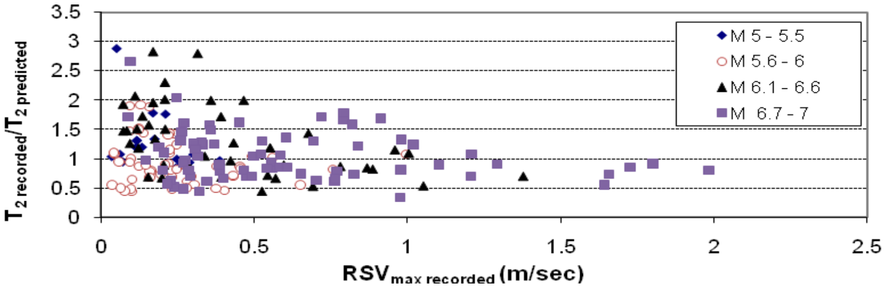


Figure 6 Magnitude dependence of second corner period values

Finally, Figure 9 presents the value of RSD_{max} as observed from the individual response spectra calculated from the recorded accelerograms. The values shown have been normalised with respect to the respective predicted values. The predictions were based on equations (3a) and (3b) which are primarily based on recommendations in Lam *et al.* (2000b). It is shown in Figure 9 that there is no overall bias in the predictions (ie. μ ~ 1.0).

$$RSD_{\max}(mm) = RSV_{\max}(mm) \times \frac{T_2}{2\pi} \tag{3a}$$

$$RSV_{\max}(mm/s) = 70\left(0.35 + 0.65(M-5)^{1.8}\right) \times \frac{30}{R}^{1=0.005R} \times 1.5 \times 1.4 \quad \text{for stiff soil sites} \tag{3b}$$

where the value of T_2 is calculated from equation (1); M is the moment magnitude; and R is the site source distance in km.

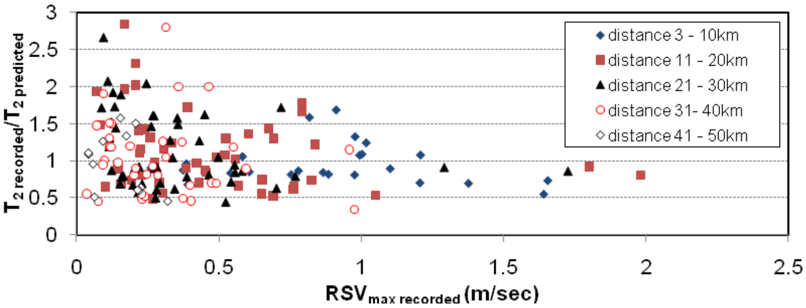


Figure 7 Distance dependence of second corner period values

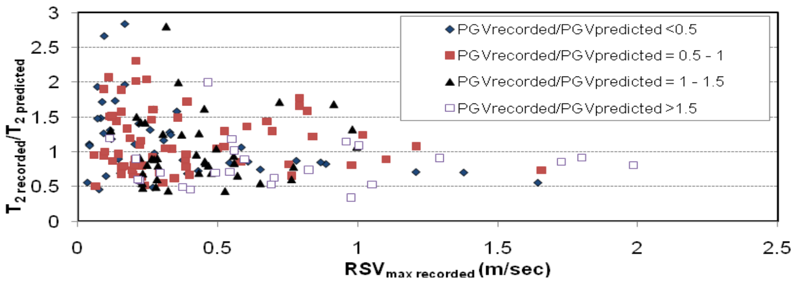


Figure 8 Stress parameter dependence of second corner period values

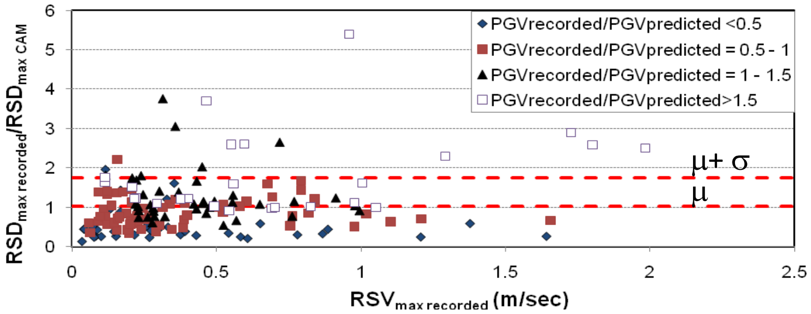


Figure 9 Evaluation of recommended values for D_{max}

5. Conclusions

a) The potential seismic performance of vulnerable structures including buildings with a soft-storey, free-standing components and gravity structures can be readily assessed using a displacement-based approach if the value of T_2 is known. Most structures at ultimate conditions will have their effective periods considerably smaller than the period limit of 5 seconds. Consequently, definitions for the second corner period should be based upon the bi-linear displacement response spectrum which has been truncated at the period limit of 5 seconds. Alternative definitions for T_2 that are associated with a higher truncation limit would only be applicable to systems with exceptionally high natural periods and hence too restrictive in their application potentials.

b) Equation (1) which was developed from stochastic simulations of the seismological model of *Central and Eastern North America* is also supported by analyses undertaken in this study using accelerograms collected from a diverse database of shallow earthquakes. All the accelerograms incorporated into the study were recorded on stiff soil sites and within 50 km from the epicentres of earthquakes with magnitude of up to M7. Current provisions for T_2 in both the *Australian Standard for Seismic Actions* and in *Eurocode 8* are generally consistent with equation (1) and hence supported by the evaluation.

c) Equation (2) developed by Faccioli is roughly consistent with equation (1) in the low magnitude range but is strongly influenced by accelerograms recorded from the Taiwan M7.6 Chi Chi earthquake of 1999, which has resulted in larger T_2 values for all higher magnitude events.

d) A new trend showing the dependence of the value of T_2 on regional stress drop properties has also been identified.

6. References

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