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Displacement Controlled Behaviour of Asymmetrical Buildings

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ABSTRACT

Seismic assessment has traditionally been based on trading off strength with displacement (ductility) to provide sufficient energy dissipation capacity to structures. However, the energy demand on a structure from small to medium magnitude earthquakes will generally subside when the structure has been displaced to a certain limit. The significance of the concept of displacement-controlled behaviour in the seismic evaluation of a structure is that the peak drift demand of the structure can be constrained by a peak displacement demand which is solely a function of the properties of the ground shaking. In this paper, this concept has been extended for the seismic evaluation of asymmetrical structures. It is shown that a torsional amplification factor of 1.6 and 2.0 can be applied for estimating the peak drift demands of one-way and two-way asymmetric system respectively. By applying the new approach, the drift demand at the edges of the building can be estimated without applying the conventional force-based analysis.

Keywords: earthquake; seismic assessment; displacement-controlled behaviour; non-linear time history analyses; asymmetrical building

1 Introduction

With structures in which the fundamental natural period of vibration exceeds the dominant period of excitations, the energy (velocity) demand subsides rapidly with further increase in the natural period. Effectively, the displacement demand on the structures remains constant, or decrease, with increasing natural period as illustrated in Figure 1 by the displacement response spectra of a single pulse and series of periodic pulses (representing resonance conditions on flexible soil sites).

This concept of displacement controlled behaviour has been extended to the evaluation of non-linear (inelastic) responding systems including a rocking object or a strength degrading system in which case the secant (or effective) stiffness is used for estimating the (effective) natural period of the system. The significance of this concept in the seismic evaluation of structural systems is that the peak drift (or deflection) of the structure can be constrained by a peak displacement demand (PDD) which is solely a function of the properties of the ground shaking. In other words, the structural drift demand can be estimated without the need to undertake a force-based analysis of the structure. Refer to previous publications by the authors (Lam & Chandler, 2005; Lumantarna et al. 2007, Lumantarna et al. 2008) when this concept was first introduced.

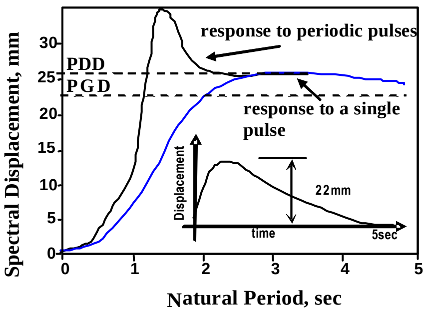


Figure 1 Displacement-controlled behaviour (Lam & Chandler, 2005)

This paper further develops this concept of displacement controlled behaviour to structural systems that are subject to torsional actions arising from asymmetry. Force-based analyses can be computationally expensive for asymmetrical structures especially when dynamic torsion is involved. By applying the drift based analysis approach, a force-based analysis of the building model can be by-passed in the estimation of the drift demand at the edges of the building. Significantly, the estimated peak drift demand is independent of the torsional resistance of the building and its eccentricity (offset of centre of resistance to centre of mass). This new approach has been tested by comparison with results from non-linear time history analyses THA of asymmetrical building models subject to unilateral and bi-directional excitations. The results from the studies were integrated to develop a simple yet reliable seismic assessment procedure for low to moderate seismicity regions.

2 Peak displacement demand of non-ductile structures

The maximum displacement demand **PDD** of single degree of freedom SDOF systems can be obtained from the elastic displacement response spectrum when linear elastic behaviour is assumed. The maximum displacement demand can be estimated as the highest displacement demand (RSD_{max}) as shown on the displacement response spectrum up to a natural period of 5 seconds.

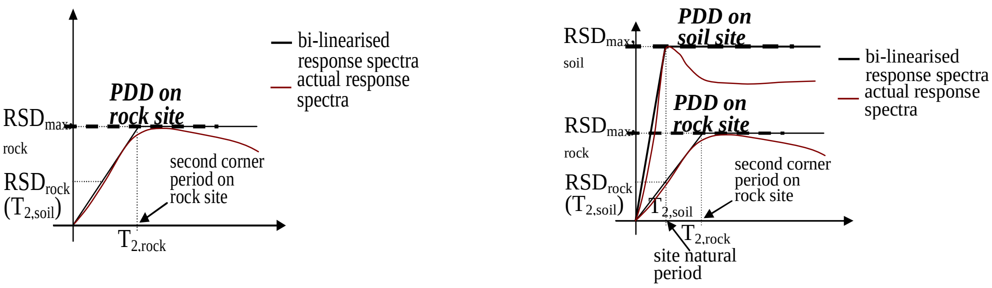
Figure 2a presents the displacement response spectrum on rock sites idealised as a bi-linear model. The maximum displacement response spectrum on rock sites $RSD_{max,rock}$ can be estimated by Equation (1):

$$RSD_{max,rock} = 1500Z\frac{T_{2,rock}}{2\pi} \quad (1)$$

where, Z is the seismic hazard factor stipulated in the Australian Standard (AS1170.4, 2007), $T_{2,rock}$ is the second corner period at which the linear part of the response spectrum intercepts the constant (flat) part of the response spectrum (Fig. 2a). $T_{2,rock}$ is related to the moment magnitude M by the linear relationship proposed by Lam et al. (2000) :

$$T_{2,rock} = 0.5 + 0.5(M-5) \quad (2)$$

for moment magnitude M less than 7. The maximum displacement demand ***PDD on rock sites*** (Fig. 2a) can be estimated for any combination of the hazard factor Z and moment magnitude M using Equations (1) and (2).



(a) Displacement response spectrum on rock sites (b) Displacement amplification on soft soil sites

Figure 2 Displacement response spectra on rock and soft sites

The maximum displacement demand can be significantly amplified when structures are founded on more onerous soil sites (***PDD on soil site*** in Fig. 2b). Studies based on shear wave analyses (Lam & Wilson, 2004) have found that the maximum displacement demand on a soil site $RSD_{max,soil}$ can be up to four times the displacement demand on a rock site at the site natural period (Fig. 2b) as shown by Equation (3):

$$RSD_{max,soil} \approx RSD_{rock}(T_{2,soil})\ 4 \quad (3)$$

where, $T_{2,soil}$ is the site natural period at which the linear and constant (flat) parts of the bi-linearised response spectrum intersects (Fig. 2b). $RSD_{rock}(T_{2,soil})$ is the response spectral displacement at the site natural period $T_{2,soil}$ as shown schematically in Figure 2b. By combining Equations (1)-(3), the maximum displacement response spectrum on soil sites can be expressed by the following equations:

$$RSD_{max,soil} \approx \frac{6000\ Z}{2\pi}T_{2,soil} \qquad \text{for } T_{2,soil} \le 0.5+0.5(M-5) \quad (4a)$$

$$RSD_{max,soil} \approx \frac{6000\ Z}{2\pi}(0.5+0.5(M-5)) \qquad \text{for } T_{2,soil} > 0.5+0.5(M-5) \quad (4b)$$

where, Z is the seismic hazard, M is the moment magnitude and $T_{2,soil}$ is the site natural period. The maximum displacement demands (RSD_{max}) associated with the seismic hazard of Australia as stipulated in the Australian Standard (AS1170.4, 2007) were estimated based on Equation (4). The estimated RSD_{max} for 500 and 2500 year return period earthquake is presented in Table 1.

Table 1 Estimates of RSD_{max} (mm) for Australian conditions

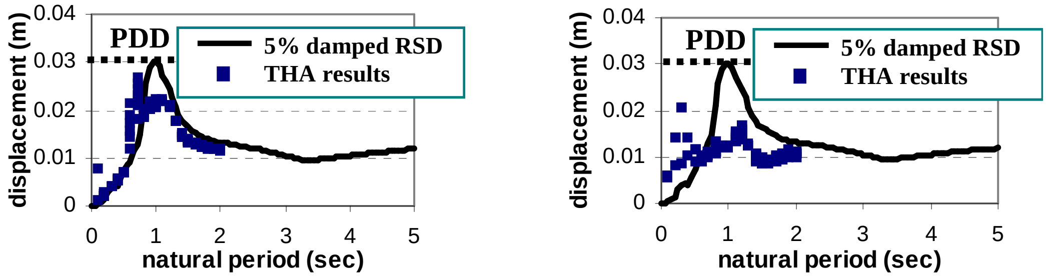
Site	Site period*	500 yr RP				2500 yr RP			
		Z = 0.06	Z = 0.08	Z = 0.10	Z = 0.12	Z = 0.06	Z = 0.08	Z = 0.10	Z = 0.12
Rock	$T_{2,rock}$ = 1.5 s**	21	29	36	43	39	52	64	77
Soil	Ts = 0.6 s	34	46	57	69	62	83	103	124
	Ts = 1.0 s	57	76	95	115	103	138	172	206
	Ts = 1.5 s	86	115	143	172	155	206	258	309

*Site period of 0.6 s, 1 s, and 1.5 s refer to the site natural period which are representative of class C, D and E sites respectively as specified in AS1170.4 (2007).

** The estimations of RSDmax on rock were based on M = 7 earthquake ($T_{2,rock}$ = 1.5 sec according to Eq 2)

Parametric studies have been undertaken based on non-linear time history analyses of SDOF systems to extend the use of displacement response spectra in estimating the maximum displacement demands of non-ductile structures (Lumantarna et al., 2007; Lumantarna et al., 2008). Hysteretic models used in the studies are representative of hysteretic behaviour of common non-ductile structures, including soft-storey buildings and unreinforced masonry buildings.

It was found from the parametric studies that despite having undergone significant strength degradation (up to 80%), SDOF systems were only displaced to a certain limit constrained by the highest point of the displacement response spectra (Fig. 3). The highest point on the displacement response spectra can be used to provide a conservative prediction of the maximum displacement demands (**PDD**) of inelastic non-ductile structures.



M=5.5, R = 17 km, site class D
a) R_μ = 2

M=5.5, R = 17 km, site class D
(b) R_μ = 4

Figure 3 Displacement demands of SDOF systems subject to generated earthquake

3 Peak displacement demand of asymmetrical building

In situations where the center of resistance (CR) of the building is offset from the center of mass (CM) (Fig. 4a), the building will translate and rotate when subject to earthquake excitations. The translation and rotation can result in displacement amplification at the edges of the building as shown in Figure 4b. The maximum displacement could occur at the flexible or the stiff edge of the building depending on the dominant mode of vibration. The peak displacement demand referred in this section represents the higher of the two values.

The maximum displacement demand of asymmetrical buildings can be estimated by applying a torsional amplification factor Γ_{DD} to the maximum response spectral displacement (**PDD** = Γ_{DD} RSD_{max}). The maximum displacement demand **PDD** of an asymmetrical building can be determined by calculating eigenvalues and eigenvectors of the equation of motion for a single storey building model as shown in Figure 4a. Γ_{DD} is defined as the ratio of the maximum displacement demand to the maximum response spectral displacement (ie. Γ_{DD}=**PDD**/RSD_{max}). Therefore, Γ_{DD} can be defined by Equation (5) when linear elastic behaviour is assumed (Lumantarna et al. 2007):

$$\Gamma_{DD} = \text{larger of } \begin{cases} \sqrt{\left(PF_1 - \hat{\theta}_1 \dfrac{b_1}{r}\right)^2 + \left((1-PF_1) + \hat{\theta}_1 \dfrac{b_1}{r}\right)^2} \\ \sqrt{\left(PF_1 + \hat{\theta}_1 \dfrac{b_2}{r}\right)^2 + \left((1-PF_1) - \hat{\theta}_1 \dfrac{b_2}{r}\right)^2} \end{cases} \quad \text{and} \quad \mathbf{PDD} = \Gamma_{DD}\, RSD_{max} \qquad (5)$$

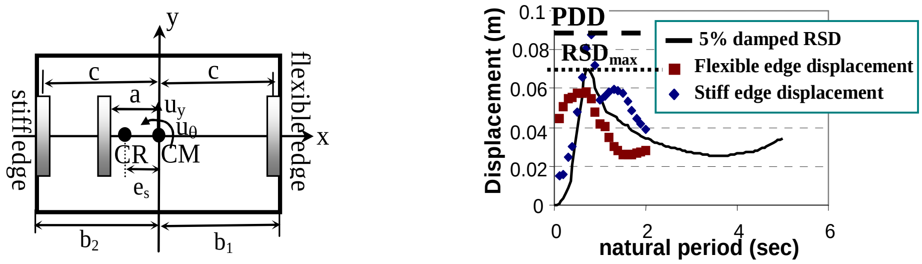
where, PF_1 is the participation factor for mode 1, $\hat{\theta}_1$ is the rotational component of mode 1 defined by Equation (6).

$$PF_1 = \frac{e^2}{e^2 + \left(1-\Omega_1^2\right)^2} \qquad (6a) \qquad\qquad \hat{\theta}_1 = -\frac{e\left(1-\Omega_1^2\right)}{e^2 + \left(1-\Omega_1^2\right)^2} \qquad (6b)$$

Ω_1 is the 1st coupled circular frequency which can be related to the uncoupled frequency ratio (ρ_k=1/r $(k_\theta/k_y)^{1/2}$) and the distance from the center of resistance to the center of mass normalised to the mass radius of gyration (e) as shown by:

$$\Omega_1 = \frac{1+\rho_k^2+e^2}{2} - \sqrt{\frac{\left(1+\rho_k^2+e^2\right)^2}{4} - \rho_k^2} \qquad (7)$$

From Equations (5), (6) and (7), the value of Γ_{DD} can be determined for any combination of e (eccentricity) and ρ_k (uncoupled frequency ratio).



(a) floor plan (b) Displacement demands from time-history-analyses

Figure 4 Single-storey building model

Field surveys on soft-storey (Wilson et al. 2005) and URM buildings (Griffith et al. 2004) showed that the eccentricity (e) and uncoupled frequency ratio (ρ_k) of the buildings (both parameters normalised to the mass radius of gyration r) were between 0.05 to 0.6 and 0.8 to 1.6 respectively. The distance from the center of mass to the flexible or stiff edge normalised to the mass radius of gyration (b/r) was less than 1.8. Equations (5) to (7) were used to determine the values of Γ_{DD} using all combination of e and ρ_k likely to be found in real buildings (Figure 5). It was found that the value of Γ_{DD} is insensitive to the variations in e and ρ_k. From the above observations, Equation (5) can be simplified to:

$$\Gamma_{DD} = \sqrt{\left(0.8 + 0.4\frac{b}{r}\right)^2 + \left(0.2 - 0.4\frac{b}{r}\right)^2} \quad \text{and} \quad \mathbf{PDD} = \Gamma_{DD}\ \mathrm{RSD}_{max} \qquad (8)$$

where, b is the distance from the building center of mass to the flexible or the stiff edge (Fig. 4b), whichever is the greater, and r is the mass radius of gyration of the building. The value of $\boldsymbol{\Gamma_{DD}}$ was estimated to be up to around 1.6 for b/r less than 1.8.

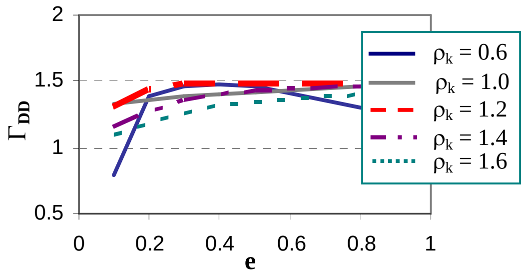
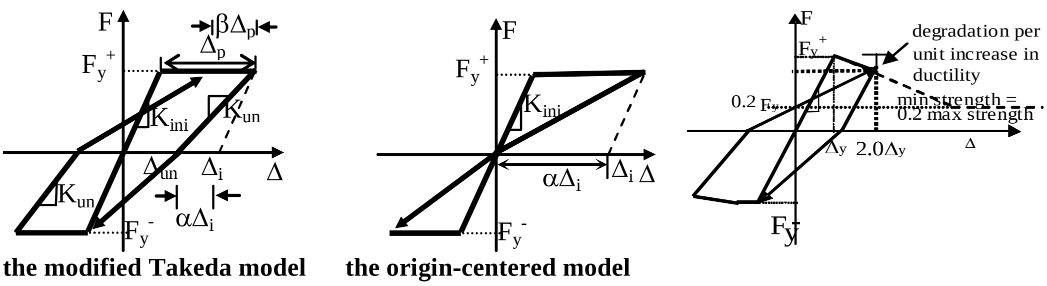


Figure 5 $\boldsymbol{\Gamma_{DD}}$ **with varying e and** $\boldsymbol{\rho_k}$ **values**

Parametric studies were undertaken based on non-linear time history analyses of a single storey building model shown in Figure 4a. The configuration of frames in the building on plan was adjusted such that the eccentricity (e) and the uncoupled frequency ratio (ρ_k) of the building was in the range 0.1 to 0.5 and 0.8 to 1.4 respectively. Hysteretic models of the frames in the building model are representative of that in unreinforced masonry walls and soft storey columns.

Hysteretic modelling needs to take into account: i) stiffness degradation and ii) strength degradation. The modified Takeda model and the origin-centered model were selected to represent the stiffness degradation of soft-storey columns and unreinforced masonry walls (Fig. 6a). The hysteretic models were calibrated to the hysteretic curves as recorded from the cyclic testing of the soft-storey columns (Rodsin et al. 2004) and unreinforced masonry walls (Griffith et al. 2007). The strength degradation was modelled to degrade with increasing ductility (displacement) demand (Fig.6b). Sensitivity of the displacement response behaviour of SDOF systems to the modelling parameters has been investigated by the authors (Vaculik et al. 2007; Lumantarna et al. 2006).



the modified Takeda model **the origin-centered model**

(a) stiffness degradation (b) strength degradation

Figure 6 Modelling of stiffness degradation

Simulated and recorded accelerograms on class C and D sites as stipulated by the Australian Standard (AS1170.4, 2007) were used in the parametric studies. These accelerograms listed in Table 2 are representative of small to medium magnitude earthquakes. The simulated accelerograms were generated based on earthquake scenarios producing a peak ground velocity of 60 mm/sec on rock (Lam et al. 2005).

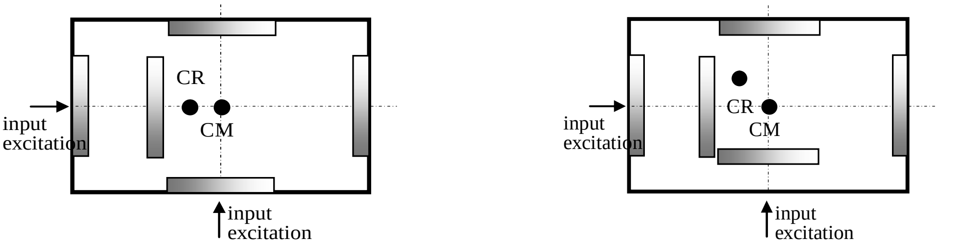
Table 2 List of accelerograms

No	Event	M	R	PGV	Site
1	Generated	6.5	40	60	Class C
2	Generated	6.5	40	60	Class D
3	Generated	5.5	17	60	Class C
4	Generated	5.5	17	60	Class D
5	Generated	7	90	60	Class C
6	Generated	7	90	60	Class D
7	Friuli aftershock	5	7	100	Class C
8	San Fernando	6.5	25	80	Class D

Non-linear time history analyses were performed on the single storey building model using the modified Takeda model with parameters: $\alpha = 0$ and $\beta = 0$. The initial stiffness of each frame was adjusted to produce initial uncoupled periods of the building which range between 0.2 and 2 secs. The yield strength of each frame was scaled such that the strengths of the building models were exceeded by the strength demands from the earthquake excitations by a factor (R_μ) of 2 to 4 (Lumantarna et al. 2008). The rate of strength degradation was varied between 9% and 22% per unit increase in ductility demand (Fig. 6b). The strength was modelled to start degrading when the yield limit has been exceeded and continue to degrade until reaching the minimum residual strength (when 80% of the maximum strength has been degraded) (Fig. 6b).

The maximum displacement demands at the edges of the buildings were plotted against their initial uncoupled periods in Figure 8. It was found that despite significant degradation of the torsional resistance of the building (up to 80%), the maximum displacement demands of the asymmetrical buildings analysed in the study were constrained by the peak displacement demand (**PDD)** as estimated by Equation (8).

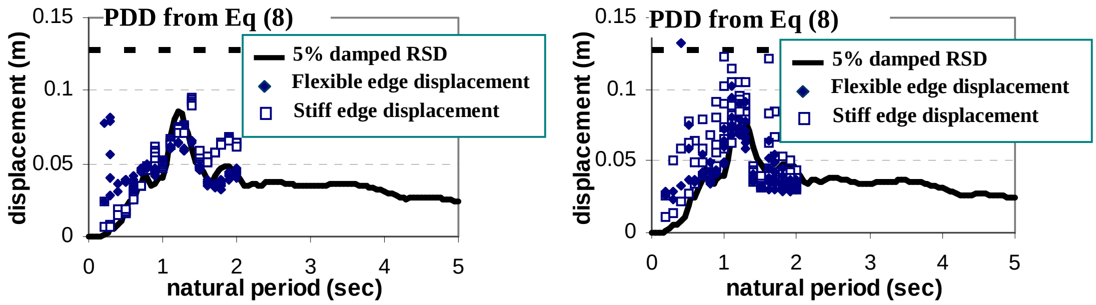
The torsional amplification factor as defined by Equation (8) and the non-linear THA results presented in Figure 8 were from a simplified model (based on parallel frames subject to uni-lateral excitation) (Fig. 4a). The contributions of load resisting elements and ground excitations in the orthogonal direction have been neglected. The maximum displacement demands from the simplified analyses based on unilateral excitations were compared to results from analyses based on bi-directional excitations. It was shown in Figure 9 that the response of realistic asymmetrical buildings to concurrent bi-directional excitations can be represented by the estimated response to uni-lateral excitations, provided that the center of resistance in the orthogonal direction is in alignment with the building center of mass. This type of asymmetry is denoted herein as Type A asymmetry as shown in Figure 7a.



One-way asymmetric system (a) Type A asymmetry

Two-way asymmetric system (b) Type B asymmetry

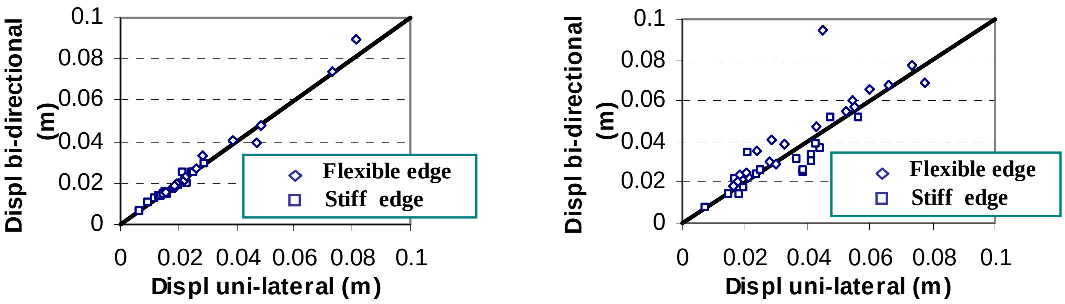
Figure 7 Plan view of building models



M = 7, R = 90 km, site class D (a) $R_\mu = 2$

M = 7, R = 90 km, site class D (b) $R_\mu = 4$

Figure 8 Displacement demands of buildings with Type A asymmetry (e = 0.5, ρ_k = 0.8), subject to uni-lateral excitations



M=6.5, R=40 km, site class C (a) $R_\mu = 2$

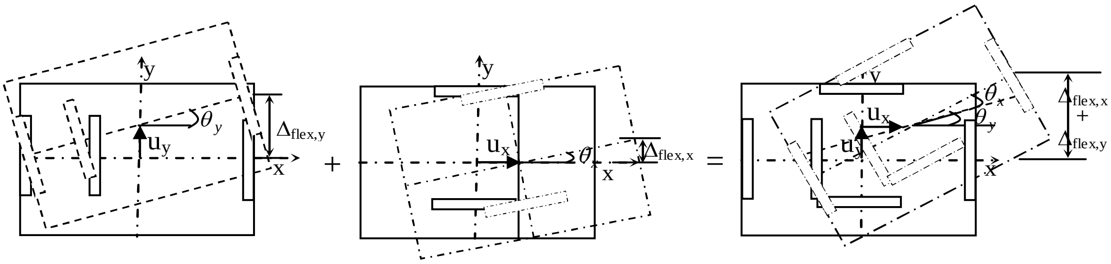
M=6.5, R=40 km, site class C (b) $R_\mu = 4$

Figure 9 Displacement demands of Type A asymmetry (e = 0.5, ρ_k = 1.3), subject to uni-lateral and bi-directional excitations

However, buildings with Type B asymmetry (Fig. 7b) were shown to be subject to higher displacement demands (compare Figs. 11 and 8). This is as a result of the asymmetry in the orthogonal direction which contributes to additional displacement demand at the edge of the building (Fig. 10). Equation (8) can be modified by adding the rotational component in the orthogonal direction (Fig. 10b) as defined by Equation (6b) to the maximum displacement demand defined by Equation (5). By substituting common values of e and ρ_k obtained from the field surveys (Wilson et al. 2005; Griffith et al. 2004), the torsional amplification factor (Γ_{DD}) which takes into account asymmetries in the orthogonal directions can be expressed as follows:

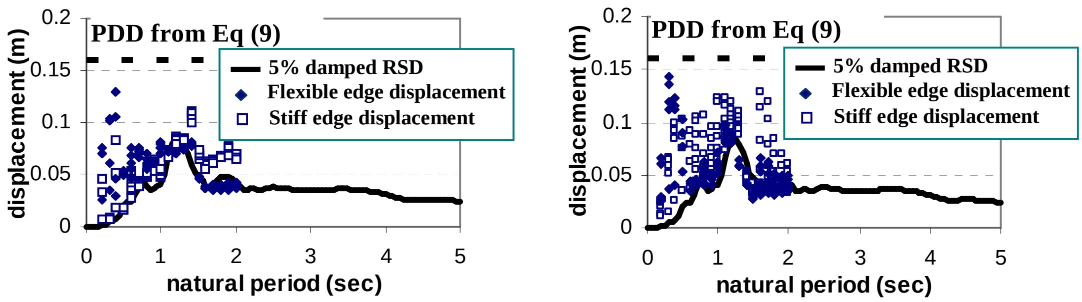
$$\Gamma_{DD} = \sqrt{\left(0.8 + 0.4\frac{b}{r}\right)^2 + \left(0.2 - 0.4\frac{b}{r}\right)^2 + 2\left(0.5\frac{b}{r}\right)^2} \quad \text{and} \quad \mathbf{PDD} = \Gamma_{DD}\ \mathrm{RSD}_{max} \qquad (9)$$

where, b is the distance measured from the center of mass to the edges of the building and r is the mass radius of gyration of the building. The value of Γ_{DD} was estimated to be up to around 2.0 for b/r less than 1.8. It was shown in Figure 11 that Equation (9) could provide conservative estimates of the maximum displacement demands (**PDD**) of buildings with asymmetries in the orthogonal directions (ie. Type B asymmetry as illustrated in Fig. 7b).



(a) with parallel frames only (b) with orthogonal frames only (c) with frames in both directions

Figure 10 Schematic response of asymmetrical buildings



M = 7, R = 90 km, site class D (a) $R_\mu = 2$ **M = 7, R = 90 km, site class D** (b) $R_\mu = 4$

Figure 11 Displacement demands of buildings with Type B asymmetry (e = 0.5, ρ_k = 0.8) subject to bi-directional excitations

4 Concluding remarks

A seismic assessment of asymmetrical structures in regions of low to moderate seismicity has been developed based on the concept of displacement-controlled behaviour. The potential seismic performance of a structure can be assessed by comparison of the displacement demand with the displacement capacity of the structure.

The maximum response spectral displacement RSD_{max} associated with the seismic hazard of Australia was presented. Parametric studies based on non-linear time history analyses of SDOF systems revealed that the highest point on the displacement response spectrum can provide conservative predictions of the maximum displacement demand of the structure (**PDD** = RSD_{max}). Further studies on an asymmetrical single-storey building model resulted in a simple analytical model which can be used to estimate the torsional effect on asymmetrical buildings. One interesting finding is that the peak drift demands were found to be insensitive the torsional resistance of the building and its eccentricity. The proposed mathematical model can be extrapolated to estimate the peak drift demands of multi-storey buildings in which different levels could have varying torsional resistance and eccentricity. It was found that a torsional amplification factor (Γ_{DD}) of 1.6 and 2.0 can provide conservative estimates of the peak drift demands (**PDD** = Γ_{DD} RSD_{max}) of one-way and two-way asymmetric system respectively. The maximum displacement demand **PDD** can be compared to the displacement capacity of a building for a quick assessment of its potential seismic risks to collapse.

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