1. INTRODUCTION

The ground motion characteristics of a soil site can be highly dependent on conditions of the overlying Quaternary sediments. Response spectrum models stipulated by contemporary codes of practices specify site factors for different site classes and hence enable site effects to be predicted without calculations, or with very simple manual calculations. Site classification schemes adopted by major codes of practices typically parameterize soil dynamic properties on the basis of the shear wave velocity averaged over a certain depth in the sediment. With this modelling approach, which is based on the statistical analyses of large volumes of empirical data, parameters representing details of the soil layers have been averaged. Consequently, the effects of multiple reflections at the boundaries of the soil medium (pertaining to resonance behaviour) have not been parameterized.

The significance of the soil resonance phenomenon depends on soil conditions, level of seismic hazard, seismic performance criterion of the structure and its ductility level. The resonance phenomenon deserves special attention with flexible soil sediments with high impedance contrast at the interface with bedrock, and more so in regions of low and moderate seismicity typified by structures with limited ductility which accentuates the effects of resonance. The effects of resonance results in high displacement (drift) demand on structures and are best represented by the displacement response spectrum. The new Australian Standard for seismic actions uses the fundamental natural period of the site as a key parameter for site classification. The soil resonance phenomenon is hence addressed albeit in a very broad sense. However, the impedance contrast between the soil and the underlying bedrock which controls the extent of the resonance behaviour has not been parameterized.

A range of analytical software has been developed to model a multitude of site modification mechanisms including the effects of soil resonance at varying levels of sophistication. Whilst one-dimensional non-linear wave analysis approach is the lowest tier approach it is operated with the most widely used analytical tool, for example, program *SHAKE* (Schnabel *et al.*, 1972) due to its relative ease of use and the low demand on input information in comparison with the higher tier programs. Even then, programs like *SHAKE* are still not well known to practising professionals in low and moderate seismic regions. The main difficulties with the use of these time-history analysis programs is the lack of knowledge on the ground motion time-histories and hence uncertainties as to what accelerogram data is considered representative and suitable for input into the analysis.

This paper presents the development of a simple (hand-calculation) model for predicting site effects characterized by soil resonance behaviour as described above. Importantly, the impedance ratio between the bedrock and the overlying soil has been introduced as a parameter in the calculation along with damping parameters. It is noted that many of the expressions used in developing the proposed calculation procedure are based on well established wave theories. The original contributions of this paper are in combining these expressions for the direct estimation of the displacement response amplification factor and for constructing a displacement response spectrum model which accounts for the effects of soil resonance. The predicted amplification has been shown to be very consistent with results obtained from analyses using program *SHAKE*. The proposed calculation procedure which is in its early stage of development is based on modelling the soil sediment as a homogenous material overlying bedrock. Intuitively, non-homogenous soil layers may also be analysed using this method by weighted averaging the soil shear wave velocity and density. Further study is now underway to further develop this method for handling complex layering conditions.

2. THEORETICAL DEVELOPMENT

2.1 Basic parameters and expressions

The objective of the proposed calculation procedure is to estimate the spectral ratio (*SR*) which is defined herein as the ratio of the maximum response spectral displacement on the surface of the soil (RSD_{max}) and the corresponding response spectral displacement on the adjacent rock outcrop (RSD_{Tg}) at the fundamental natural period of the site (T_g). The value of T_g can be estimated using the well known quarter-wavelength approximation as represented by equation (1).

$$T_g = \frac{4H}{V_S} \tag{1}$$

where H is the depth of the soil and the V_s is the weighted average shear wave velocity.

Structures found on the soil site and possessing this natural period will experience resonance behaviour and hence this period is most critical in terms of the seismic displacement demand. Refer Figure 1 for a schematic illustration. The amplification from RSD_{Tg} to RSD_{max} is modelled in two parts: (i) amplification of the peak displacement demand at the bedrock surface to that at the soil surface as represented by the peak displacement ratio (*PDR*) and (ii) response amplification of an elastic single-degree-of-freedom system when subject to periodic motion at the soil surface and is represented by the resonance factor (f). The relationship between *SR* and the amplification factors is defined by equation (2).

$$SR = \frac{RSD_{\max}}{RSD_{Tg}} = PDR \cdot f \tag{2}$$

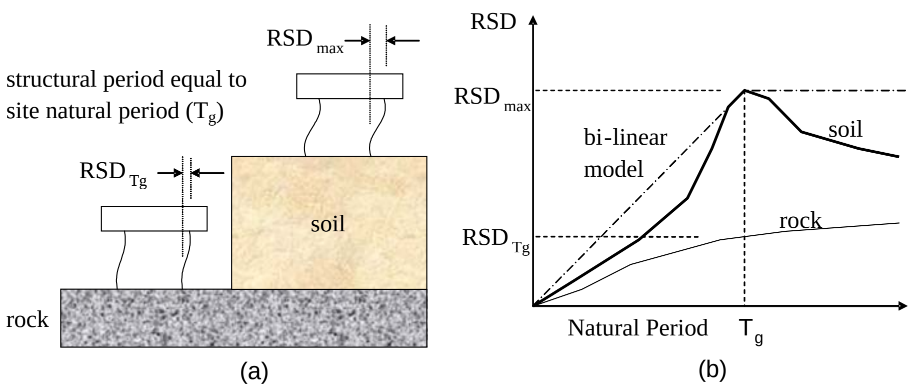

Figure 1 Schematic representation of displacement amplification on a flexible soil site.

Modelling of the *PDR* is based on three principal mechanisms: (i) transmission of seismic waves across the interface between two media (the bedrock and soil media), (ii) reflection of seismic waves at the two boundaries of the soil medium (i.e. boundary with rock and that with air), and (iii) hysteretic energy dissipation during wave transmission within the soil medium.

As upward propagation seismic waves reach the interface between the bedrock and the soil, only part of the wave energy is transmitted into the soil whilst the rest is reflected back into the half-space of the bedrock. The displacement amplitude of the transmitted wave (A_T) and the reflected wave (A_R) can be calculated using equations (3a & 3b) based on elementary wave theory for zero angle of incidence (approach the interface at 90° angle).

$$A_R = \frac{\alpha - 1}{\alpha + 1} A_i \quad \text{and} \quad A_T = \frac{2\alpha}{1+\alpha} A_i \qquad \text{(3a) and (3b)}$$

where A_i is the amplitude of the incident wave and α the impedance ratio as defined by equation (4).

$$\alpha = \frac{\rho_R V_R}{\rho_S V_S} \quad (4)$$

where ρ and V are the weighted-average of the density and the shear wave velocity of the respective layers (the subscripts R and S represent the rock and soil layers respectively).

From equation (3b), the amplitude of the seismic waves is amplified by a factor of between 1 and 2 when transmitted into the soil medium. This same equation can be used to model the amplification of seismic waves reaching the soil surface in which case the value of α is equal to infinity (based on considering the soil and air as two media separated by the interface). This surface amplification factor is accordingly equal to 2 (as is widely known).

When seismic waves are amplified by a factor of 2 as they reach the soil surface, there are waves reflecting back down into the soil medium. The amplitude of the downward propagating reflected waves is accordingly equal in amplitude and sign to the incident wave based on equation (3a) but with the value of α made equal to infinity.

The reflected seismic waves will then reach bedrock for the second time when reflection will again occur. Equation (3a) may, yet again, be used for modelling seismic waves reflecting from the bedrock-soil interface back up into the soil medium, but the value of α is reciprocal to that defined by equation (4) due to the change in direction of the wave transmission. The ratio of the amplitude of the reflected and incident waves, which is defined as the wave reflection coefficient (R), can be calculated using equation (5).

$$R = \frac{\frac{1}{\alpha} - 1}{\frac{1}{\alpha} + 1} = \frac{1 - \alpha}{1 + \alpha} \quad (5)$$

From equation (5), the value of R varies between 0 and 1 (as α varies between 1 and infinity) and with a change in sign which means that the sense (or polarity) of the waves will also change. The de-amplification of the seismic waves ($R < 1$) reflected back up from the bedrock surface is sometimes described as radiation damping. The effects of the four modifications occurring at the boundaries of the soil medium as described above is summarised in Table 1.

Unlike boundary mechanisms, hysteretic damping occurred within the soil medium modifies wave amplitude continuously. The de-amplification of the wave amplitude can be expressed as an exponential function of the number of wave cycles experienced during the wave transmission. The de-amplification factor (β) for half the wave-cycle is given by equation (6).

$$\beta = \exp(-\pi\zeta) \quad (6)$$

where ζ is the critical damping ratio (as is widely used to represent damping in structures). The dependence of the shaking level (nonlinearity) in site response is accounted for by this soil damping ratio. A model for estimating soil damping ratio for given intensity of shaking has been developed in Tsang and Chandler (2005). Details are not presented herein. From equation (1), seismic wave components possessing the site natural period (T_g) will experience quarter-of-a-cycle periodic motion during the transmission of the waves through the thickness of the soil medium. The reduction in the wave amplitude is accordingly represented by equation (7).

$$A_0 = \beta^{\frac{1}{2}} A_T \tag{7}$$

where A_0 is the wave amplitude reaching the soil surface, β and A_T have been defined in equations (6) and (3b), respectively.

Table 1 Summary of wave modification mechanisms

Modification no.	Description	Equation no.	α	Amplitude ratio	Change in sign
1	Initial transmission from bedrock into soil	3b	Eqn.4	1-2	no
2	Amplification on soil surface	3b	infinity	2	no
3	Reflection from soil surface	3a	infinity	1	yes
4	Reflection from bedrock	5	Eqn.4	<1	no
5	Hysteretic damping	7	NA	<1	no

2.2 Modelling the peak displacement ratio

The amplification of seismic waves reaching the ground surface depends on the modifications (see Table 1) which the wavefront has experienced since entering the soil medium. Table 2 (to be read in conjunction with Figure 2) demonstrates the calculation for the amplitude along the travel path of the wavefront. In estimating the amplitude at wavefront positions (i) and (ii), equations 3b & 7 can be used to take into account the effects of impedance contrast (modification no.1) and hysteretic damping (modification no.5) as explained in Section 2.1. Given that reflection at the soil surface (modification no. 3) does not have an effect on the wave amplitude, the change in amplitude between wavefront positions (ii) – (iii) is only due to hysteretic damping as represented by the $\beta^{1/2}$ factor. In estimating the amplitude at wavefront position (iv), the "R" factor is introduced to take into account radiation damping at the rock-soil interface (modification no. 4). A further $\beta^{1/2}$ factor is introduced to estimate the amplitude at wavefront position (v), and finally, a surface amplification factor of 2 (modification no. 2) is introduced to calculate the amplitude of the soil surface motion as experienced by the above-ground structures.

Table 2 Amplitude at different positions on the wavefront

Position of wavefront (refer labels in Fig. 2)	Modification nos. as listed in Table 1	Expressions for estimating wave amplitude	Remarks
(i)	1	$\frac{2\alpha}{1+\alpha} A_i \quad (= A_T)$	as defined by eqn.3b
(ii)	1-5	$\beta^{\frac{1}{2}} A_T \quad (= A_0)$	as defined by eqn.7
(iii)	1-5-3-5	$\beta^{\frac{1}{2}} A_o$	Eqn.8a
(iv)	1-5-3-5-4	$R\,\beta^{\frac{1}{2}} A_o$	Eqn.8b
(v)	1-5-3-5-4-5	$R\beta A_o$	Eqn.8c
(vi)	1-5-3-5-4-5-2	$2R\beta A_o$	Eqn.8d

The upwardly propagating S-waves after reflecting from the soil-bedrock interface will reach the soil surface to complete half a cycle of wave motion. The displacement amplitude is defined by equation (9a).

$$A_{\frac{Tg}{2}} = 2R\beta A_0 \tag{9a}$$

The same modifications will be experienced by the waves when undergoing another half a cycle of motion (with yet another change in the wave polarity). On completion of the two half-cycles, the displacement amplitude of the wave reaching the soil surface is defined by equation (9b).

$$A_{Tg} = R\beta A_{\frac{Tg}{2}} = 2R^2\beta^2 A_0 \tag{9b}$$

Equations (7) & (9b) represent the displacement amplitude of the wave when reaching the soil surface at time $T = 0$ and $T = T_g$ (ie. n = 0 and 1) respectively. The polarity of the wavefront at both instances have the same polarity.

Wavefronts with time-lag will superpose as they are reflected onto the soil surface repetitively. The amplitude of wave components as defined by equation (7) and (9a), for n = 0 and 1 respectively, can be aggregated using equation (10a) which satisfies the principle of the conservation of energy.

$$\tilde{A}_{Tg} = 2\sqrt{A_0^{\ 2} + A_{Tg}^{\ 2}} = 2A_0\sqrt{1+R^4\beta^4} \tag{10a}$$

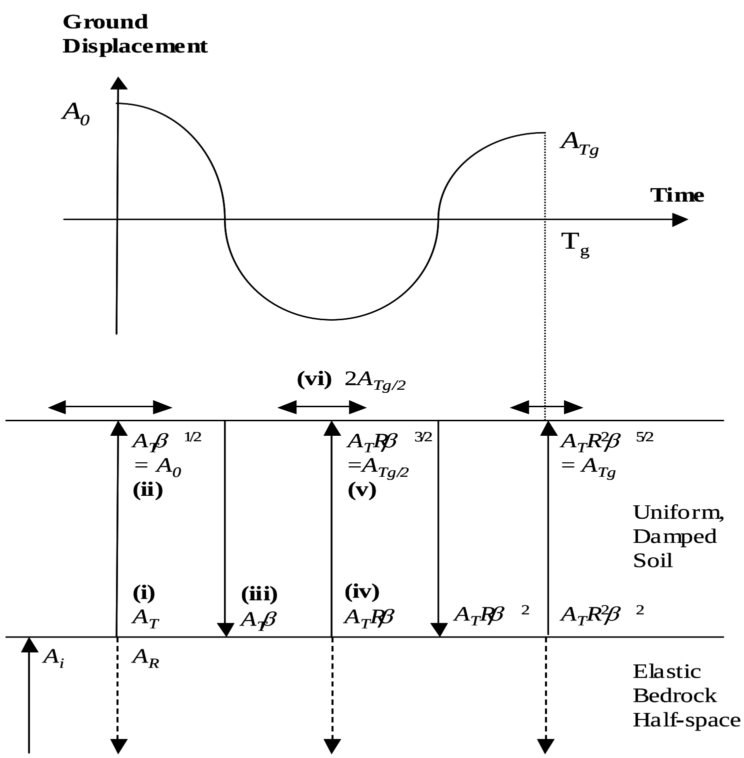

Figure 2 Illustration of the concept of the site fundamental natural period, multiple wave reflections, material and radiation damping (as summarized in Table 2)

The aggregation of the wave components can be extended to the limit of n = infinity as represented by equation (10b) which is based on the summation of infinite number of terms in a geometric series.

$$A_{soil-surface} = \sqrt{\sum_{n=0}^{\infty} A_{nTg}^{\ 2}} = 2A_o \sqrt{\sum_{n=0}^{\infty} \left(R^{2n}\beta^{2n}\right)^2} \tag{10b}$$

where n is the number of wave cycles (of period T_g).

Given that the value of $R^{2n}\beta^{2n}$ are less than unity (as α and ζ are both less than unity), equation (10b) can be re-written as :

$$A_{soil-surface} = 2A_0 \sqrt{\frac{1}{1-R^4\beta^4}} \tag{11}$$

Substituting equations (3b) and (7) into Eq. (11) results in equation (12)

$$A_{soil-surface} = A_i \frac{4\alpha}{1+\alpha} \sqrt{\frac{\beta}{1-R^4\beta^4}} \tag{12}$$

In comparison, the amplitude of ground motions experienced by structures founded directly on the rock surface can be represented by equation (13).

$$A_{rock-surface} = 2A_i \tag{13}$$

where the factor of 2 represents the surface effects at the interface between rock and air.

The peak displacement ratio (*PDR*) which is the ratio of the wave amplitude as calculated from equations (12) and (13) is hence represented by equation (14).

$$PDR = \frac{A_{\text{soil-surface}}}{A_{rock-\text{surface}}} = \frac{2\alpha}{1+\alpha} \sqrt{\frac{\beta}{1-R^4\beta^4}} \tag{14}$$

The *PDR* calculated from equation (14) can be substituted into equation (2) for obtaining an estimate for the response spectral factor (*SR*).

2.3 Resonance Factor (*f*)

The response of linear elastic single-degree-of-freedom (SDOF) systems found on the soil surface is considered next. The modelling is based on systems with natural period matching the site natural period. The amplification of the system's response, which is represented by the "*f* " factor in equation (2), has been found to be sensitive to the rate of energy dissipation in both the soil and the structure. The empirical function of equation (15) was developed by the authors (Tsang *et al.*, 2005) in a parametric study to study the trends.

$$f(\alpha) = \alpha^{0.3} \leq 2.3 \tag{15}$$

The upper limit of 2.3 is to reflect the observation that f becomes insensitive to changes in the value of α when α > 16.

Equation (16) is finally obtained by combining equations 14 & 15 to provide an estimate for SR

$$SR = f(\alpha) \cdot \frac{2\alpha}{1+\alpha} \sqrt{\frac{\beta}{1-R^2\beta^2}} \tag{16}$$

3. VERIFICATION OF MODEL

Shear wave analyses using program *SHAKE* have been undertaken on some twenty soil columns to analyse the value of *SR* for comparison with results obtained using equation 16. The analyses covered the following parameter values: (i) bedrock spectral velocity (RSV_{Tg} = 20 – 400 mm/s)

(ii) initial soil shear wave velocities (V_s = 100, 150, 200, 300 and 500 m/s), (iii) initial site natural period (T_i = 0.12 sec - 2.4 sec), (iii) soil plasticity indices (PI = 0, 15, 30 and 50 %) and (iv) shear wave velocity of the bedrock half-space (V_R = 500 – 3500 m/s). The correlation between results obtained from equations (14) and (16) and that obtained from *SHAKE* provide support for the proposed model (refer Figures 3a and 3b). Further verification has been provided in Tsang *et al.* (2005), by comparing the proposed model with 1994 Northridge earthquake recordings compiled in Borcherdt (2002).

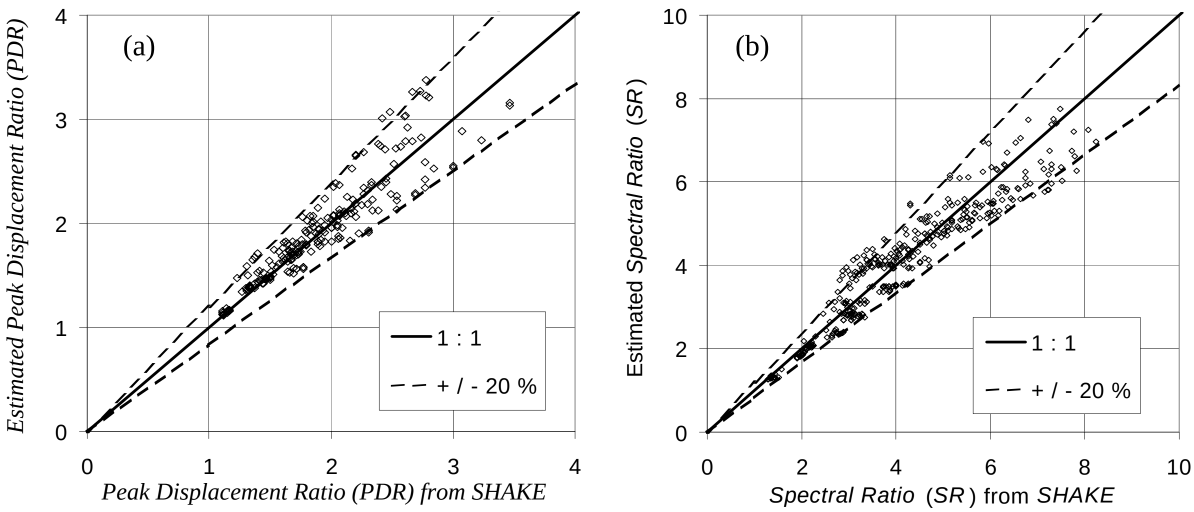

Figure 3. Correlation of (a) peak displacement ratio (*PDR*) and (b) spectral ratio (*SR*) [defined in equation (2)] estimated using equations (14) and (16) and the computed values from *SHAKE*.

4. CONCLUSIONS

(i) The effects of soil resonance on the site seismic hazard can be represented by the spectral ratio which is defined by equation (2). The model proposed in this paper enables the value of SR to be predicted by a simple hand-calculation procedure and a simple displacement spectrum to be constructed.

(ii) In the proposed procedure, *SR* is expressed as the product of peak displacement ratio *PDR* and resonance factor *f*. *PDR* can be estimated as a simple function of the impedance contrast ratio α (equation 4) and the hysteretic damping factor β (equation 6), whilst f can be calculated using equation (15).

(iii) Verification analyses based on comparison with results obtained from program *SHAKE* have been undertaken to support the proposed model (refer Figure 3).

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