Effects of uncertain earthquake source parameters on ground motion simulation using the empirical Green's function method

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Abstract

A combined stochastic and Green's function approach was developed to simulate strong ground motions in Southwest Western Australia (SWWA) in a previous study. Although it was proven that adopting the source parameters derived from other regions yielded reasonable simulation of ground motions in SWWA as compared with a few available strong motion records, the effect of source parameter variations on simulated ground motions was not known. This paper performs a statistical study of the effects of random fluctuations of the seismic source parameters on simulated strong ground motions. The uncertain source parameters, i.e., stress drop ratio, rupture velocity and rise time, corresponding to the empirical source models are assumed to be the mean value and normally distributed with an assumed coefficient of variation. An ML6.0 and epicentral distance 100 km event is simulated using Rosenblueth's point estimate method to estimate the mean and standard deviation of PGA, PGV and response spectrum. The accuracy of the Rosenblueth's approach is proved by Monte Carlo simulations.

Keyword: uncertain earthquake source parameters, empirical Green's function method

1. Introduction

Because of the lack of strong ground motion records, the studies of seismic effects on structures in Southwest Western Australia (SWWA) are based primary on strong ground motion models and records elsewhere, especially those in Central East North America (CENA). Recently, Hao and Gaull (2004) compared the various CENA models and a few recorded ground motion time histories in SWWA and found that none of the CENA models gave very satisfactory predictions of the recorded motions in SWWA. Because the number of available records in SWWA is limited and it is not practical to use those records alone to develop a reliable ground motion model for SWWA, Hao and Gaull then modified the Atkinson and Boore model (1995) based on available ground motion records to derive a model for SWWA. The modified model was proven yielded better prediction of local recorded motions (Hao and Gaull 2004). However, since most recorded motions in SWWA are associated with minor earthquakes of magnitude less than ML4.5. The modified model may be biased towards ground motion characteristics of small earthquakes. To overcome this problem, a combination of Green's function and stochastic method has been developed to generate strong ground motion time histories (Liang *et al.*, 2006). In the latter method, ground motion time histories at various epicentral distances from small earthquakes are stochastically simulated. The simulated ground motion time histories, together with the recorded motions from small earthquakes, are used as input to simulate ground motions of large earthquakes with the empirical Green's function method. By comparing with the only records available in SWWA from two relatively large earthquakes, i.e., an ML5.5 event and an ML6.2 event, it was demonstrated that the combined method gave reasonably good prediction of ground motions in SWWA from large earthquakes (Liang *et al.*, 2006).

Because study of earthquake source and path parameters in SWWA is limited, many of the CENA parameters are adopted in the latter study (Liang *et al.*, 2006). The reliability of using these parameters was proven only with four strong ground motion records in two earthquake events. In reality, many uncertainties exist in the seismic source and path parameters. Variations of these parameters may greatly affect the simulated ground motions. This paper analyses the effects of variations of uncertain earthquake source parameters on the simulated ground motions using the empirical Green's function approach. Statistical variations of the various source parameters are considered in the simulation and their effects on the simulated ground motions are examined. An ML6.0 and epicentral distance 100 km event is simulated as an example. Each source parameter is assumed statistically varying with a normal distribution. The respective source parameter value from the empirical model is taken as the mean value with an assumed standard deviation in this study. Rosenblueth's point estimate method (Rosenblueth, 1981) is used for statistical calculations. The accuracy of the Rosenblueth's point estimate method is verified by Monte Carlo simulations. The Monte Carlo simulation results are also used to derive the distribution types of the parameters of the simulated ground motion time histories.

2. The Green's Function Method and the Seismic Source Parameters

The equation of empirical Green's function method (Irikura *et al.*, 1997) is given as

$$U(t) = \sum_{i=1}^{N} \sum_{j=1}^{N} (r / r_{ij}) \cdot F(t) * (C \cdot u(t)) \tag{1}$$

$$F(t - t_{ij}) = \delta(t - t_{ij}) +$$
$$\{ (1/n')(1 - \exp(-1)) \} \sum_{k=1}^{(N-1)n'} \left[\exp\{-(k-1)/(N-1)n' \} \times \delta\{t - t_{ij} - (k-1)T/(N-1)n' \} \right] \tag{2}$$

and

$$t_{ij} = \left| r_{ij} - r_0 \right| / V_s + \left(r_{ij} - r_0 \right) / V_r + \varepsilon \tag{3}$$

where '*' means convolution, $U(t)$ is the ground motion of large event; r is the distance between the hypocenter of small event and the receiver; r_{ij} is the distance between the subfault (i,j) and the receiver; r_0 is the distance between the subfault (i,j) and the hypocenter of large event; $F(t)$ is the slip-time filtering function; C is the stress drop ratio; V_s is the shear wave velocity; V_r is the rupture velocity; $u(t)$ is the contribution of the jth sub event; $\delta(t-t_{ij})$ is the Dirac delta function; t_{ij} is the phase delay term. T is the rise time for large event. ε is a small random time for each cell to remove periodicities caused by the cell size. n' is a properly selected integer to eliminate spurious periodicity. N is the scaling between large and small event, which is derived from the study of Kanamori and Anderson (1975). Irikura (1986) introduced the following Eq. (4) for different stress drop between small and large event.

$$L / L_e = W / W_e = T / \tau = (M_O / Cm_O)^{1/3} = N, \qquad D / d = CN \tag{4}$$

In the above model, the earthquake source is characterized by a set of source parameters, i.e., stress drop, fault dimensions, rupture velocity and rise time. These parameters affect the simulated ground motions. Many authors have studied these parameters and various empirical relations for these source parameters have been proposed (Celler, 1976, Wells and Coppersmith, 1994, Dowrick and Rhoades, 2004). Many unknown and/or uncertain factors influence these parameters. In most previous studies, however, they are assumed as deterministic. The effects of their variations on earthquake ground motions are not properly studied yet.

In this study, the source parameters, i.e., the stress drop ratio, rupture velocity or phase delay and rise time are assumed to vary randomly. The fault size is closely related to the seismic magnitude. It is assumed as deterministic in this study. As can be seen in Eq. (1) to Eq. (4), the stress drop ratio affects the N value which causes the change of superposition times. The rise time of large event determines the corner frequency of spectrum. Phase delay term has an effect on phase spectrum. It should be noted that the variations of the path parameters are not explicitly considered in this study. However they are implicitly included in stochastic simulations of ground motions because the simulations are carried out according to the target ground motion spectrum, which usually are the mean spectrum of the expected ground motions.

3. Variations of the Seismic Source Parameters

Earthquakes in SWWA are intraplate events and the stress drop does not seem to be constant in small magnitude event, which can be observed in the data of Burakin earthquake (Allen *et al.*, 2006). However, for earthquakes of magnitude above 5, compared with the recorded data, Liang *et al.* (2006) found that the constant-stress scaling law seems suitable because simulated motions based on a constant stress drop assumption well fit with the recorded motions. To study the effect of stress drop variation on ground motions in this study, the constant stress-drop ratio is assumed with a normal distribution.

Another parameter that significantly affects the ground motion is the phase delay which is also assumed to vary randomly. Rupture velocity is a main factor that affects phase delay. In this study, the mean rupture velocity is taken as 0.8 times of the shear wave velocity of 3.91 km/sec of the seismic source in SWWA (Dentith *et al.*, 2000). The normal distribution is also assumed.

The mean value of rise time was computed using Eq. (5) (Somerville *et al.*1993). It is also assumed to vary randomly. It should be noted that the stress-drop ratio, rupture velocity and rise time may be inter related. In this study, however, they are assumed to be statistically independent of each other owing to the lack of information on their cross correlation.

$$T = 1.72 \times 10^{-9} \left(M_0\right)^{1/3} \qquad (5)$$

4. Ground Motion Simulation with Uncertain Seismic Source Parameters

As an example, ground motion time history from an ML6.0 and epicentral distance 100 km event is simulated from a small event of ML4.5 and the same epicentral distance. The mean value, coefficient of variation, and distribution type of the three random source parameters are defined in Table 1. Monte Carlo simulation and Rosenblueth's point estimate method were applied in the simulation to calculate the mean value and standard deviation of the PGA, PGV and response spectrum of the simulated ground motion time histories. The Monte Carlo simulation results are also used to determine the distribution types of these parameters of the simulated time history.

Table 1.Random variables and their distribution

Parameters	Mean	C.O.V. %	Distribution
Stress drop ratio	1	10	normal
Phase delay term (rupture velocity)	3.1 (km/sec)	10	normal
rise time	0.39(sec)	10	normal

4.1 Monte Carlo simulation

Ground motion simulations are carried out with many randomly selected stress-drop ratio, phase delay and rise time. For each simulated time history, the PGA, PGV and response spectrum are determined. A probability density function or cumulative density function of PGA, PGV and response spectrum can then be determined from a large number of simulations. The mean value and standard deviation of PGA, PGV and response spectrum can then be determined.

A convergence test is conducted to check the number of Monte Carlo simulations required to obtain converged simulation results. The simulated ground motion PGA, PGV and response spectrum values at 0.1sec, 1.0sec, 2.5sec and 5sec are used as the quantity for the before convergence test. It is found that the mean value and standard deviation of PGA, PGV and the response spectrum amplitudes remained virtually unchanged after 600 simulations as shown in Figure 1 and Figure 2. Therefore, in the subsequent calculations 600 simulations are performed for each case. The 600 simulated data for PGA, PGV and response spectrum values at the selected periods all display a lognormal distribution. To verify these observations, a Kolmorogov–Smirnov goodness-of-fit test (K–S test) is carried out. The significance level alpha for the test is 0.01 in this study.

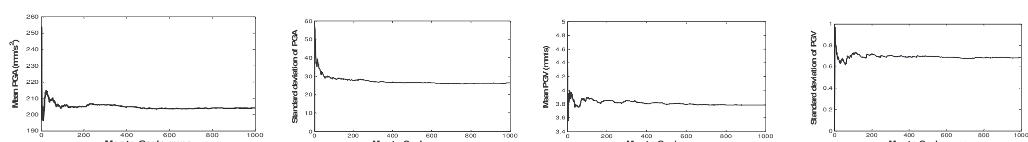

Figure 1. Mean value, standard deviation of PGA and PGV

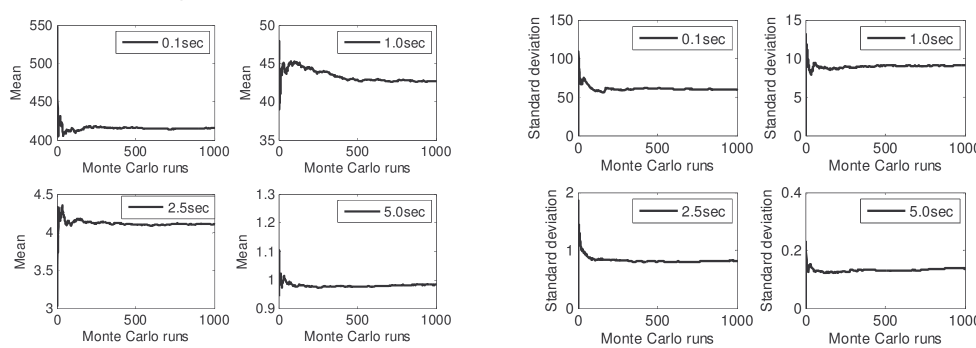

Figure 2. Mean value, standard deviation of response spectrum at 0.1sec, 1.0sec, 2.5sec and 5sec

Figure 3 illustrates the density histograms of the PGA, PGV and response spectrum at 0.1sec, 1.0sec, 2.5sec and 5sec, and the corresponding lognormal distribution function. As shown, lognormal distribution function fits the simulated data well. All parameters pass the K-S test with a 1% significance level, indicating a very good-fit. Table 2 gives the results of Monte Carlo simulation and K-S test.

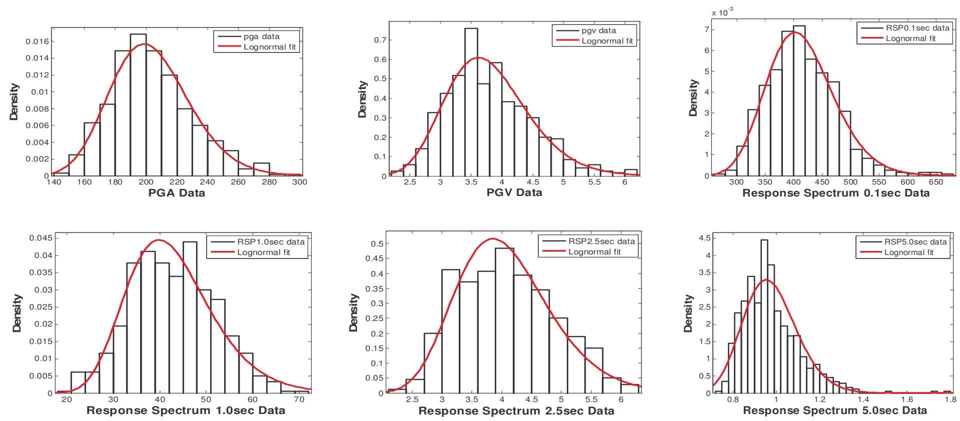

Figure 3. Probability density for PGA, PGV and response spectrum at 0.1sec, 1.0sec, 2.5sec and 5sec and the corresponding lognormal distribution function

Table 2. Monte Carlo simulation and K-S test result for PGA, PGV and response spectrum

	Mean	Standard deviation	The test statistic	The critical value
PGA (mm/s^2)	203.481	26.00	0.0362	0.0661
PGV(mm/s)	3.79	0.68	0.0392	0.0661
RSP0.1sec(mm/s^2)	414.99	59.64	0.0183	0.0661
RSP1.0sec(mm/s^2)	42.83	9.53	0.0542	0.0661
RSP2.5sec(mm/s^2)	4.09	0.81	0.0402	0.0661
RSP5.0sec(mm/s^2)	0.98	0.12	0.0657	0.0661

4.2. Rosenblueth's point estimate method

Monte Carlo simulation is straightforward to use and can give reliable estimations of statistical parameters of the simulated ground motion time histories. However, it is extremely time consuming and needs a large number of simulations, e.g., 600 simulations in this case, to get the converged estimations. The Rosenblueth's point estimate method (Rosenblueth, 1981) allows a direct estimation of the mean response and standard deviation. Because it is computationally more efficient than the Monte Carlo simulation method, in this study, the Rosenblueth's point estimate method is also used. Its reliability is verified by using the Monte Carlo simulation results.

To use the Rosenblueth's point estimate method, 8 simulations are needed for three random variables in this study. Use PGA as an example, PGA_{+++} is the PGA value of the ground motion simulated with mean value (μ) plus one standard deviation (s) of the three random source parameters, i.e, stress-drop ratio, rupture velocity and rise time. Similarly, PGA_{---} is the PGA value of the time history simulated using mean value minus one standard deviation of the three random source parameters. In this study, it is assumed that the three variables are independent of each other. Then the point-mass weights are given as:

$$P_{+++} = P_{---} = P_{+--} = P_{++-} = P_{-++} = P_{--+} = P_{+-+} = P_{-+-} = \left(\frac{1}{8}\right) = 0.125 \tag{6}$$

The mean PGA□ μ_{PGA}, and the variance of PGA, σ_{PGA}, are

$$\begin{aligned}\mu_{PGA} = {} & P_{+++}PGA_{+++} + P_{---}PGA_{---} + P_{+--}PGA_{+--} + P_{++-}PGA_{++-} + P_{-++}PGA_{-++} + P_{--+}PGA_{--+} \\ & + P_{+-+}PGA_{+-+} + P_{-+-}PGA_{-+-}\end{aligned} \tag{7}$$

$$\begin{aligned}\sigma^2_{PGA} \approx {} & P_{+++}PGA^2_{+++} + P_{---}PGA^2_{---} + P_{+--}PGA^2_{+--} + P_{++-}PGA^2_{++-} + P_{-++}PGA^2_{-++} + P_{--+}PGA^2_{--+} \\ & + P_{+-+}PGA^2_{+-+} + P_{-+-}PGA^2_{-+-} - \mu^2_{PGA}\end{aligned} \tag{8}$$

Following the above point estimate method, the statistical parameters of PGA, PGV and response spectrum are derived and listed in Table 3. As shown, the Rosenblueth's method yields reliable estimations of the ground motion statistics. The results also indicate that the source parameter uncertainty significantly influences the simulated ground motions, and the source parameter uncertainty level amplifies. With a 10% variation in source parameters, the variations of the simulated ground motion parameters are in general more than 10%. The variation in response spectrum could be around 20%. These observations imply the importance of reliably determining the source parameters when using Green's function method to simulate ground motions.

Table 3. Point estimation and Monte Carlo simulation results for PGA , PGV and response spectrum in 0.1sec, 1.0sec, 2.5sec and 5sec

	Point estimate method		Monte Carlo simulation	
	Mean	C.O.V (%)	Mean	C.O.V (%)
PGA (mm/s^2)	209.95	13.8	203.48	12.8
PGV (mm/s)	3.41	13.2	3.79	17.4
RSP0.1sec(mm/s^2)	401.35	11.3	414.99	14.4
RSP1.0sec(mm/s^2)	45.48	19.6	42.83	22.3
RSP2.5sec(mm/s^2)	4.43	16.9	4.09	19.8
RSP5.0sec(mm/s^2)	0.97	6.2	0.98	12.2

5. Discussion and conclusion

This paper studies the effects of uncertain source parameters on ground motions simulated using Green's function method. Assuming the three earthquake source parameters, i.e., stress drop ratio, phase delay and rise time, have normal distribution and a 10% coefficient of variation, the statistics of PGA, PGV and response spectrum of the simulated ground motion are calculated by Monte Carlo simulation and Rosenblueth's point estimate method. It has proven that the Rosenblueth's point estimate method gives similar results as the Monte Carlo simulations. Numerical results also indicated that variations of the earthquake source parameters significantly affect the simulated ground motions. With 10% variations in source parameters, the variation of PGA, PGV and response spectrum of the simulated ground motion are all more than 10%, indicating the importance of reliably determining the earthquake source parameters in ground motion simulations.

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