Experimental Evaluation of Reduced Models of Large Structural Systems for Active Vibration Control

J. Boffa and N. Zhang
Faculty of Engineering,
University of Technology, Sydney.
PO Box 123, BROADWAY NSW 2007, AUSTRALIA.
john.boffa@eng.uts.edu.au

ABSTRACT

This paper assesses the performance of reduced plant models of large and flexible structures obtained from using two different model reduction methods in vibration analysis and active control. The Dynamic model reduction method and the Guyan method are compared using experimental test results. A tall building model with 20 degrees of freedom was used as the plant, with a linear motor installed at the top storey for the purposes of active-damping. Although the results of simulations would suggest that both models perform sufficiently well, experimental testing proved that only the Dynamic model performs adequately for this specific application of active control. The problem associated with the Guyan method, and with most other model reduction methods, is that they assume that the system behaves strictly according to linear elastic theory. The versatility of the Dynamic model reduction method is such that it provides the option of obtaining system parameters from experiment, not just from theory. The experimental procedure ensures that the Dynamic model reduction method forms an accurate description of the real system dynamics. The applicability of this method for obtaining low-order plant models in the active vibration control of flexible structures was demonstrated through physical testing of the structure, while it was subject to sinusoidal excitation. The tests have shown that the Dynamic model reduction method can be used as an alternative approach for model reduction of structural systems for the purpose of active vibration control.

1. INTRODUCTION

For the active vibration control of complicated mechanical or structural systems a reduced dynamic model with a very limited number of degrees of freedom and yet sufficient accuracy is often required. One of the typical applications is the active vibration control of high rise and flexible building structures subject to earthquake excitations and wind loads that have mainly low frequency components. In this case, the dynamic responses of the structural systems concerned contain mainly the contributions made by a few of their lowest modes of vibration. Consequently, the vibrations can be effectively controlled based on a reduced-order plant model that contains only a few of the lowest modes of these structures (Seto et al, 1991).

The use of a reduced plant model within the controller can minimise the computation time for determining the feed-back gains required by the actuators and therefore improve the overall performance of the combined plant-controller system (Matsumoto et al, 1998; Kajiwara et al, 1992). Seto et al, 1991 and Matsumoto et al, 1998 pointed out the importance of having reduced plant models in terms of meaningful physical parameters such as mass, damping, stiffness parameters and presented a few successful applications of the active vibration of flexible structures based on reduced physical low-order plant models. Ma and Hagiwara, 1991 developed the Mode-displacement method for obtaining the reduced model of a large structural system. The resultant models often perform well in structural analysis. Zhang, 1995 presented a dynamic model reduction method that produces reduced models of systems with a large number degrees of freedom for dynamic analysis. The reduced models are formulated from condensed mass, damping and stiffness coefficient matrices and retain a small number of the lowest modes of the original system. Care needs to be taken in choosing the reference frequency for taking into account the dynamic effect of the high modes, and in choosing the master coordinates that are retained in the reduced models.

This paper begins with a description of the two different model reduction techniques that were used in this investigation, and then presents their simulated performance. The theoretical models are compared next, against an experiment-based plant model, and then the final test results of the real tall building model are presented. The two different plant model reduction techniques are: the Guyan and the Dynamic model reduction method (DMRM). From the results, it is clear that the DMRM is superior particularly when applied to the active vibration control of real (physical) large structures.

2. DESCRIPTION OF THREE DIFFERENT MODEL REDUCTION METHODS

2.1 The Guyan Model Reduction Method

The Guyan model reduction method is the most common procedure for reducing the size of mass and stiffness matrices that form the reduced model of a plant. While the reduced stiffness matrix preserves its accuracy, the reduced mass matrix produced by this method does not. The reason for this inaccuracy is that the Guyan method uses a static transformation between the eliminated and retained coordinates for obtaining the reduced mass matrix. This static transformation ignores the dynamic effect of the applied loads and creates an increasing error as the frequency of excitation is increased. For this reason, the Guyan model reduction method is only accurate in the low frequency range, and this has been previously demonstrated by simulation.

2.2 The Dynamic Model Reduction Method (DMRM)

In order for the DMRM (Zhang, 1995) condensed models to best approximate the original one, the condensed model retains n_c number of natural frequencies and the corresponding modes at the chosen master coordinates of interest from the original model. For the same unique harmonic forces applied at the master coordinates, the response matrix X_c determined from the condensed model must also be the same as that determined from the original model. To achieve the first requirement, the system matrix $M_c^{-1}K_c$ is determined as

$$B_c = M_c^{-1}K_c = \Phi\Lambda\Phi^{-1}, \tag{1}$$

where Λ is the eigenvalue matrix and Φ is the corresponding modal matrix from the full-size model, (all damping is ignored here for simplicity). To meet the second requirement, the response matrix X_c must be determined from the original structural system which has a large number of degrees of freedom, or alternatively from vibration testing. The mass matrix of the reduced model can then be determined as,

$$M_c = X_c^{-1}(-\omega^2 I + M_c^{-1}K_c)^{-1} \qquad (2)$$

Consequently, the stiffness matrix is determined as,

$$K_c = M_c B_c. \qquad (3)$$

After the condensed model is obtained, the responses at the master coordinates due to the applied forces can then be computed, and hence the dynamic responses at those eliminated coordinates can also be obtained in terms of the computed responses at the master coordinates.

As damping always exists in actual structural systems and is difficult to be modelled accurately, modal damping is therefore used for the reduced models. The level of the modal damping is determined by experience or by experimental modal testing on the systems.

Unlike the Guyan method, the DMRM includes the dynamic effect of the applied loads when formulating the reduced mass matrix, and has therefore much greater accuracy at the high frequency range of excitation. Both the Guyan and DMRM use real coordinates, and this is their greatest advantage over the Mode-displacement method, particularly for closed-loop control applications (Boffa et al, 2005).

3. EARTHQUAKE EXCITATION SIMULATION, CLOSED-LOOP RESPONSE

Although the linear quadratic regulator is a more common control method for this application, the pole placement control technique was used here because of its simplicity. The pole placement control was configured so that parameters such as the desired closed-loop damping ratios (active-damping ratios) and the desired closed-loop natural frequency of the active mass could be adjusted. By increasing these parameters, more control force is produced. This same pole-placement technique was also used in the final active-control testing of the real building model.

For all simulations reported below, the recorded El Centro earthquake data was used. The original data had many dominant low frequency components and was sampled at 50Hz. The original sampling frequency was scaled up by a factor of 8 (i.e., 400Hz sampling frequency) in order to shift the dominant frequency components to a higher range. In doing so, the higher modes of the reduced plant models were also excited under the modified earthquake input. The 400Hz sampled earthquake input has major dominant frequencies between 9 and 17Hz, with some minor dominant frequencies occurring between 17 to 47Hz. At this sampling frequency, the total frequency content of the earthquake ranges from zero to approximately 130Hz.

The accelerations of the top storey of the building were plotted in Figures 1b to 1d. The Mode-displacement 7dof model was used here as the plant model, because an independent plant was required to assess the closed-loop performance of the Guyan and DMRM observer models. Both of these models were derived from the basic linear-elastic Finite Element Method (FEM), and therefore they perform very well when compared against the Mode-displacement plant model, because it is also based on FEM theory. For a more comprehensive set of simulation results, refer to Boffa et al, 2005.

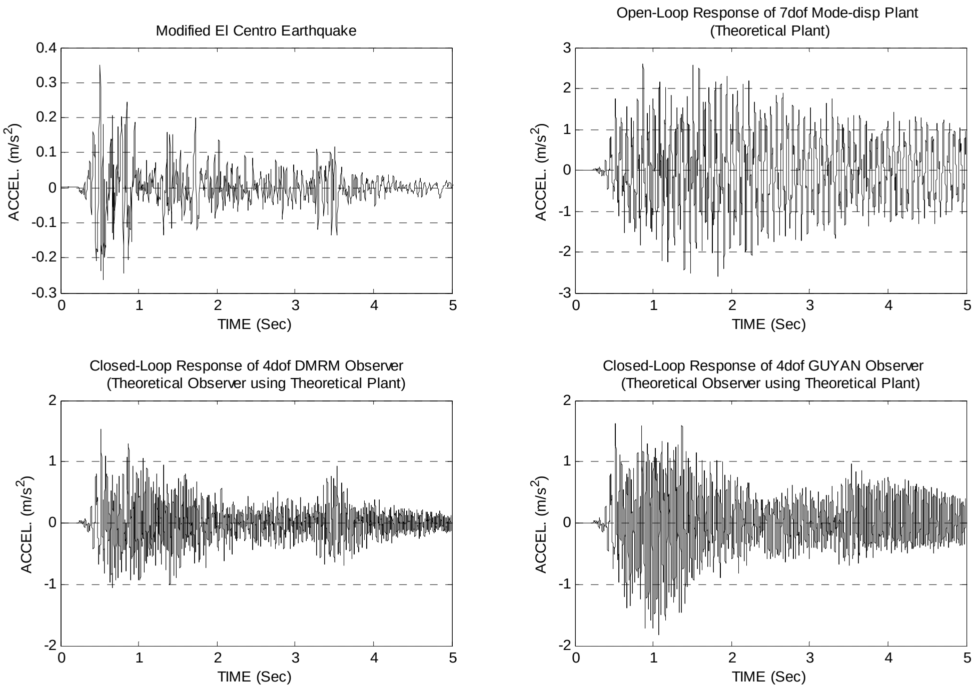
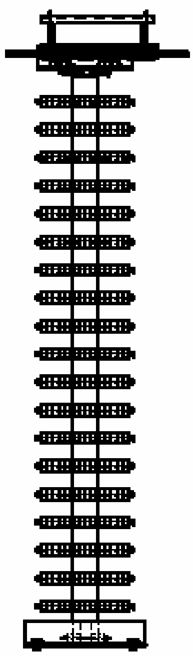


Fig.1a: (Upper-Left): Earthquake ground acceleration simulation, using a data sample frequency of 400Hz.
Fig.1b: (Upper-Right): Open-loop simulated plant response of 7dof theoretical Mode-displacement model
Fig.1c: (Lower-Left): Closed -loop simulation, 4dof DMRM theoretical observer, using theoretical plant.
Fig.1d: (Lower-Right): Closed -loop simulation, 4dof Guyan theoretical observer, using theoretical plant.

Fig.2:
Structural Model



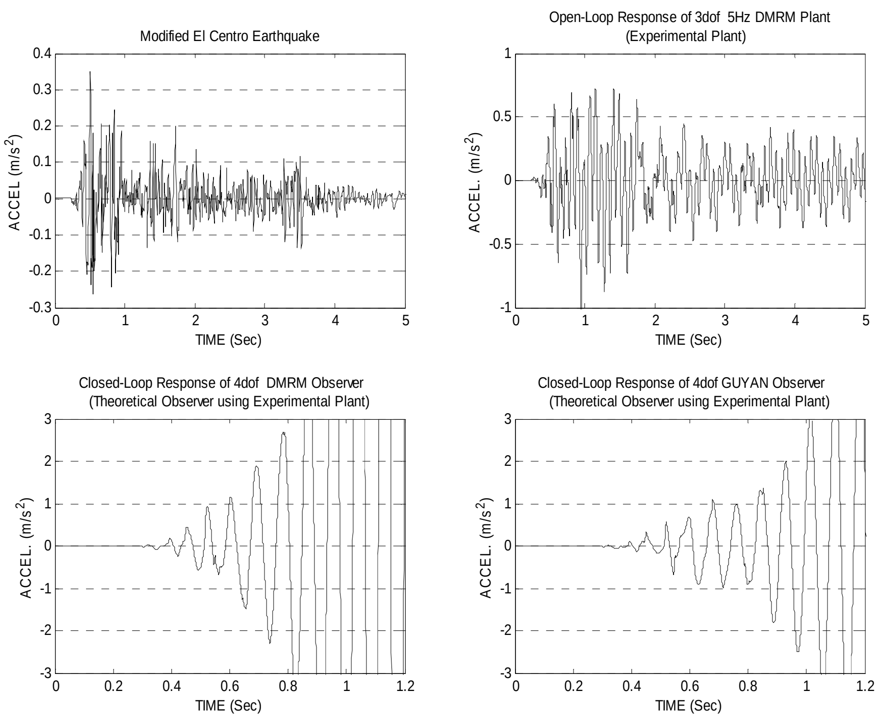


Fig.3a: Upper -Left Earthquake ground acceleration simulation, using a data sample frequency of 400Hz.
Fig.3b:Upper-Right Open-loop simulated plant response of 3dof DMRM (using real plant 5Hz test model).
Fig.3c:Lower-Left Closed -loop simulation, of 4dof DMRM theoretical observer, using real plant model.
Fig.3d:Lower-Right Closed -loop simulation, of 4dof Guyan theoretical observer, using real plant model.

In Figure 3b (above), the Mode-displacement plant is replaced with a 5Hz DMRM real plant model, which is based on the experimental procedure outlined under section 2.2. It is only when the theoretical observer models (in Figure 3c and 3d) are compared against a real plant model such as this, that we notice the active control become obviously unstable. The same situation occurs for any chosen DSF (not just 400Hz). The reason for this instability is twofold: Firstly the natural frequencies and modal shapes of all of the theoretical models deviate considerably from the real (physical) building model, and secondly, their forced response characteristics also deviate significantly from the real model. These discrepancies have occurred because the theoretical models are based on the linear elastic spring theory, which is not always closely followed in reality.

To solve this problem of instability, the Guyan model is replaced with a static DMRM model (in Figure 4d), which was derived from a specific experimental procedure. The displacements were measured at all master coordinates of the building while a range of static loads were applied separately to each master co-ordinate at a time. Linear graphs were obtained, the slopes of which describe the displacement per unit force elements of the static response matrix X_c, (that was mentioned above under section 2.2). In addition to this, the theory-based DMRM model was replaced by a 5Hz DMRM (in Figure 4c), which was also derived from experiment. Instead of a static load, this time a sinusoidal load at a frequency of 5Hz was applied to each master coordinate of the building. All displacements were measured again, and this produced a dynamic response matrix X_c, describing an alternative reduced model.

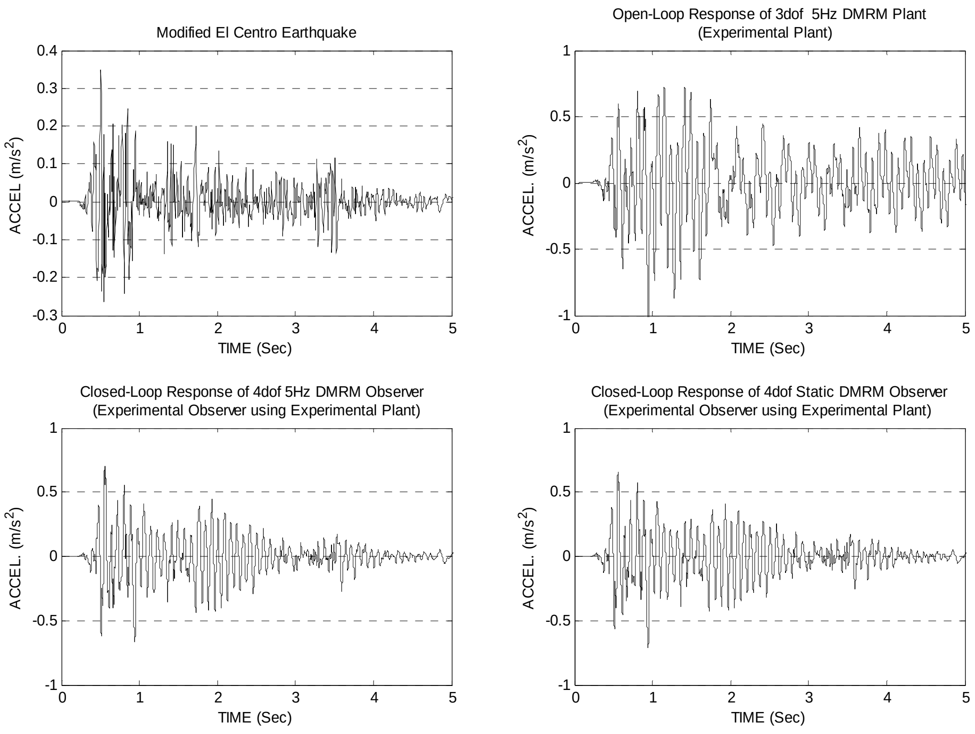


Fig.4a:(Upper-Left): Earthquake ground acceleration simulation using a data sample frequency of 400Hz.
Fig.4b:(Upper-Right): Open-loop simulated plant response of 3dof DMRM (using real plant 5Hz model)
Fig.4c:(Lower-Left): Closed-loop simulation, 4dof 5Hz DMRM real observer, using real plant model.
Fig.4d:(Lower-Right): Closed-loop simulation, 4dof Static DMRM real observer, using real plant model.

The test frequencies of zero and 5Hz were both chosen specifically, because they fall well below the third natural frequency of the structure, and are close to, but lower than the dominant frequency of excitation. The third natural frequency is significant here, because it is the highest mode of vibration that we are trying to control. As can be seen in Figure 4, both experiment based DMRM models perform well as observers, when compared against a real (physical) plant model.

4. SINUSOIDAL ACTIVE-CONTROL TESTING OF PHYSICAL STRUCTURE

The following graphs prove that the Static DMRM and the 5Hz DMRM reduced models both effectively mitigate oscillations during active vibration testing on a real 2.5m high, 20 storey model structure. Both graphs compare the active-damping mode against the passive-damping mode of the system when a sinusoidal force is applied to 18th floor. At a frequency of excitation of 12.1Hz, the 5Hz model performs in a very similar manner to the static model. But when excited at 24.2Hz, the 5Hz model performs much better than the static one, because it describes the higher vibration modes more accurately. The opposite was also proven by experiment, that at much lower frequencies of excitation, the static DMRM performs better than the 5Hz model, for similar reasons. The pole-placement control technique was used here throughout.

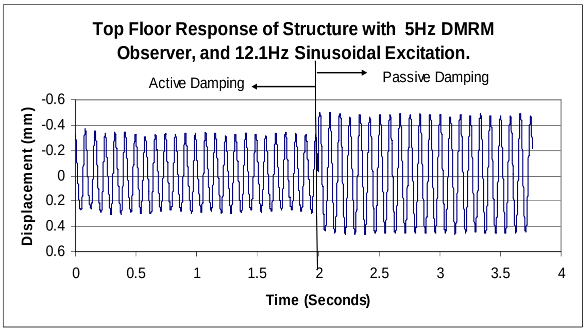


Figure 5a: 12.1Hz Sinusoidal Experimental Test Results of 5Hz DMRM observer

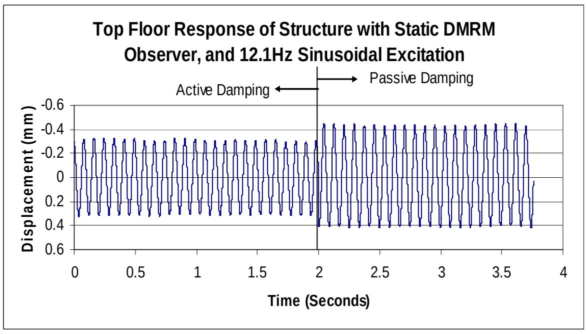


Figure 5b: 12.1Hz Sinusoidal Experimental Test Results of Static DMRM observer

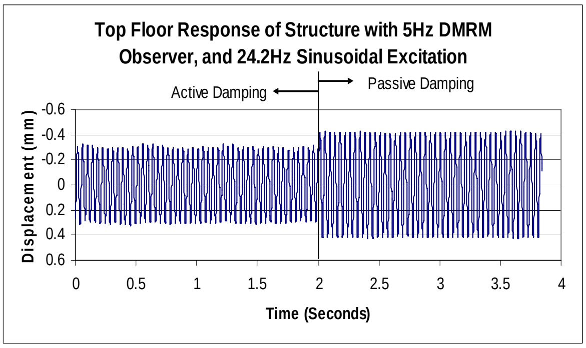


Figure 6a: 24.2Hz Sinusoidal Experimental Test Results of 5Hz DMRM observer

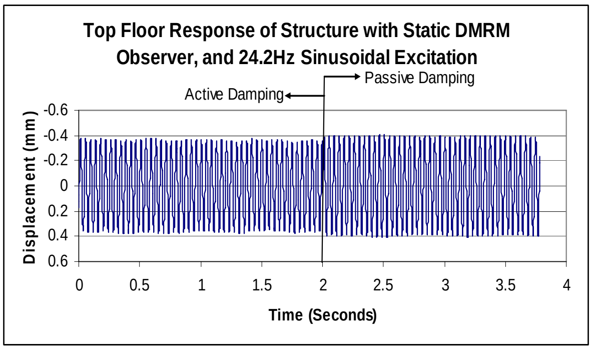


Figure 6b: 24.2 Hz Sinusoidal Experimental Test Results of Static DMRM observer

5. CONCLUDING REMARKS

The applicability of the Dynamic model reduction method to the active vibration control of large structural systems has been demonstrated from the presented test results. The versatility of the Dynamic model reduction method is such that it provides the option of obtaining system parameters from experiment, not just from theory. The problem with theory based model reduction techniques is that they rely on the linear elastic assumption. This assumption resulted in a drastic deviation in performance from the real structural model, as determined from physical testing on it, and produced unstable observers for active control. The experimental procedure outlined in this paper ensures that the Dynamic model reduction method forms an accurate description of the real system dynamics, and can be performed at any convenient frequency including zero. Care needs to be taken when choosing this test frequency, as it should be as close as possible to, but lower than, the predicted dominant frequency of the excitation force. Further attempts at improving the active-damping effect of the real structural system are currently being attempted; they are being directed towards better control algorithms.

6. ACKNOWLEDGEMENTS

Financial support for this research was provided by the University of Technology, Sydney.

7. REFERENCES

Boffa, J., Zhang, N. and Samali, B., (2005) Study on Model Reduction of Large Structural Systems for Active Vibration Control, Proceedings of Australian Congress on Applied Mechanics, 16-18 February, Melbourne, Australia, pp 293-298.

Kajiwara, I., Nakamatsu, A. and Inagaki, T., (1992) Reduced Modeling of Structure of Large Degrees-of-Freedom and Optimum Design of Its Control System, Trans. of Measurement and Automatic Control, Vol 28, pp 383-391.

Ma, Z. and Hagiwara, I., (1991) Development of New Mode-Superposition Technique for Truncating the Lower-and/or Higher Frequency Modes, JSME Trans. C, Vol 57, pp 74-81.

Matsumoto, Y., Doi, F. and Seto, K., (1998) Active Vibration Control of Multiple Buildings Connected with Active Control Bridges, JSEM Trans. C, Vol 64, pp 56-62.

Seto, K. and Mitsuta, S., (1991) A New Method for Making a Reduced-Order Model of Flexible Structures Using Unobservability and Uncontrollability and Its Application in Vibration Control, JSME Trans. C, Vol 57, pp 281-287.

Zhang, N., (1995) Dynamic Condensation of Mass and Stiffness Matrices, Journal of Sound and Vibration, Vol 188(4), pp 601-615.