# Simulation of strong ground motions with a combined Green's function and stochastic approach

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# Abstract

A combination of Green's function and stochastic method is developed to generate strong ground motion time histories for engineering applications. This approach is an example of obtaining strong motion data in a region without strong seismicity. Hao and Gaull (2004) proposed a stochastic model based on recorded motion from minor-large magnitude earthquakes on SWWA rock sites. This model is found to be apt to generate moderate-sized earthquake events and below over different epicentral distances. In this study, the strong ground motions from large earthquake events are generated using the Green's function method. The results of this are validated by comparing the Fourier spectrum of the simulated events with the larger magnitude recordings (The  $M_L$  6.2 event in Cadoux in June 1979 and a  $M_L$  5.5 event recorded in Meckering in January 1990).

# Introduction

Because there are very few recordings from significant southwest Western Australia (SWWA) earthquakes ( $M_L$  4 or more), it is difficult to derive a reliable strong ground motion attenuation model for this region. Two previously established PGA and PGV attenuation models that are based on local data are Gaull and Michael-Leiba (1987) and Gaull (1988). Gaull (1988) warned of the large uncertainties using his relations due to the limited database. Other models used in SWWA in more recent times are mainly from central and eastern North America (CENA) (Atkinson and Boore 1995, Atkinson and Boore 1997 and Toro et al. 1997). These models are believed to be reliable because both CENA and SWWA are located in the stable continental intraplate region. However, Hao and Gaull (2004) and Kennedy et al. (2005) showed that none of these models yielded very satisfactory prediction of the recorded strong ground motions in SWWA.

Hao and Gaull modified the Atkinson and Boore (1995) model and incorporated SWWA seismological parameters into the model which resulted in a better correlation with the existing SGM data. However, its reliability in representing larger SWWA earthquakes is not known due to the lack of data from events of large magnitude. This means the model could be biased to the ground motion characteristics associated with small events and narrower frequency band biased towards the high frequency region. Furthermore, this cannot be verified because of the lack of such data.

To overcome the problem of paucity of data from large magnitude events, it was decided to use a two-stage approach: 1) Simulating ground motion time histories from minor earthquake events using the stochastic model (Hao and Gaull 2004) and then 2) These time histories will be used to simulate time histories of large magnitude events using empirical Green's Function method. This approach is an example of obtaining strong motion data in a region without strong seismicity. The validity of this approach is verified by comparing the simulated Fourier spectrum with the recorded motions in SWWA from a M<sub>L</sub> 6.2 event centred in Cadoux in June 1979 and a M<sub>L</sub> 5.5 event in Meckering in January 1990. It should be noted that these are the only two events that are large enough for this validation process.

# Simulation of strong ground motion

#### Methodology

There are two main approaches of simulating strong ground motions. One is stochastic approach (Hanks and McGuire 1981, Boore 1983, Boore and Atkinson 1987, Boore 2003, Hao and Gaull 2004), which is based on a set of assumptions regarding the earthquake source spectrum, propagation path and site conditions. The modified model by Hao and Gaull (2004) was verified yielding reliable prediction of recorded motions in SWWA. However, because most of the recorded motions are from minor earthquakes, this modified model may be biased towards events of this magnitude and may bias the prediction of ground motions for large or greater magnitude earthquakes.

Green's Function method is the other method based on the representation theorem for a kinematic dislocation model. In the empirical Green's function technique of Irikura (1986), the large event has been modelled from the aftershocks that may not be well distributed within the rupture plane. Irikura (1986) divided the mainshock fault plane into subfaults plane to satisfy the scaling law of the source spectral. The size of the mainfault and subfault corresponds to the rupture area of main event and small event respectively. Because the frequency contents of a small event are usually not the same as those of a large event, Irikura et al. (1997) modified an exponential slip function to boost the low-frequency energy in the simulation. Their equation is given as

$$U(t) = \sum_{i=1}^{N} \sum_{j=1}^{N} (r/r_{ij}) \cdot F(t) * (C \cdot u(t))$$
(1)

$$F(t - t_{ij}) = \delta(t - t_{ij}) +$$

$$\left\{ (1/n')(1 - \exp(-1)) \right\} \sum_{k=1}^{(N-1)n'} \left[ \exp\left\{-(k-1)/(N-1)n' \right\} \times \delta\left\{t - t_{ij} - (k-1)T/(N-1)n' \right\} \right]$$

$$(2) \text{ and}$$

$$t_{ij} = \left|r_{ij} - r_{0}\right|/V_{s} + (r_{ij} - r_{0})/V_{r}$$

$$(3)$$

where \* means convolution, U(t) is the ground motion of large event; r the distance between the hypocenter of small event and the receiver;  $r_{ij}$  the distance between the subfault(i,j) and the receiver;  $r_0$  the distance between the subfault(i,j) and the hypocenter of large event; F(t) the slip-time filtering function; C the stress drop ratio;  $V_s$ the rupture velocity;  $V_r$  the shear wave velocity; u(t) the contribution of the jth sub event;  $\delta(t-t_{ij})$  the Dirac delta function;  $t_{ij}$  the phase delay term. N is the scaling law between large and small event, which is derived from the study of Kanamori and Anderson (1975) and Aki (1967) and Brune (1970). n' is an appropriate integer to eliminate spurious periodicity. The reader is reminded that small events will be generated by using Hao and Gaull (2004)'s stochastic model and Green's Function will then use these to simulate ground motions from large events.

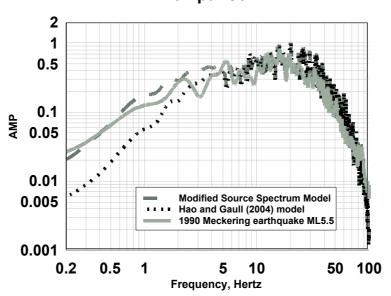
### **Case study**

#### Parameter modification

In Figure 1, the predicted FFT spectrum of the ground motion from a  $M_L$  5.5 event by Hao and Gaull (2004)'s model is compared with the recorded motion of the  $M_L$  5.5 Meckering event. It can be seen that the Hao and Gaull (2004) model underestimates the ground motion energy at frequencies lower than 2 Hz. This observation implies that the corner frequency used in the model should be lower in order to more reliably predict motions in SWWA. To achieve this, the source spectrum used in the model is modified as

$$S_{a}(f) = \frac{k(1-\varepsilon)}{1+k(f/f_{a})^{2}} + \frac{k\varepsilon}{1+k(f/f_{b})^{2}}, \qquad S_{b}(f) = 1.0$$
(4)

where k is a modification parameter. It is found that, by trial and error, k=10 gives better prediction of the recorded motion. As shown in Figure 1, using the stochastic model and the modified source spectrum yields good prediction of the recorded motion.



FFT Comparison

Fig. 1 FFT Comparison of the recorded and simulated  $M_L$  5.5 Meckering earthquake motion

Gaull and Michael-Leiba (1987) used the following relation to estimate the length of fault in accordance with surface-wave magnitude.

 $\log L = 3.2 + 0.5 M_s$ 

(5)

where *L* is the length of the fault in cm and  $M_s$  is the surface-wave magnitude.

Because the earthquake source parameters in SWWA are not well studied and the geophysical conditions of CENA are relatively similar to that of SWWA, many CENA parameters are adopted here. Equation (6) was given by Somerville et al. (2001). It will be used to estimate rupture area.

$$A = 8.9 \times 10^{-16} \times M_0^{2/3}$$

(6)

where A is the rupture area in  $km^2$  and  $M_0$  is the seismic moment in dyne-cm.

Boore and Atkinson (1987) indicated that constant-stress model appears to be supported by CENA data. Following this idea, the constant-stress model is used in this work. The shear wave velocity  $\beta$  in SWWA was found around 3.91km/s (Dentith et al. 2000), which is used in this study.

Because no magnitude conversion relation that is specifically for SWWA earthquakes is available, a popularly used conversion relation for CENA earthquakes (Hanks and Kanamori 1979) is used in this study. It has the form

$$Log_{10}M_0 = 1.5M_w + 16.05$$

(7)

where  $M_0$  is the seismic moment and  $M_w$  is the moment magnitude. This conversion relation was also used in Hao and Gaull (2004). There is no reliable relation between  $M_L$ and  $M_w$  for SWWA. For the magnitudes below its saturation point ( $M_L$  6.5),  $M_L$  and  $M_w$  are almost equivalent. However, more accurate estimation of equivalent  $M_w$  and  $M_L$  is needed in the future.

The rise time was computed using Equation (8) of Somerville et al. (1993).  $T = 1.72 \times 10^{-9} (M_0)^{1/3}$  (8) where *T* is rise time in sec.

#### Simulation

The Green's function method has been cited many times in its application to simulation of large earthquakes from smaller ones, such as Sinadinovski et al. (1996); Frankel (1995); Joyner and Boore (1986), Sinadinovski et al. (2005). For SWWA events, the validity of the method will be verified by comparing the simulated and recorded motions in SWWA. The two events used for the comparison were the 1979  $M_L$  6.2 Cadoux earthquake that was recorded near Meckering, 96 km distant and the 1990  $M_L$  5.5 Meckering event that was recorded near Dowerin, some 78 km distant.

Hao and Gaull (2004)'s stochastic model was firstly used to simulate events of  $M_L$  4.5 with the same epicentral distances as those events used for comparison. The Green's Function empirical method with modifications from Irikura et al. (1997) was then used to increase the magnitude of these simulated  $M_L$ 4.5 events to  $M_L$ 5.5 and  $M_L$ 6.2 respectively. Figures 2 compare the FFT spectrum of the simulated with the recorded ground motion time histories. These simulated ground motions agree well with the recorded motions.

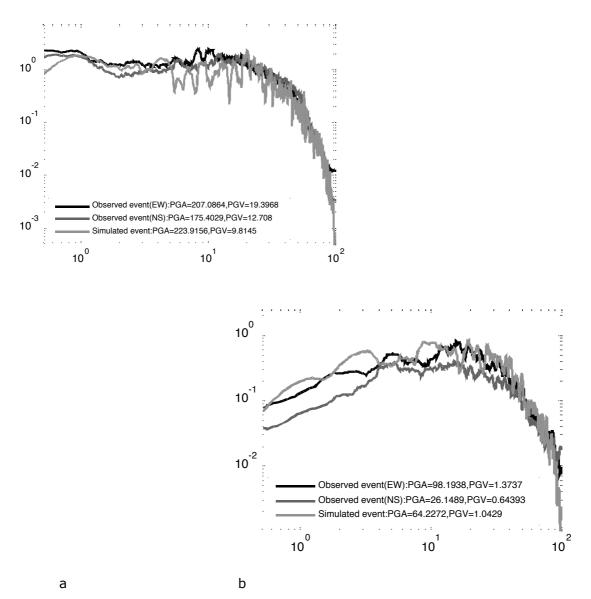


Fig. 2 FFT of the simulated and recorded ground motions (a.  $M_L$  6.2, epicentral distance 96 km, b.  $M_L$  5.5, epicentral distance 78 km)

It should be noted that simply summing small events, as the original Green's function method, with delay time will underestimate low-frequency signal. The modified slip

function proposed by Irikura et al. (1997) effectively overcomes this problem. The two simulated motions agree well with the recorded motions in a wide frequency band, as shown in Figures 2.

	1979 Cadoux event, <i>M</i> <sup><i>L</i></sup> 6.2, 96km		1990 Meckering event, $M_L$ 5.5, 78km	
	Observed	Simulated	Observed	Simulated
	record	record	record	record
PGA (mm/s <sup>2</sup> )	191.25	223.92	62.17	64.23
PGV (mm/s)	16.06	9.81	1.38	1.04

Table 1. The peak value of the observed record and simulated record

The peak values of the observed record and simulated record are shown in Table 1. It should be noted that the substantially larger PGV value of the recorded  $M_L$  6.2 event may be caused by an abnormal low-frequency peak in the recorded time history at about 0.8 Hz, as shown in Figure 3. Because this record was hand digitised, the scope for anomalous low-frequency data is increased.

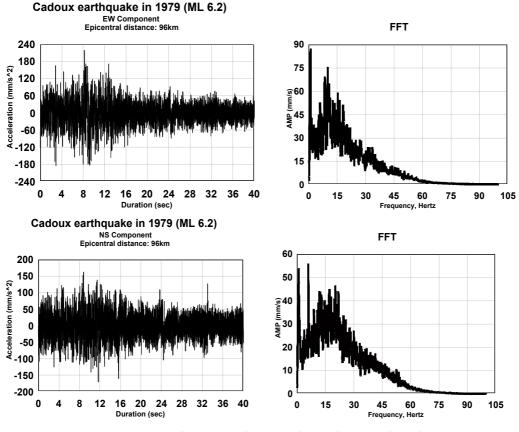


Fig. 3 Time history and FFT of the Cadoux earthquake in 1979

The exact reason why this record has a double peak is not known. It is believed that this low-frequency peak is not the normal free-field ground motion. Other than this significant difference between the simulated and recorded PGV of the  $M_L$  6.2 event, the simulated peak values agree well with the recorded motions. Unfortunately, there is no other record to further verify the validity of the simulation method. This method will be used in further study to simulate ground motions.

## Conclusions

This paper presents a method that combines empirical Green's function and stochastic model for strong ground motion simulation. The main conclusions of the study are as follows.

1. By comparing the model with the recorded data of the two earthquake events in SWWA, this method, with the assumption of constant-stress scaling, was proved to be suitable as most of simulated curves fit well against the curves of observed records.

2. Although some parameters referred from ENA data because of limited study of the parameters of the source in SWWA, we still can find satisfaction in the simulation result.

3. Matching the longer periods is also possible if enough data are available for all range of depths. However, there are too few data from large earthquakes in WA to prove that this scaling is correct over the entire magnitude range.

4. It is suggested that this method with these parameters can be used in further study to simulate ground motions that will be added to the SWWA database.

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