Modelling ocean waves and their effects on offshore structures

N. Haritos

Civil & Environmental Engineering,
The University of Melbourne, Parkville, Victoria 3010
Email: nharitos@unimelb.edu.au

Abstract

Unlike the waves associated with ground motion, waves in an ocean environment are predominantly generated by winds, so can be considered as virtually continuously varying phenomena. Irregular wind generated waves in deep water can be characterised by a power spectrum that incorporates a directional spreading term. The frequency-wavelength characteristics of ocean waves in this spectral mix are dependent upon the depth of water for waves in shallower depths. The design of offshore structures (such as oil and gas platforms) that interact with ocean waves requires an understanding of ocean wave characteristics and their effects in promoting their dynamic response. The modelling of such waves, can assume a number of layers of sophistication from considering a single regular (so-called "design") wave, to a highly irregular three-dimensional sea state conforming to a given spectral description and directional spreading characteristic using a spectral random phase model. Models in between these two extremes include uni-directional random waves via a spectral-random phase model and the use of a limited combination of constant-amplitude regular waves with random phase, which are especially chosen to "capture" the essential statistical properties of the ocean waves under consideration.

In this paper, the author describes a number of ocean wave generation models and offers a few, drawn from his own initiative and experience, that have proven to be particularly efficient and useful in both analytical and physical model studies on offshore structures.

Keywords: wind, ocean waves, frequency, wave statistics, dynamic response

1. INTRODUCTION

The character of the dynamic response of offshore structures to excitation from the effects of environmental loading is dependent upon a number of obvious factors that include: the geographic location of the site, the environmental loading itself that essentially stems from wind/wave/current/earthquake conditions thereat, the depth of water and the particular characteristics of the design of the offshore structure itself, (Haritos, 2007).

Often, the dominant excitation source for an offshore structure (such as an oil and gas platform) stems from wind-generated waves and their interaction with the structure concerned in promoting its dynamic response. Unlike the waves associated with ground motion, waves in an ocean environment can be considered as virtually continuously varying phenomena. Irregular wind generated waves with surface elevation profile $\eta(x,t)$ located in deep water can be characterised by a power spectrum, $S_\eta(f,\theta)$, that incorporates a directional spreading term, $D(\theta)$, (in which θ is measured from the dominant wave direction), in combination with the wave elevation spectrum, $S_\eta(f)$, eg:

$$S_\eta(f,\theta) = D(\theta)\, S_\eta(f) \tag{1}$$

The frequency-wavelength characteristics of ocean waves in this spectral mix are dependent upon the depth of water in which these waves are located and this dependency is referred to as the dispersion relationship, viz:

$$\begin{aligned} \omega^2 &= g\kappa \tanh(\kappa h) \\ &\approx g\kappa \qquad \text{(for deep water where } \kappa h > \pi\text{)} \end{aligned} \tag{2}$$

in which ω is the circular frequency (= $2\pi f$), and κ is the wave number (= $2\pi/\lambda$ where λ is the wavelength) of the wave or wavelet under consideration in this sea state.

A typical expression for the directional spreading function, $D(\theta)$, is given by

$$D(\theta) = \cos^{2s}(\theta) \qquad (-\pi/2 < \theta < \pi/2) \tag{3}$$

where $s = 1$ corresponds to a widely-spread sea state and $s = 3$, one that is quite narrow.

A widely adopted formulation of the fully-developed wind generated wave elevation spectrum, $S_\eta(f)$, is that of Pierson-Moskowitz (P-M), (Janssen 2010) which can be expressed as:

$$S_\eta(f) = \frac{0.0005}{f^5} e^{-\frac{5}{4}\left(\frac{f_p}{f}\right)^4} \tag{4}$$

in which f_p is the frequency at peak wave energy which is related to the mean wind speed at 19.5m above Mean Water Level (MWL), $U = U_{19.5}$, via $f_p = 1.37/U$. Figure 1 depicts the form of this spectrum for a range of wind speeds. The variance in the irregular wave elevation is the area under the spectrum and equates to $0.0001/f_p^4$.

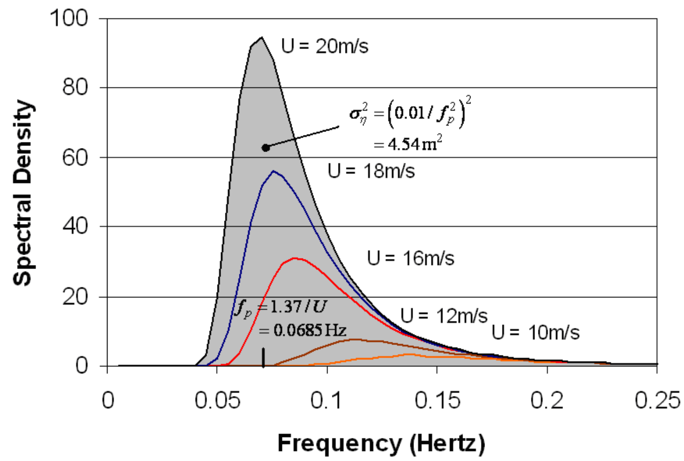

Figure 1: Pierson Moskowitz spectrum and its relationship to mean wind speed U.

2. MODELLING IRREGULAR SEA STATES

A Fourier series can be used to model an irregular uni-directional sea state, $\eta(t)$, at a fixed location ($x = 0$), as a time series of length T, and N points dt apart ($T = N.dt$) as follows:

$$\begin{aligned}\eta(t) &= \sum_{n=0}^{N/2}\left(a_n \sin\left(\frac{2\pi nt}{T}\right) + b_n \sin\left(\frac{2\pi nt}{T}\right)\right) \\ &= \sum_{n=0}^{N/2}\left(A_n \sin\left(\frac{2\pi nt}{T} - \varphi_n\right)\right) = \sum_{n=0}^{N/2}\left(A_n \sin\left(2\pi f_n t - \varphi_n\right)\right)\end{aligned} \tag{5}$$

$$\text{where: } f_n = \frac{1}{T};\ A_n = \sqrt{a_n^2 + b_n^2};\ \varphi_n = \text{atan}\left(\frac{b_n}{a_n}\right)$$

Equation (5) suggests that the n^{th} wavelet in this series has a circular frequency $\omega_n = 2\pi f_n$, and satisfies the dispersion relationship of Eq (2). In addition, it can be shown that the amplitude of this wavelet, A_n, is given by:

$$A_n = \sqrt{2 S_\eta(f_n)\, df} = \sqrt{2}\,\sqrt{S_\eta(f_n)\, df} \tag{6}$$

which can be interpreted to be $\sqrt{2}$ times the RMS value of this wavelet.

2.1 Random phase Inverse Fast Fourier Transform model

Equations (5) and (6) can be used, under a random phase modelling assumption, (ie where $\varphi_n = Ran(0 - 2\pi)$) to simulate a time series of the surface elevation that conforms to the selected wave spectrum, $S_\eta(f)$, via an Inverse Fast Fourier Transform (IFFT) algorithm. The *Data Analysis* option under the *Tools* tab in EXCEL offers a *Fourier Analysis* option that allows an IFFT to be performed on a suitably prepared data series interpretation of Eqs (5) and (6) that can produce such a simulated wave elevation trace.

2.2 Design Wave model

When the lowest natural frequency of an offshore structure, f_o, is significantly greater than f_p, the wave frequency at peak wave energy in the wave elevation spectrum, (say for example $f_o > 4 f_p$), then a *Design Wave* can be formulated that can be inferred from the design surface elevation spectrum and used to determine the design wave loading on the structure and hence its quasi-static design response to this loading.

A single wave interpretation of the model depicted by Eq(6) would suggest an amplitude of $A_I = \sqrt{2}\,\sigma_\eta$ which is essentially a wave with an RMS value equal to that of the irregular sea state. Statistically, under a Normal probability assumption, this would correspond to an amplitude that is exceeded by 15.9 % of the waves in this irregular sea state. For design, a factor is instead applied to the *Significant Wave* amplitude given by $A_s = H_s / 2 \approx 1.98\sigma_\eta$ in which H_s is the *Significant Wave Height* (crest to trough height of wave) which corresponds to the mean of the topmost 1/3 of the waves in the sea state. The factor used on A_s for the *Design Wave* amplitude depends upon what Limit State condition is being investigated for the design under consideration.

The frequency associated with the Design Wave can be taken as f_p, the frequency corresponding to peak wave energy. Alternatively f_m, the *mean* wave frequency or f_h, the frequency that evenly splits the spectrum into equal areas (or energy) can be used, ie

$$f_m = \int_0^\infty f\, S_\eta(f)\, df \,/\, \sigma_\eta^2 = 0.000106\left(\frac{f^3}{f_p^4}\right)\Gamma\left(0.75, \frac{5}{4}\left(\frac{f_p}{f}\right)^4\right) / \sigma_\eta^2 \approx 1.295 f_p \qquad (7)$$

$$f_h; \text{ where } \int_{f_h}^\infty S_\eta(f)\, df = \sigma_\eta^2 / 2 \rightarrow f_h = 1.16 f_p \text{ for P-M spectrum} \qquad (8)$$

2.3 Equal amplitude random phase wavelet model

Haritos (1988) suggested a simplified Fourier series representation of the surface elevation time series based upon an interpretation of Eq(5) in which the number N of wavelets in the series is dramatically reduced (typically N = 1024, 2048 or 4096 using the IFFT approach), to say as little as N = 8 whilst still being able to capture the characteristics of the sea state being simulated to an acceptable accuracy. In this interpretation, the wavelets are taken to have equal amplitudes, by partitioning the P-M spectrum into equal areas. The corresponding frequency of the wavelet is taken to be at the half area position within the area segment associated with the wavelet sequence number, n, under consideration.

For N wavelets in the series the amplitude of each, A_n, simply becomes $A_n = \sqrt{2/N}\,\sigma_\eta$, which for N = 8 equates to $\sigma_\eta/2$. The frequency for the nth wavelet in the series, in the case of a P-M wave surface elevation spectrum, is then given by:

$$\int_{f_n}^\infty S_\eta(f)\, df = \frac{2n-1}{2N}\sigma_\eta^2 \;\rightarrow\; f_n = \left(\frac{5}{4}\ln\left(\frac{n}{n-1}\right)\right)^{0.25} f_p \qquad (9)$$

The absolute maximum surface elevation in this model, η_{max}, occurs when all wavelets are in phase (a rare event in the case of N > 4), so that $\eta_{max} = \sqrt{2N}\,\sigma_\eta$, which for N = 8 equates to 4 times the RMS value. Values exceeding 4 times the RMS value are statistically quite rare. Consequently, adopting an equal amplitude random phase model for numerically simulating an irregular sea state, becomes quite efficient when compared with the IFFT approach, especially when this approach needs be exercised at regular intervals such as when dealing with non-linear behaviour requiring iterative techniques to be able to solve the problem being investigated. Figure 2 depicts traces generated over 1024 seconds at 0.5 second intervals (2048 data points) conforming to a P-M spectrum with U = 30 m/s, using the IFFT and equal amplitude methods respectively, both with random phase (0 - 2π). Shown alongside the traces is the result of an upcrossing analysis on the respective trace used to predict the expected maximum surface elevation in a 4-hour period. The respective lines of best fit in the graphical representation of the upcrossings for both traces are closely similar and both lead to a near identical peak surface elevation prediction for a 4-hour duration of the sea state of close to 16.2 m or approx. 3.43 standard deviations (η_{RMS}) of 4.73 m.

3. MODELLING "SPECIAL" SEA STATE CONDITIONS

3.1 Swept Sine Waves

The Swept Sine Wave (SSW) concept is where a *chirp* style signal (one in which the frequency of the wave changes linearly in time from f_1 to f_2), Eq (10), is adopted in combination with a time-varying amplitude, $A(t)$, to generate the resultant time series, (Haritos, 1988). SSW's are particularly useful in the study of dynamic phenomena (eg in structural dynamics applications) as they allow clear identification of resonant behaviour in the dynamic response – whether this response is simulated from numerical code or is measured in physical experiments of say wave-structure interaction effects.

$$\eta(t) = A(t)\sin\left(\left(2\pi f_1 + \pi\frac{(f_2 - f_1)}{\tau}t\right)t\right) \quad \tau : \text{time length of record} \qquad (10)$$

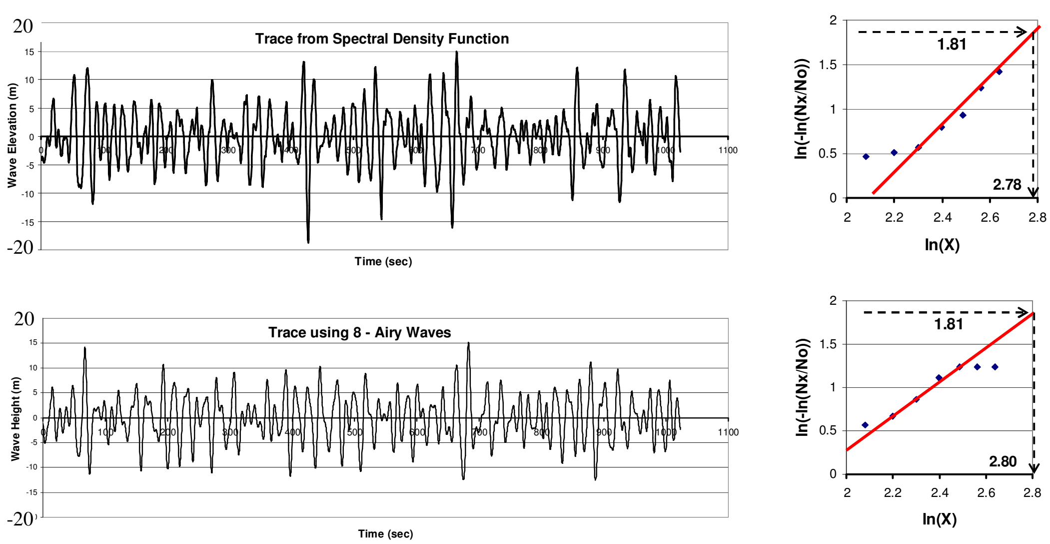

Figure 2: Comparison of wave traces using IFFT and 8 equal wave amplitude wavelets

Figure 3 depicts an SSW designed to produce constant amplitude inertia wave forcing on an instrumented multi-segment bottom-pivoted vertical cylinder that was tested in the wave basin at the National Research Council (NRC) facility centre in Ottawa by the author. The cylinder is 2.4m long and had provision of being supported by either rigid rods or a set of springs in series (to vary the degree of compliancy of the assemblage) in mutually orthogonal directions at cylinder tip. Data over some 30 channels of measurement was recorded at a 10 Hz sampling rate for 8192 data points for each test configuration for several wave types and cylinder compliancy conditions. Figure 4 shows a segment of the wave trace close to the natural frequency of approx. 0.55Hz for a selected spring series for this test cylinder tested in a 2m water depth, together with the corresponding in-line (X) and transverse (Y) responses at the cylinder tip. The traces clearly depict the condition of resonance and the triggering of vortex shedding (as demonstrated by the Y-direction motion) once large amplitudes of vibration were induced in the in-line X-direction. This situation clearly demonstrates the powerful role SSW's can play in the study of complex frequency dependent phenomena.

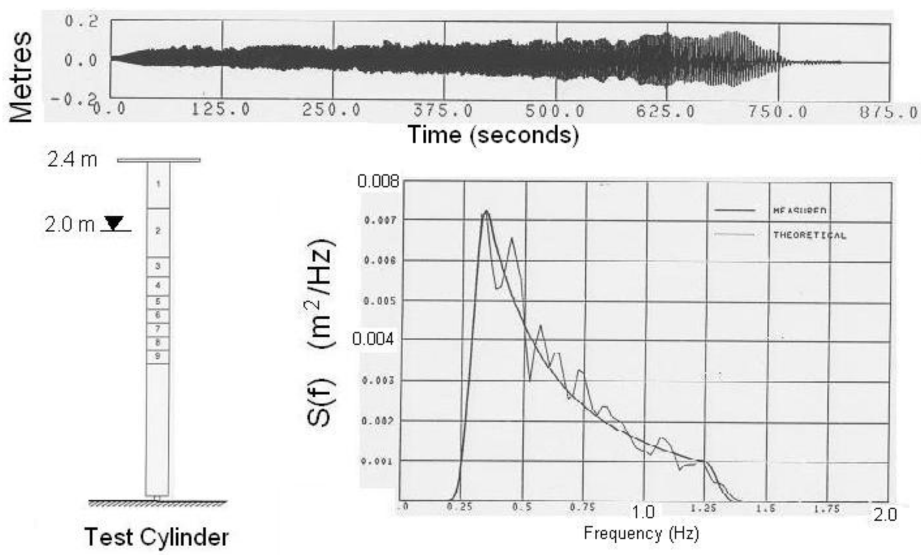

Figure 3: SSW wave with constant Inertia force characteristics on test cylinder

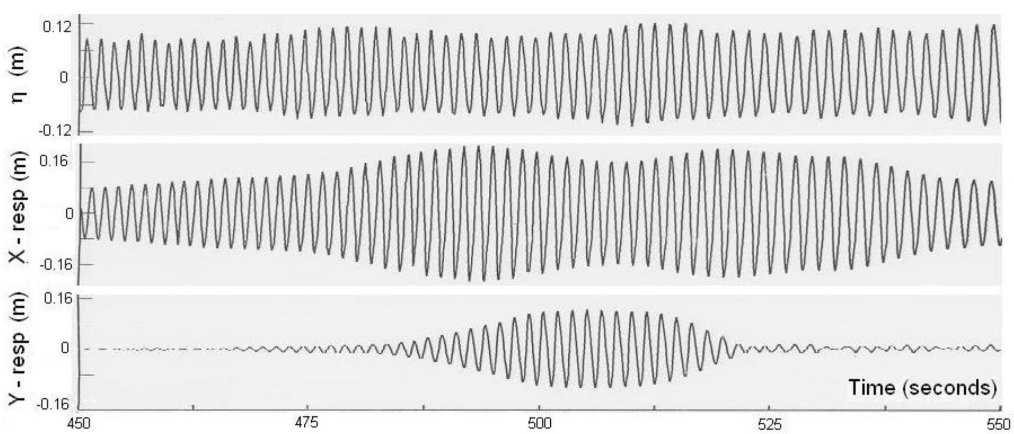

Figure 4 Resonant response of test cylinder from constant Inertia force SSW test

3.2 Freak or rogue waves

So-called *freak* or *rogue* waves are referred to what are believed to be naturally occurring waves which can assume very large proportions as rare events in a random sea state. They occur when several wavelet components superpose at a particular location with only a small, (close to zero), phase shift relative to each other to produce these rather large amplitude proportions. Such waves have been suggested to be responsible for capsizing large sea-going vessels and even a semi-submersible offshore oil platform – the Ocean Ranger Platform in 1982 operating off the Canadian coast near Newfoundland, (http://en.wikipedia.org/wiki/Ocean_Ranger, Moan, 2007).

One can replicate the condition of the superposition of a sequence of waves at a nominated point x from the generation point ($x = 0$), by making use of the relationships in Eq(2) and Eq(5) for a limited number of sequential waves. The idea here is to nominate a wave frequency ω_1 and corresponding period T_1, calculate the time t_1 taken by that wave to reach the target point x with celerity ω_1/κ_1 and to then generate that wave. The next wave in the generation process will require a time period of ($t_1 - T_1$) to reach the target point so its ω_2, (T_2 and wave celerity ω_2/κ_2) need be selected for this condition to be satisfied via Eq(2). The procedure can be repeated for subsequent wavelets until it's no longer possible to satisfy a wave celerity that can achieve coincident superposition with the other wavelets in the mix. Figure 5 illustrates the numerical simulation, through a series of snapshots, of such a freak wave state intended for generation in the Michell laboratory wave tank at the University of Melbourne for 1m water depth and x = 10m using four equal waveheight wavelets of 0.1m.

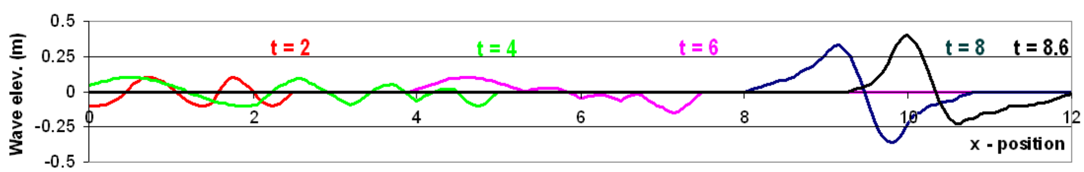

Figure 5: Freak wave generation leading to a breaking wave 10m from wave generator

4. CONCLUDING REMARKS

The author has overviewed a number of modelling techniques for simulating ocean waves in numerical and laboratory based studies. The SSW testing technique and the equal amplitude random phase wavelet model have been found to be particularly useful to the study of wave-structure interaction phenomena exhibiting non-linear features.

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