Simulation of spatially varying ground motions with non-uniform intensities and frequency content

Kaiming Bi[1], Hong Hao[2]

[1]PhD student, School of Civil and Resource Engineering, The University of Western Australia, WA. Email: bkm@civil.uwa.edu.au

[2]Professor, School of Civil and Resource Engineering, The University of Western Australia, WA. Email: hao@civil.uwa.edu.au

Abstract

Many researchers have studied and proposed methods to simulate spatially varying earthquake ground motions. These methods usually assume the spatially varying ground motions have the same power spectral density or response spectrum, i.e., they have the same amplitudes and frequency content. The only variation is the loss of coherency and a phase delay between spatial ground motions owing to seismic wave propagation. This assumption is valid for spatial ground motions in a flat site with uniform site properties. For a canyon site, due to site amplification effect, the spatial ground motions at various points on the canyon surface will not have the same power spectral densities. This paper presents a stochastic method to simulate spatially varying earthquake ground motions on surface of a canyon site. The spatial ground motions at various locations have different amplitudes and frequency content, which are determined by considering the base rock motion propagating through the soil site. The base rock motion is modelled by the filtered Tajimi-Kanai power spectral density function and a theoretical coherency loss function. The site amplification effect is modelled by the transfer function derived from the deterministic wave propagation method.

Keywords: Ground motion simulation, ground motion spatial variation, site amplification

1. Introduction

For large dimensional structures, such as long span bridges, pipelines, communication transmission systems, the ground motions at different stations during an earthquake are inevitably different owing to seismic wave propagation and local site conditions, and hence, resulting in different structural responses as compared to the uniform excitation. In the past two decades, more and more earthquake engineers have realised the importance of this phenomenon and many studies on modelling of ground motion spatial variations have been reported especially after the installation of the SMART-1 array in Taiwan. However, in all these studies, the power spectral densities of ground motions at various locations are assumed to be the same and ground motion spatial variation is represented by a coherency loss function and a phase delay. A few models of ground motion spatial variations have been proposed. Zerva and Zervas (2002) overviewed these models. Some methods have also been developed to simulate spatially varying ground motions based on these models (Hao, et al. 1989). These methods were developed based on the assumption that all the spatially varying ground motions have the same power spectral density, i.e., the same amplitude and frequency content. The simulated ground motion time histories in these approaches are compatible with a chosen spatial variation model, i.e., the ground motion coherency loss model and a phase delay, and compatible with the same ground motion power spectral density function or response spectrum. If the considered site is flat with the same soil properties, the uniform ground motion power spectral density assumption for spatial ground motions in the site is reasonable. However, for a canyon site or a site with different soil properties, because local site conditions affect the wave propagation hence the ground motion intensity and frequency content, the uniform ground motion power spectral density assumption is no longer valid. Deodatis (1996) developed a method to simulate spatial ground motions with different power spectral densities at different locations. The method is based on a spectral-representation algorithm to generate sample functions of a non-stationary, multivariate stochastic process with evolutionary power spectrum. The considered varying spectral densities are filtered white noise functions with different central frequency and damping ratio. When base rock motion propagates through a soil site to ground surface, site properties strongly affect the wave propagation and the motion on ground surface. Filtered white noise function does not necessarily represent the power spectral density of ground motion on a canyon surface. Moreover, trying to establish an analytical expression for a realistic evolutionary power spectrum is quite difficult since very limited information is available on the spectral characteristics of propagating seismic waves.

On the other hand, to model the site amplification, Wolf (1985) presented an approach in the frequency domain to analyze the site responses based on the wave propagation theory and finite element model. Wolf (1988) and Safak (1995) also presented methods to deal with the propagating shear waves in layered media in the time domain. Ground motion time histories on surface of the site can be calculated from these methods. But these approaches only calculate ground motion at one point on ground surface. Ground motion spatial variation is not considered.

This paper presents a method to simulate ground motion time histories on surface of a canyon site with different power spectral densities derived from base rock ground motion propagation through a layered soil site. The spatially varying ground motion at the base rock is modelled by the filtered Tajimi-Kanai power spectral density function and a coherency loss function; the amplification effect of local soil site is modelled by the wave propagation theory. The auto power spectral density functions of ground motions at various points on ground surface and the cross power spectral density functions between ground motions at any two points are derived. The spectral-representation method is used to generate spatially varying ground motion time histories with different intensities and frequency content, besides the loss of coherency and phase delay. This method has important advantages compared to the method proposed by Deodatis (1996): (1) the power spectral densities at different locations of a canyon site are derived based on the wave propagation theory, which realistically reflect the effect of local soil conditions on seismic ground motions; (2) different types of incoming waves, which have great influence on the motions on the ground surface, can be considered. The simulated ground motion time histories can be used as input to multiple supports of long structures crossing a canyon site, or a site of different soil properties.

2. Ground motion simulation

We assume the earthquake ground motions on the base rock are stationary random processes with zero mean values and have the same power spectral density function. This is a reasonable assumption since the distance from the source to the site is usually much larger than the dimension of the structure. We let the base rock motion propagates through the site to ground surface. The matrix of the cross-power spectral density functions of ground motions at *n* locations on surface of a canyon site can be expressed as

$$S(i\omega)=\begin{bmatrix} S_{11}(\omega) & S_{12}(i\omega) & \cdots & S_{1n}(i\omega) \\ S_{21}(i\omega) & S_{22}(\omega) & \cdots & S_{2n}(i\omega) \\ \cdots & \cdots & \cdots & \cdots \\ S_{n1}(i\omega) & S_{n2}(i\omega) & \cdots & S_{nn}(\omega) \end{bmatrix} \tag{1}$$

The elements of the cross-power spectral density function are

$$S_{ij}(i\omega)=H_i(i\omega)H_j^*(i\omega)S_g(\omega)\gamma_{i'j'}(d_{i'j'},i\omega) \qquad i,j=1,2,...,n \tag{2}$$

where $H_i(i\omega)$, $H_j(i\omega)$ are the site amplification spectra at sites *i* and *j*, respectively, which can be estimated by using the wave propagation theory proposed by Wolf [2]. Superscript '*' denotes complex conjugate. $S_g(\omega)$ is the power spectral density function of ground motion at the base rock, $\gamma_{i'j'}(d_{i'j'},i\omega)$ is the coherency loss function of the motion at the base rock, which is related to the distance between location i' and j' and circular frequency ω.

The matrix $S(i\omega)$ given in Equation (1) is Hermitian and positive definite, it can always be decomposed into the multiplication of a complex lower triangular matrix $L(i\omega)$ and its Hermitian $L^H(i\omega)$ as

$$S(i\omega)=L(i\omega)L^H(i\omega) \tag{3}$$

After obtaining $L(i\omega)$, the stationary time series $u_i(t), i=1,2,...,n$, can be simulated in the frequency domain as (Hao et al. 1989):

$$U_i(i\omega_n)=\sum_{m=1}^{i}B_{im}(\omega_n)\cos[\cos\alpha_{im}(\omega_n)+i\sin\alpha_{im}(\omega_n)], \qquad n=1,2,...,N \tag{4}$$

where

$$B_{im}(\omega_n) = \sqrt{\Delta\omega}\left|L_{im}(i\omega)\right|, \qquad 0 \le \omega \le \omega_N$$

$$\alpha_{im}(\omega_n) = \beta_{im}(\omega_n) + \varphi_{mn}(\omega_n) = \tan^{-1}\left(\frac{\mathrm{Im}[L_{im}(i\omega)]}{\mathrm{Re}[L_{im}(i\omega)]}\right) + \varphi_{mn}(\omega_n), \qquad 0 \le \omega \le \omega_N \tag{5}$$

are the amplitudes and phase angles to ensure the proper correlation relations satisfying Equation (1). $\varphi_{mn}(\omega_n)$ is a random phase angle uniformly distributed over the range $[0,2\pi]$, φ_{mn} and φ_{rs} should be statistically independent unless $m = r$ and $n = s$. ω_N represents an upper cut-off frequency beyond which the elements of the cross-power spectral density matrix given in Equation 3 is assumed to be zero. $\Delta\omega$ is the resolution in the frequency domain. $\omega_n = n\Delta\omega$ is the *n*th discrete frequency.

The stationary time series $u_i(t)$ can be obtained by inverse transforming $U_i(i\omega_n)$ into the time domain. In order to obtained the non-stationary time histories, an envelope function $\zeta(t)$ can be applied to $u_i(t)$. The non-stationary time histories at different locations are thus

$$f_i(t) = \zeta(t)u_i(t), \qquad i = 1,2,\ldots,n \tag{6}$$

3. Numerical example

To illustrate the proposed algorithm in the paper, a numerical example is presented to simulate non-stationary ground motion time histories at different locations either on the base rock or ground surface of a canyon site with multiple soil layers as shown in Figure 1. The damping ratio for all the soil layers and the base rock are assumed to be 0.05.

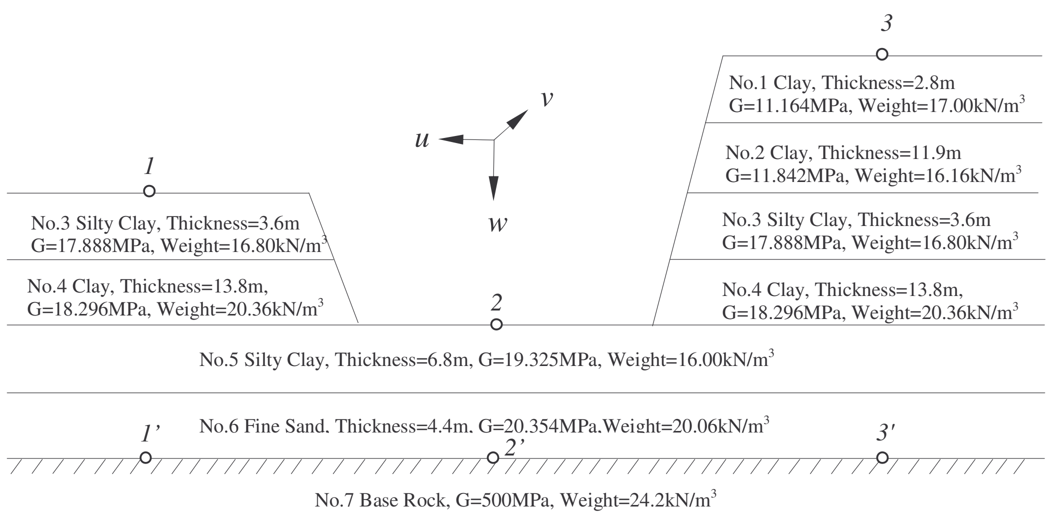

Figure 1. A canyon site with multiple soil layers

The filtered Tajimi-Kanai power spectral density function in the form of Equation (7) is chosen as the input on the base rock,

$$S_g(\omega) = |H_P(\omega)| S_0(\omega) = \frac{\omega^4}{(\omega_f^2 - \omega^2)^2 + (2\omega_f \omega \xi_f)^2} \frac{1 + 4\xi_g^2 \omega_g^2 \omega^2}{(\omega_g^2 - \omega^2)^2 + 4\xi_g^2 \omega_g^2 \omega^2} \Gamma \qquad (7)$$

with $\omega_g = 6\pi rad/s$, $\xi_g = 0.6$, $\omega_f = 0.5\pi$, $\xi_f = 0.6$, $\Gamma = 0.00565 m^2/s^3$. These parameters correspond to the PGA on the base rock as $0.2g$ based on the standard random vibration method (Der Kiureghian 1980).

The Sobczyk model (1991) is selected to describe the coherency loss between the ground motions at points i' and j' ($i \neq j$) on the base rock:

$$\gamma_{i'j'}(i\omega) = \exp(-\beta \omega d_{i'j'}^2 / v_{app}) \cdot \exp(-i\omega d_{i'j'} \cos\alpha / v_{app}) \qquad (8)$$

where β is a coefficient which reflects the level of coherency loss, $\beta = 0.002$ is used in the present paper. $d_{i'j'}$ is the distance between the points i' and j', and $d_{1'2'} = d_{2'3'} = 100m$ is assumed. v_{app} is the apparent wave velocity, which is set to be $v_{app} = 2500 m/s$. α is the incident angle of the incoming wave on the base rock.

The Jennings envelope function is used for the modulating purpose:

$$\zeta(t) = \begin{cases} (t/t_0)^2 & 0 \le t \le t_0 \\ 1 & t_0 < t \le t_n \\ \exp[-0.155(t - t_n)] & t_n < t \le T \end{cases} \qquad (9)$$

with $t_0 = 2s$ and $t_n = 10s$.

For conciseness, only the SH wave is assumed as the incoming wave on the base rock. Other types of waves can be straightforwardly considered by modifying the dynamic stiffness matrix based on the wave propagation theory. Figure 2 and 5 show the generated time histories on the base rock and the ground surface respectively with a wave incident angle of 60°. Figure 3 and 6 are the comparison of the power spectral densities of the generated ground motion time histories and the prescribed model. Figure 4 and 7 compare the corresponding model coherency loss function and that of the simulated time histories, and Figure 8 shows the site amplification spectra.

It is obvious that the soil layers amplify the amplitudes and filter the frequency content of the incoming wave on the base rock. The PGAs of the simulated motions on the base rock are 2.01, 2.25, 1.96 m/s^2 respectively, which is very close to the value of $0.2g$ estimated by the standard random vibration method. However, the corresponding values on the ground surface reach 3.31, 4.33, 3.61 m/s^2 respectively. Both the power spectral density and coherency loss functions of the generated time histories are compatible with the prescribed models. The generated time histories can be used as inputs to multiple supports of long span structures crossing a canyon site.

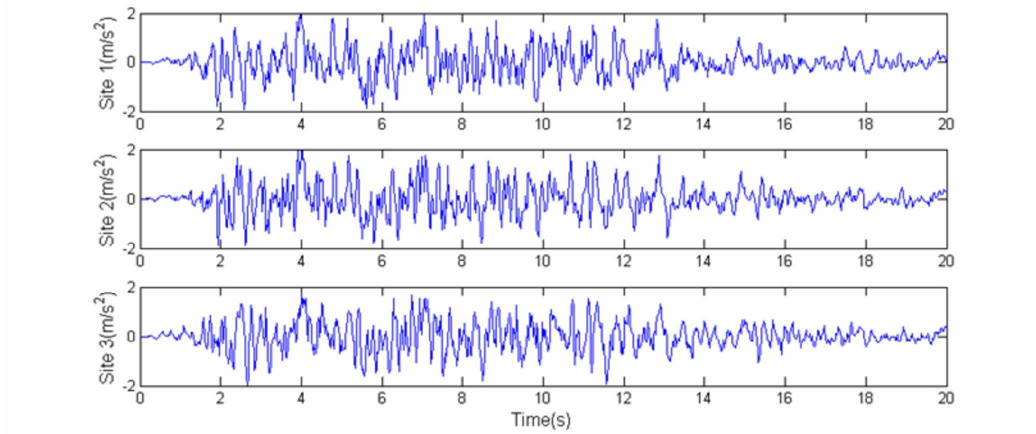

Figure 2. Generated base rock motions

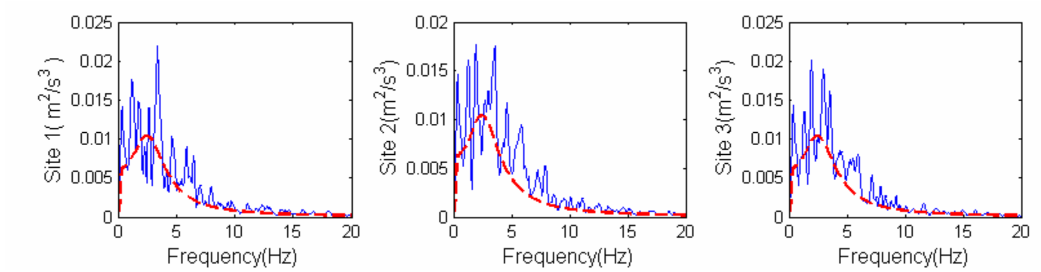

Figure 3. Comparison of the power spectral densities of the generated base rock motions (solid line) with the model power spectral density (dashed line)

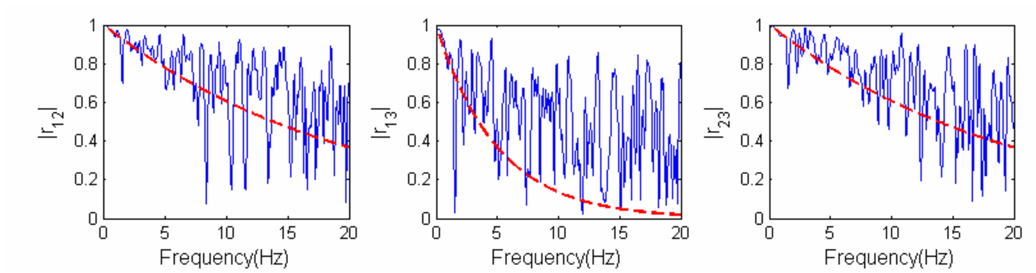

Figure 4. Comparison of the coherency loss between the generated base rock time histories (solid line) with the model coherency loss function (dashed line)

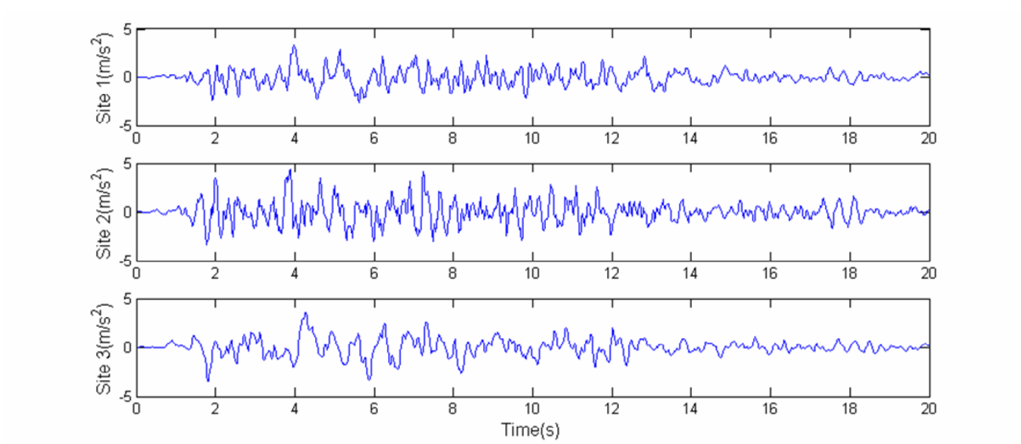

Figure 5. Generated out-of-plane motions on ground surface

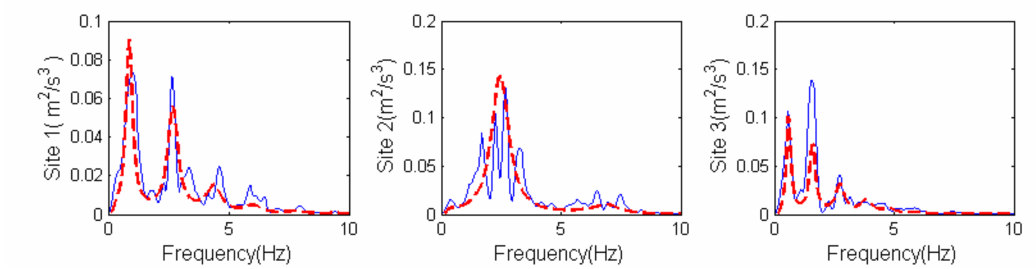

Figure 6. Comparison of the power spectral densities of the generated out-of-plane motions on ground surface (solid line) with the model power spectral density functions (dashed line)

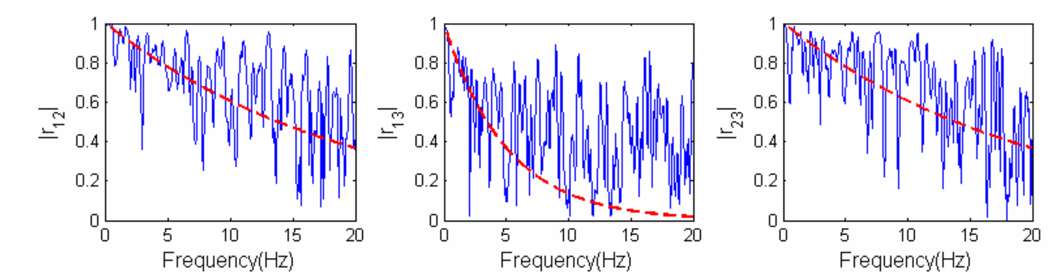

Figure 7. Comparison of the coherency loss of the generated out-of-plane motions on ground surface (solid line) with the model coherency loss function (dashed line)

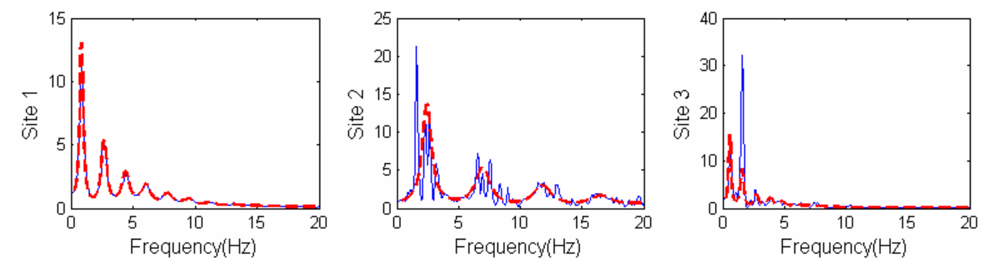

Figure 8. Comparison of the amplification spectra of the out-of-plane motions on ground surface (solid line) with the theoretical model (dashed line)

4. Conclusions

This paper presents a method to generate spatial ground motion time histories to be compatible with different power spectral density functions and a coherency loss function. It is applied to generate spatial ground motions on surface of a canyon site with multiple soil layers. The generated time histories are compatible with power spectral density functions of ground motions at various points on base rock or on ground surface of a canyon site, which can be theoretically derived based on wave propagation theory. The simulated spatial ground motions are also compatible with a prescribed coherency loss function. The generated spatial ground motion time histories can be used as multiple inputs to long span structures crossing a canyon site.

5. References

Deodatis G. (1996). Non-stationary stochastic vector process: seismic ground motion applications. *Probabilistic Engineering Mechanics*, 11: 149-168.

Der Kiureghian A. (1980). Structural response to stationary excitation. *Journal of Engineering Mechanics*, 106(6): 1195-1213.
Hao H, Oliveira CS, Penzien J. (1989). Multiple-station ground motion processing and simulation based on SMART-1 Array data. *Numerical Engineering and Design*, 111: 293-310.
Safak E. Discrete-time analysis of seismic site amplification. Journal of Engineering Mechanics 1995; 121(7): 801-809.
Sobczky K. (1991).Stochastic wave propagation. Kluwer Academic Publishers, the Netherlands.
Wolf JP. Dynamic soil-structure interaction. Englewood Cliffs, NJ: Prentice Hall, 1985.
Wolf JP. Soil-structure interaction analysis in time domain. Englewood Cliffs, NJ: Prentice Hall, 1988.
Zerva A, Zervas V. (2002). Spatial variation of seismic ground motions: An overview. *Applied Mechanics Reviews*, 56(3): 271-297.