Energy-based design method for seismic retrofitting with passive energy dissipation systems

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Abstract

Passive energy dissipation is a reliable, effective and relatively inexpensive technique for mitigating seismic risks to civil structures. Significant amount of seismic input energy can safely be dissipated by designated sacrificial energy dissipation devices (EDDs), leaving the main structural elements relatively intact. The most common types of passive EDDs are metallic devices that utilize inherent material ductility for energy dissipation. For the past nine years, a metallic device exploiting the plastic response of a diaphragm steel plate in pure shear has been investigated analytically and experimentally at the University of Queensland. This device, the yielding shear panel device (YSPD), is compact, inexpensive and easy to fabricate and offers excellent and stable energy dissipation response.

Implementation of EDDs in general and YSPD in particular, as a seismic retrofit solution would require nonlinear time-history analysis of the parent structure subjected to ground acceleration. This type of analysis is complex and time consuming. For this purpose a new practical design method is proposed for seismic retrofitting with EDDs. Conventional seismic design accounts for maximum earthquake load and maximum displacement and does not provide enough information on the accumulated plastic behavior of the structure. In contrast the proposed design method is energy-based and accounts for accumulated damage.

1. Introduction

In spite of the steady development of EDDs, implementation of EDDs as a retrofit strategy for mitigating seismic risk in existing structures is still lacking. To facilitate the adoption and implementation of EDDs, the development of suitable design methods is necessary. Aguirre [1] proposed an iterative design method based on linear static analysis while Williams and Albermani [2] proposed a design method based on equivalent viscous damping. These force-based methods rely on a force reduction factor that is controversial, does not directly address the inelastic nature of the structure during the earthquake and the resulting displacement is only checked at the end of the design process to satisfy serviceability criteria. Furthermore, structural (and non-structural) damage experienced during an earthquake is primarily due to lateral displacements. Therefore, a force-based design method may not provide a reliable indication of damage potential.
To resolve these shortcomings, displacement-based seismic design methods have been proposed such as FEMA-273 coefficient method [3], ATC-40 capacity-spectrum method [4] and the direct displacement-based design method [5]. Chen et al [6] proposed a seismic design approach for steel portal frame piers with buckling-restrained braces. They utilized

displacement-based as well as strain-based indices as pre-determined target performance at the beginning of design.

The structural damage caused by earthquake ground motion results not only from the maximum response but also from accumulated plastic deformations. However current seismic design practice, which accounts only for the maximum earthquake load and maximum lateral displacement, does not provide enough information on the inelastic response of the structure. In this regard energy-based seismic design methods, which utilize hysteretic energy as the main design parameter and account for damage accumulation, have been considered as potential alternative to the conventional force or displacement-based seismic design methods.

In this paper, a step-wise multi-mode energy-based design method for seismic retrofitting of frame structures with EDDs is proposed. Nonlinear time-history analysis is used as a baseline case against which the proposed design method is validated. The method has been applied to frame structures with different heights and number of stories and under different earthquake ground motions. However, since the objective of this paper is the presentation of the proposed design method, only one frame structure (a 9-story moment resisting frame (MRF)) under one earthquake record (1940 Imperial Valley El Centro LA02) is presented.

2. Seismic Energy Demand (SED)

Chou and Uang [7] had outlined a procedure for determining SED at each floor of a multi degree of freedom (MDOF) system. This procedure forms the basis of the proposed step-wise design method.

2.1 Constant-ductility response spectra for SDOF System

The energy equation for an inelastic SDOF system

$$E_k + E_\xi + E_a = E_i \tag{1}$$

Where

$$E_k = \frac{m(\dot{u}_t)^2}{2} \tag{1a}$$

$$E_\xi = \int_0^t (c\dot{u})du \tag{1b}$$

$$E_i = \int_0^t (m\ddot{u}_t)du_g \tag{1c}$$

$$E_a = E_s + E_h = \int_0^t f du \tag{1d}$$

In which E_k, E_ξ and E_i are the kinetic, viscous damping and input energy, respectively, E_a is the absorbed energy that consists of recoverable strain energy, E_s, and the irrecoverable hysteretic energy, E_h. The other variables in eq 1 are; ground displacement, u_g, relative displacement, u, total displacement, $u_t = u + u_g$, mass m, viscous damping, c, restoring force, f, and superscript ($\dot{}$) indicates $\partial/\partial t$.

The equivalent velocity, V_a, is used as a parameter for energy demand since it converges to pseudo-velocity in the elastic case (ie for ductility factor less than one).

$$V_a = \sqrt{\frac{2E_a}{m}} \tag{2}$$

For a given modal ductility factor, μ, and damping ratio, ξ, nonlinear dynamic analysis of the inelastic SDOF is conducted and the yield force, f_y, and yield displacement, D_y, are evaluated. From these the normalized yield strength coefficient, C_y, and maximum displacement, D_s, are obtained;

$$C_y = \frac{f_y}{mg} \tag{3a}$$

$$D_s = \mu D_y \tag{3b}$$

In this work, the 1940 Imperial Valley El Centro (LA02) earthquake ground motion is used to construct the constant-ductility response spectra (D_s and C_y) for an elastic-perfectly plastic SDOF system with 0% -40% viscous damping. Fig 1 shows sample D_s and C_y spectra for ξ=5, 10 and 15%. From nonlinear time-history analysis of the SDOF system with C_y values at different ductility levels, the maximum value of E_a for different damping ratios is calculated and the V_a spectra of the absorbed energy is constructed as shown in Fig 1. For any earthquake record, constant-ductility response spectra similar to those in Fig 1 can be generated using a Matlab code written for this purpose.

2.2 Equivalent SDOF systems

In this work the SED of the MDOF system is evaluated using two modes which have the highest participation factors. These two modes are determined first using elastic modal analysis of the MDOF. Using each of these two modes, modal pushover analysis of the MDOF is conducted and the response is converted to an equivalent single degree of freedom (ESDOF) to determine C_y and hence μ from C_y spectra (Fig 1). Using μ for each of these two modes, E_a is determined from V_a spectra and summed up to calculate the total absorbed energy of the MDOF system.

3. Stepwise energy-based design method

This section outlines the proposed stepwise energy-based design method for seismic retrofitting of frame structures with passive energy dissipation systems (PEDS).

Step 1:
This step involves modal analysis (sub-step 1a), push-over analysis (sub-step 1b) and initialization (sub-step 1c).

1a- Performing elastic modal analysis of the original frame structure (ie before retrofitting) and determine the two modes (i=1 and 2) which have the highest participation factor. For each of these two modes, the period is T_i, the normalized modal shape vector is φ_i (eq 4a) and the participation factor is Γ_i (eq 4b)

$$m_i = \varphi_i^T \mathrm{M} \varphi_i = 1 \tag{4a}$$

$$\Gamma_i = \varphi_i^T \mathrm{M} 1 / \mathrm{m_i} \tag{4b}$$

Where M is the diagonal mass matrix and **1** is a unit vector.

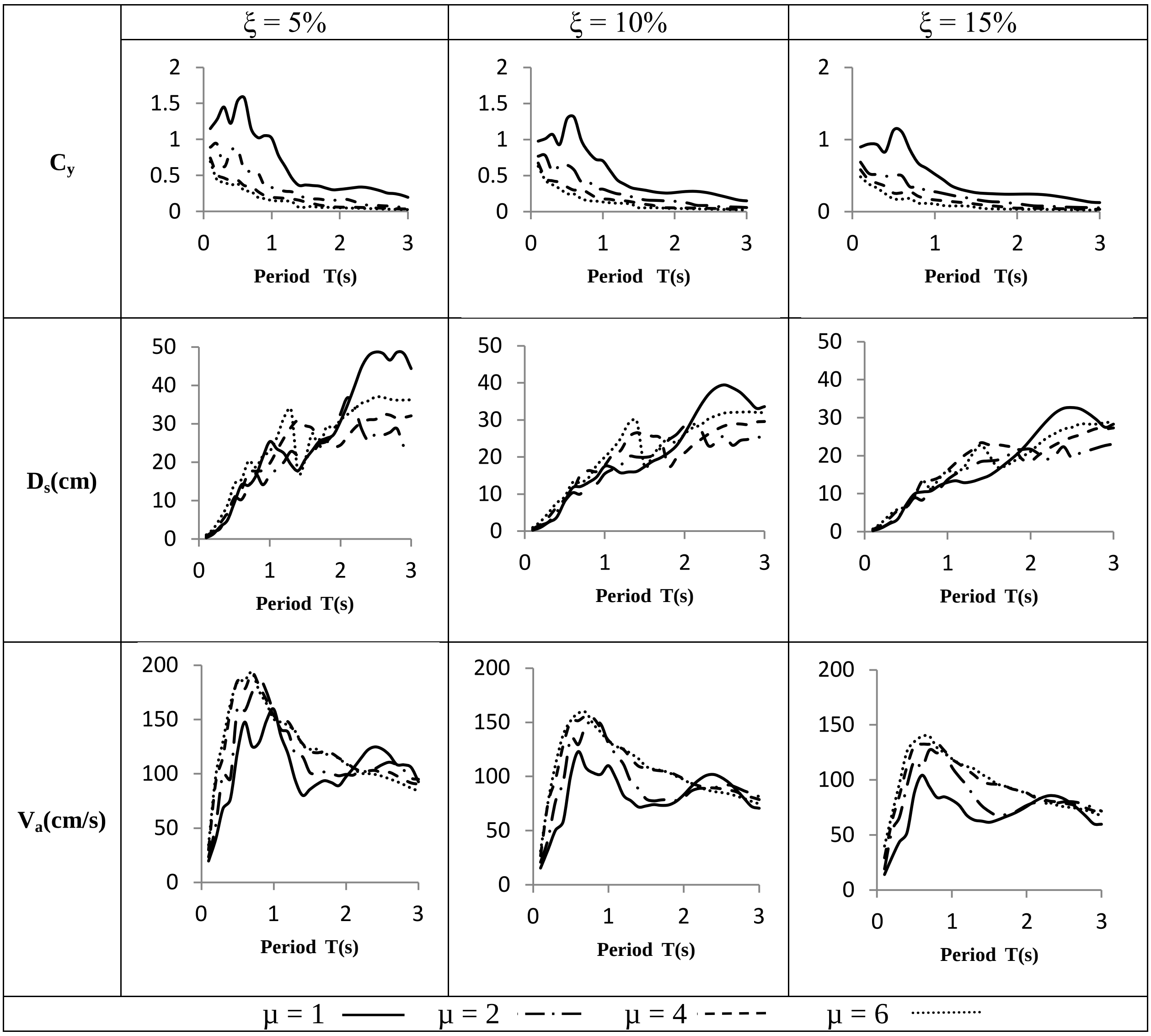

Figure 1: Generated response spectra for LA02

1b- For each mode, i, perform modal pushover analysis of the original frame structure and calculate the yield force F_{yi} then transform this to the corresponding force f_{yi} in the ESDOF

$$f_{yi} = \frac{F_{yi}}{\Gamma_i} \tag{5a}$$

Using f_{yi} to calculate the yield strength coefficient C_{yi}

$$C_{yi} = \frac{f_{yi}}{\Gamma_i m_i g} \tag{5b}$$

1c- Assign a value for the damping ratio ξ to start design iterations. The starting value of ξ depends on the level of acceptable plastic deformations (following an earthquake) in the original structure. In this work ξ=5% is assumed which corresponds to nearly elastic response of the original structure under seismic action.

Step 2:

For a damping ratio ξ with T_i and C_{yi} of each mode, determine the ductility factor μ_i from C_y spectra (Fig 1). This value can be find by interpolation between spectra. Note that when the ductility factor (μ) is less than 1 (from C_y-spectra), C_y value for μ =1 and specific period (T) is first determined, then the ratio of this C_y to that from step 1b is the system ductility factor for the elastic response.

Step 3:

Using ξ, μ_i and T_i, determine the equivalent velocity of absorbed energy V_{ai} from V_a spectra (Fig 1).

Step 4:

In this step the absorbed energy and the elastic strain energy are calculated;
4a- Calculate the absorbed energy Ea_i and the total absorbed energy E_{aT}

$$E_{a_i} = \frac{m_i\left(\Gamma_i V_{ai}\right)^2}{2} \tag{6a}$$

$$E_{a_T} = \sum_{i=1}^{2} E_{a_i} \tag{6b}$$

4b- This step is performed once only at the first design iteration,

$$\overline{E_{a_i}} = E_{a_i} \tag{7a}$$

Using D_s spectra (Fig 1) with ξ, μ_1 and T_1, obtain D_s for i=1 and calculate the elastic strain energy Es

$$E_s = \frac{2\pi^2}{T_1^2} m_1 \left(\frac{\Gamma_1 D_{s1}}{\mu_1}\right)^2 \tag{7b}$$

It is assumed that E_s is dominated by the 1st mode and it will not change during design iterations since it represents the elastic capacity of the original structure.

Step 5:

Calculate the plastic strain energy that need to be dissipated, E_D

$$E_D = E_{a_T} - E_s \tag{8}$$

For the 1st iteration, if eq 8 gives $E_D \leq 0$ then the structural response is elastic and no need for EDDs, the design method is terminated. Otherwise the damping demand ξ_D is

$$\xi_D = \frac{E_D}{4\pi E_s} \tag{9}$$

update the damping ratio accordingly

$$\xi = \xi + \xi_D \tag{10}$$

Step 6:

Compare the dissipative plastic energy E_D with the elastic strain energy (E_s)

If $\frac{E_D}{E_s} > \varepsilon$ go back to step 2, otherwise

$$E_{Di} = \overline{E_{a_i}} - E_{ai} \tag{11}$$

where ε is a convergence tolerance that can be adjusted according to design requirements.

Step 7:

Distribute E_{Di} over the height of the structure according to the energy profile of the corresponding mode i to obtain the absorbed energy at each storey E_{Dis} (see Appendix). Calculate the total absorbed energy at each story from both modes E_{DTS}

$$E_{D_{Ts}} = \sum_{i=1}^{2} E_{D_{is}} \tag{12}$$

Step 8:

Using the D_s spectra (Fig 1) with ξ, μ_1 and T_1, obtain D_{s1} (maximum displacement of ESDOF), convert to MDOF system ($D_{s1}\Gamma_1$) and distribute over the height of the structure according to the modal shape vector φ_1 then calculate the drift at each story, Δ_s.

Step 9:

For each story, using E_{DTs} (step 7) and Δ_s (step 8), determine the required EDDs based on the device characteristic displacement u_y and device strength F_{yd} (device stiffness $K_d = F_{yd}/u_y$).

Step 10:

Design the bracing members according to the required axial force in each brace, F_{Brace}

$$F_{Brace} > \frac{F_{yd}}{\cos\alpha} \quad \text{for metallic yielding devices} \tag{13a}$$

$$F_{Brace} > F_{yd} \quad \text{for other devices} \tag{13b}$$

Where α is the brace angle with the horizontal axis. The brace is designed as a strut member with adequate capacity to preclude buckling.

4. Verification

To demonstrate the implementation of the design method outlined in Sec 3, a 9 story moment resiting frame (MRF) is used in this section. The original structure (Fig 2 and Tables 2 and 3) has steel MRF in both directions (shown as solid lines in Fig 2) and was designed by SAC-commissioned consulting firm, according to 1994 UBC [8] and was used by many researchers as a benchmark case study [9, 10]. The designed nominal yield strength of the beams and columns were 248 (339) MPa and 345 (397) MPa, respectively (values in brackets are the expected yield strengths and were used to compute the members' capacity in dynamic analysis). The computer program SAP2000 [11] was used for the analysis assuming a bi-linear moment-rotation relationship with 0 and 3% strain hardening to model the response of the beams and columns, respectively, and 5% Rayleigh damping for the first two modes. To start the design method modal analysis and modal pushover analysis is performed (Table 1 Fig A1).

Table 1: Modal properties

Frame	T_1 (sec)	T_2 (sec)	Γ_1	Γ_2
9 story	2.13	0.80	1931.60	709.82

Nonlinear time history analysis is used to verify the accuracy of the proposed design method. The 1940 Imperial Valley El Centro (LA02, PGA 662.88 cm/sec^2) is used and the analysis is conducted using SAP2000 [11] with implicit integration using Hilber-Hughes-Taylor method [12].

Four different analyses were conducted, these are;

a- Original structure with 5% Rayleigh damping

b- Original structure with amplified 15% Rayleigh damping

c- Dissipative structure: original structure retrofitted with EDDs (metallic yielding) according to the proposed design method

d- Stiffened structure: original structure retrofitted with braces only (no EDDs) from case c

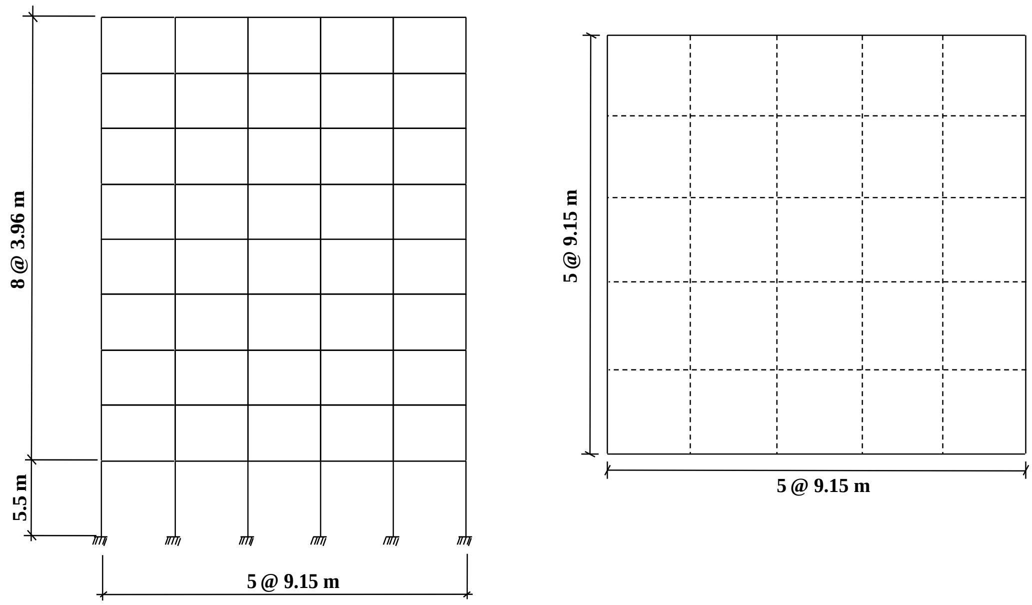

Figure 2: 9-story MRF

Table 2: Summary of design parameters and beam-member sections for the 9-story frame

Floor	Weight (kN)	Design Load (kN)	Drift (cm)	Drift Limit (cm)	Beam members
2nd	5053	21.92	1.71	2.0	W36×160
3rd	5053	58.85	1.34	1.44	W36×160
4th	5053	111.24	1.41	1.44	W36×135
5th	5053	178.14	1.49	1.44	W36×135
6th	5053	258.87	1.47	1.44	W36×135
7th	5053	352.94	1.41	1.44	W36×135
8th	5053	459.95	1.43	1.44	W30×99
9th	5053	579.55	1.35	1.44	W27×84
Roof	3790	533.6	0.94	1.44	W24×68

Table 3: Column-member sections and normalized modal shape vectors for the 9-story frame

Story	**Column members**		φ_1 ($\times 10^{-3}$)	φ_2 ($\times 10^{-3}$)
	Interior	**Exterior**		
1st	W14×500	W14×370	0.11	-0.28
2nd	W14×500	W14×370	0.20	-0.45
	W14×455			
3rd	W14×455	W14×370	0.28	-0.55
4th	W14×455	W14×370	0.37	-0.57
	W14×370	W14×283		
5th	W14×370	W14×283	0.45	-0.47
6th	W14×370	W14×283	0.53	-0.25
	W14×283	W14×257		
7th	W14×283	W14×257	0.60	0.09
8th	W14×283	W14×257	0.67	0.49
	W14×257	W14×233		
9th	W14×257	W14×233	0.71	0.81

The results of the nonlinear time history analysis are summarised in Table 5. Table 5 compares the maximum absorbed energy from time history analysis ($E_{a,\,max}$) with that from the design method, E_{aT} (eq 6) for case (a) and (b). From Table 5 it is clear that for different damping ratios (5% and 15%), the amount of the absorbed energy predicted by the design method is very close to that obtained from nonlinear time history analysis. The design method over predicts the amount of the absorbed energy by 3% and 6% for damping ratio of 15% and 5% respectively.

The evolution of plastic hinges in any member (beams, columns and braces) is defined according to FEMA 356 (Tables 5-6 and 5-7) [13]. Three different performance levels are defined. These are; immediate occupancy (IO), life safety (LS) and collapse prevention (CP).The result at the end of the time-history analysis of the original structure with 5% Rayleigh damping shows that 42 LS and 11 IO plastic hinges have already formed in the beam members in addition to 3 IO plastic hinges formed at the base of the column members at the ground floor (a total of 56 plastic hinges). With 15% viscous damping (case b) the number of plastic hinges has reduced to 7 hinges in the beam members only and out of these 7 hinges, 6 are at IO level and only one has reached the LS level.

The time history analysis of the dissipative structure (case c) indicates that the structure remains elastic and the damaging part of the seismic energy is dissipated by the EDDs. The dissipative frame in case c is designed based on 15% hysteretic damping (Table 4), however time history analysis shows that, unlike case b (15% viscous damping), the dissipative frame remains elastic (while case b has 7 hinges). The superior performance of the dissipative frame (case c) in comparison to case b is due to proper distribution of EDDs along the height of the frame according to energy demand at each story. Furthermore, the dissipated energy of the EDD is calculated based on the largest hysteretic cycle executed by the device (based on maximum story drift). During design earthquake excitation, the device may undergo some smaller cycles in addition to the largest design one. This makes the design method to be conservative.

In order to demonstrate the effectiveness of the EDDs designed according to the proposed retrofitting method, the original frame with added bracing system (stiffened frame without EDDs) is considered in case d. The same bracing members used in case c are utilised in case d. The results show that all bracing members would fail during the earthquake; this is followed by plastic hinge formation in the original frame structure with 25 hinges forming at beam members and 2 hinges at the base of the column members at the ground floor.

Table 4: Summary of design iterations of the proposed method

ξ	μ_1	μ_2	V_{a1} (cm/s)	V_{a2} (cm/s)	E_{a1} (kNm)	E_{a2} (kNm)	E_{aT} (kNm)	E_s (kNm)	E_D (kNm)	ξ_D	$E_{aT} \approx E_s$?
5%	1.27	0.7	105.79	128.81	2087.82	417.99	2505.81	1326.205	1179.605	7%	NO
12%	1.1	0.52	87.54	95.37	1429.61	229.13	1658.74	1326.205	332.54	2%	NO
14%	1.02	0.48	83.45	87.72	1299.14	193.85	1492.99	1326.205	166.78	1%	NO
15%	0.97	0.46	81.33	83.88	1233.97	177.24	1411.22	1326.205	85.02	_	YES*
*design iteration is terminated at ξ=15%											

At the end the results of the design method also checked with two more ground motions which are 1994 Northridge, Sylmar county Hospital (PGA =827.28 cm/sec^2) and 1989 Loma Prieta, Sratoga (PGA= 502.75 cm/sec^2). For these cases the design method continued until the last iteration (E_D=0). For Northridge ground motion the final damping ratio needs to be 46.6% to let the main structure remains elastic however for Loma Prieta this value is 11.20%. Table 6 shows the comparison between the time history analysis results and the design method for these two ground motions. We can see that the amount of the absorbed energy predicted by the design method is very close to that obtained from nonlinear time history analysis. This will confirm the accuracy of the design method which is described earlier with Imperial Valley El Centro ground motion.

Table 5: Comparison of maximum absorbed energy from nonlinear time-history analysis and the proposed design method (El Centro)

ξ	$E_{a,\,max}$ (kN.m)	E_{a1} (kN.m)	E_{a2} (kN.m)	E_{aT} (kN.m)	$\frac{E_{aT} - E_{a,max}}{E_{a,max}}$
5%	2356	2087.82	417.99	2505.81	0.06
15%	1364.45	1233.97	177.24	1411.22	0.03

5. Conclusion and discussion

A stepwise multi-mode energy-based design method for seismic retrofitting with passive energy dissipation systems is proposed. The method incorporates two modes with the highest participation factors to conduct modal push-over analysis and calculates modal yield forces and energy profiles. ESDOF is obtained and nonlinear response spectra are used to determine the ductility factor for each mode. The energy contribution of each mode is then determined and distributed over the height of the structure based on energy profiles. The required amount of energy dissipation and drift at each story are calculated and used to retrofit the structure with EDDs.

The proposed method is verified using nonlinear time-history analysis which shows that the retrofitted structure remains intact while damage is confined to the added EDDs. Very good correlation is obtained between the proposed method and nonlinear time-history analysis (Table 5). The effectiveness of the proposed retrofit design method can be seen by comparing the nonlinear time-history response of the retrofitted dissipative structure with that of the stiffened structure. The effectiveness of the proposed method can be further seen by comparing the response of the resulting dissipative system to that of the original frame with amplified viscous damping.

Although two modes are used for the calculation and distribution of the absorbed energy, only the 1st mode is used for estimating the elastic strain energy and the drift at each story. The proposed method can be easily extended to include additional modes, if required, and is applicable to any passive energy dissipation system. Although the method is presented within a retrofitting context of existing structure, it can be used for new design of dissipative structures. The method allows the designer to specify an acceptable damage in the structure and to design the dissipative system accordingly. The proposed stepwise method lends itself to spread-sheet type calculations and requires only static push-over analysis. The required response spectra (Fig 1) can be generated and tabulated for any seismic action using the developed Matlab code.

Table 6: Further comparison of maximum absorbed energy from nonlinear time-history analysis and the proposed design method (Northridge and Loma Prieta)

Ground Motion	ξ	$E_{a,\,max}$ (kN.m)	E_{a1} (kN.m)	E_{a2} (kN.m)	E_{aT} (kN.m)	$\frac{E_{aT}-E_{a,max}}{E_{a,max}}$
Loma Prieta	5%	1409.50	1406.84	77.91	1484.74	0.05
	11.2%	1172.07	1194.24	70.90	1265.14	0.08
Northridge Sylmar Hospital	5%	6795.18	6084.78	479.20	6563.98	0.03
	46.6%	965.79	957.28	88.57	1045.84	0.08

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Appendix: Energy distribution along the height of the structure

Step 7 of the proposed design method requires the distribution of the energy along the height of the frame. The energy distribution procedure proposed by Chou et al. [7] has been modified and extended for retrofitting of frame structures with EDDs.

Based on the results from modal pushover analysis of the two modes with the highest participation factors, pushover curves for each of these two modes are constructed (Fig A1). For each pushover curve, two points (A and B) are identified. Point A corresponds to the formation of the first plastic hinge in a beam member. Point B corresponds to the formation of the first plastic hinge in the base of a column member. In addition, three zones are determined on the pushover curve; from zero to A, from A to B and beyond B. In Fig A1, point 1 and 2 represent the midpoints of the first two zones while point 3 coincides with point B and represents the third zone. The first zone corresponds to elastic response, the second zone corresponds to the evolution of plastic hinges (at the lower floor levels in mode 1 and upper levels in mode 2) and the third zone corresponds to the formation of plastic hinges at the column's base at ground level in mode 1 and at the k-th story in mode 2. The k-th story is usually where significant reversal of lateral forces occurs and the first plastic hinge forms at a beam member in this story (Point A, mode 2).

For each of the three zones represented by points 1-3 on the pushover curve, the energy at each storey is calculated based on the moment-curvature response of all the members in that story. The total energy is obtained from the sum of the energy at each story over the height of the frame. Then the calculated energy at each story is normalised by the total energy. For each mode (i =1, 2), three energy profiles ψ_{ij} corresponding to each of the three zones (j =1, 2, 3) are obtained. For the 9-story building used in Sec 4, these energy profiles are shown in Fig A2.

Determine the appropriate energy profile for each mode (1 and 2) based on the drift of the first floor DR_1 (for mode 1) and k-th floor DR_2 (for mode 2). These are calculated from.

$$DR_1 = \frac{D_{s1}\ \Gamma_1\ \varphi_{11}}{\mathrm{H}_1} \tag{A1}$$

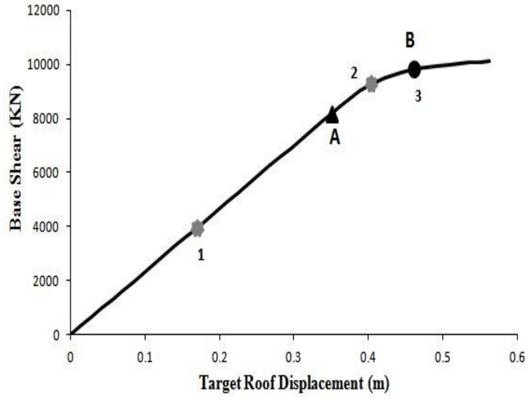
$$DR_2 = \frac{D_{s2}\ \Gamma_2\,(\varphi_{2k} - \varphi_{2(k-1)}\ )}{\mathrm{H}_k} \tag{A2}$$

D_{s1} and D_{s2} are the maximum displacements (mode 1 and 2) of the ESDOF (Fig 1). φ_{11} and φ_{2k} and $\varphi_{2(k-1)}$ are the components of the modal vectors (mode 1 and 2) at story 1, k and $k-1$ respectively. H_1, H_k are the height of 1st and the k-th story.

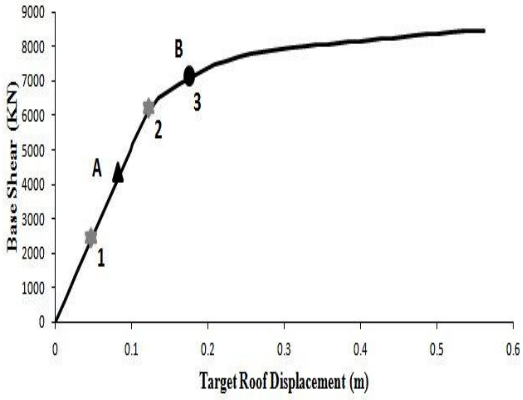
From the modal pushover analysis of each mode (i = 1 and 2) determine story drift at which the first plastic hinge form in a beam member (DR_{ibeam}) and at the base of a column member (DR_{ibase}). Select the appropriate energy profile according to

$DR_i < DR_{ibeam} \rightarrow$ Energy profile ψ_{i1}, $DR_{ibeam} \leq DR_i < DR_{ibase} \rightarrow$ Energy profile ψ_{i2}

$DR_i \geq DR_{ibase} \rightarrow$ Energy profile ψ_{i3}

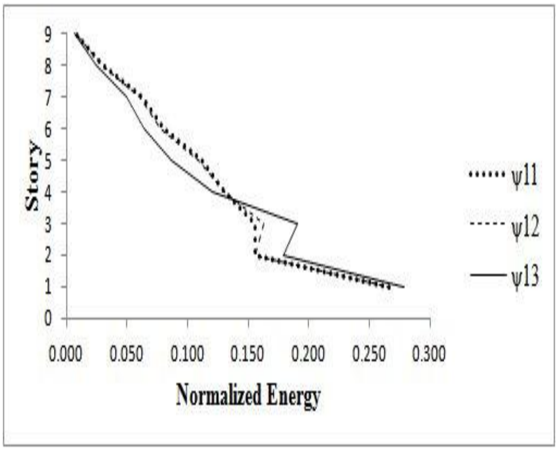


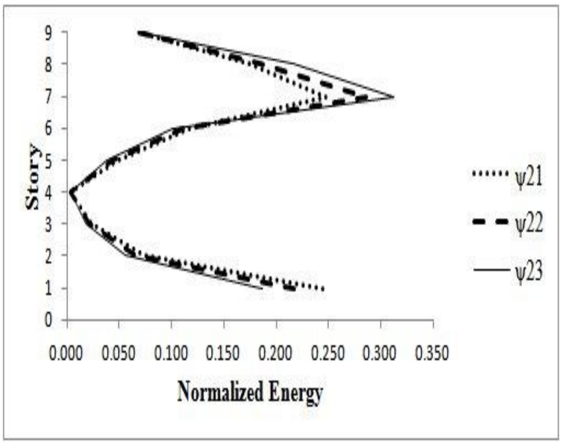
a) Mode1



b) Mode 2

Figure A1: Modal push-over curves





a) Mode1

b) Mode 2

Figure A2: Energy profiles