A NON-DESTRUCTIVE EXPERIMENTAL INVESTIGATION INTO THE IMPACT RESPONSE BEHAVIOUR OF LIGHTWEIGHT BEAMS

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ABSTRACT

This paper presents investigation by the authors on the response behaviour of a single span beam which is subject to the impact action at its mid-span position by a fallen object. Small scale experimentations were employed to study the response behavior of the beam and its maximum deflection. The investigation conducted at this stage is primarily based on linear elastic behaviour. Thus, a simple beam which is made of timber has been tested. The important effects of a *mass protection* and *cushion* in mitigating the effects of the impact are the key findings to be presented in the paper. A simple hand calculation method for predicting maximum deflection of the beam which takes into account these mitigating phenomena are also presented.

1. Introduction

Innovations involving the use of lightweight materials can have strategic advantages over traditional design in terms of operational efficiency, improved functionality and reduction in the consumption of materials and energy (including embodied energy) in the long term. With civil engineering (and building) construction, design innovations to reduce weight typically make effective use of composite actions, pre-stressing actions, membrane or shell actions and the like. The common challenges in achieving those innovations are to ensure that the strength and stiffness properties of the innovative design are either superior, or comparable, to traditional design solutions whilst possessing other advantages. For example, lightweight concrete must possess compressive strength which is at least comparable to that of traditional concrete mixes; and lightweight floor panels must be able to span the same distance as conventionally designed floor panels for given gravitational loads whilst satisfying strength and deflection requirements. It is noted that parameters used for measuring structural performance objectives are typically based upon static, or quasi-static, conditions.

Limitations with analyses based on quasi-static conditions for modeling impact actions have been recognized for a long time. Whilst, collision loads are still commonly represented as quasi-static loads by designers, alternative methods of analysis based on equating energy have also been used. For example, highway codes of practices have also adopted the equal energy method in estimating the deflection demand (Δ) and strength demand (F) on the *target* (e.g. vehicular barrier, column in support of a over-bridge) when subject to the impact action of a vehicle. In essence, Eq. (1.1) and Eq. (1.2) can be used for frontal impact if linear elastic behaviour of the *target* is assumed. Refer review presented by the authors in Yang *et al.* [in press] for literature references.

$$\Delta = \frac{mV_o}{\sqrt{km}}$$
(1.1)

$$F = \sqrt{km}V_o$$
(1.2)

Where "m" is the mass of the vehicle, "V_0" is the cruising velocity immediately prior to contact with the surface of the target, and "k" is the elastic lateral stiffness of the structural element in support of the target in the direction of impact.

Equations (1.1) and (1.2) are based simply on equating the kinetic energy of the impacting object (the vehicle) with the amount of energy absorbed by the structural element in support of the target. Whilst these equations are easy to derive they have not taken into account the amount of energy that has been lost (and hence not transferred to the supporting structural system). For example, a significant amount of energy is dissipated as the impacting object (the vehicle) crumbles and sustains permanent deformation. More energy can be dissipated as the surface of the target is indented by the impacting object whilst sustaining localized permanent deformations. If the impacting object rebounds from the surface of the target following the impact, a significant amount of energy can be carried away by the rebounding impacting object in the form of kinetic energy. Similarly, significant amount of energy can be dissipated by impact on an absorber which is referred herein as "cushion". Thus, the amount of energy that is left to be absorbed by the structural element in support of the target can be very different to the initial kinetic energy carried by the *projectile* (as assumed in the derivation of Eq. (1.1) and Eq. (1.2). Importantly, this fraction of energy which is transferred

into strain energy can be very sensitive to the effective mass of the target, or the mass ratio (α) which is defined as the target effective mass divided by the mass of the impacting object (*m*). Consequently, the impact resistant capacity of a lightweight structural element (beam, column or floor panel) can be very different to its conventional design counterpart even if the two designs feature comparable static strength and stiffness properties. This distribution of energy that occurs on impact must be modeled accurately if the resistant capacity of a structural, or protective, element is to be assessed realistically. Grossly erroneous assumptions can be made by adopting quasi-static analysis as the basis of impact resistant capacity assessment. The distribution of energy is also not modeled by the equal energy method represented in Eq. (1.1) and Eq. (1.2).

A simple hand calculation method based on considering the collision between two lumped masses is introduced in Section 2 for analyzing the distribution of energy incurred on impact and for estimating the deflection and strength demand on the structural element in support of the target. The phenomena of protection by *cushion* and by *mass* (or inertia) will be illustrated analytically in Section 2 using this simplified modeling approach and then demonstrated experimentally in Sections 3 and 4. The objective of this paper is primarily to highlight these distinct phenomena which have important implications in the design of lightweight structures for robustness.

2. Displacement Demand Estimate on Target by Lumped Masses Model

Impact, or collision, actions can be modeled by a simple model involving two lumped masses which represent the *impacting object* (of mass "*m*") and the *target* (of mass *αm*) as shown in Figure 2.1. The stiffness of the frontal spring connecting the projectile object to the target object is given the notation "k_1" whereas the stiffness of the rear spring connecting the target to its support is given the notation "k_2". Figure 2.1 is the schematic diagram showing the configuration of the lumped masses model. Stiffness value k_1 is used to represent the conditions of contact. The well known *Hertz law* can be used to calculate the value of k_1 for the idealized case of a deformable spherical object striking a hard and flat surface. To represent the conditions of a hard projectile striking the same surface, a high value of k_1 is specified. Conversely, if either the projectile or surface of the target is soft, a low value of k_1 is specified (as illustrated by the schematic diagram of Figure 2.2). Meanwhile, stiffness value k_2 is used to represent the force-deflection behaviour of the structural element in support of the target which can be a beam, column, plate or frame. With structural elements featuring distributed mass (like beams), the generalized stiffness of the element calculated using *Rayleigh* method (Eq. (2.1)) can be taken as k_2. Similarly, the generalized mass of the element calculated using the *Rayleigh* method (equation 2.2) can be taken as *αm*.

$$k_2 = \int EI \left(\frac{\partial^2 \psi}{\partial x^2} \right)^2 dx \qquad (2.1)$$

$$\alpha m = \int \rho A \psi(x)^2 dx \qquad (2.2)$$

Where *EI* is flexural stiffness of the beam or column, "$\psi(x)$" is the shape function of deflection, "ρ" is the density of the material, and "*A*" is the cross-sectional area of the element with uniform sectional properties.

Approximate values of k_2 and *αm* for simply supported beams, cantilever beams and fixed-end beams are summarized in Figure 2.3. The position of contact is assumed to be at mid-span of the beam.

The *two-degree-of-freedom* (2DOF) system introduced herein can be analyzed for its dynamic response behaviour based on the use of standard techniques involving modal analyses and impulse response functions. The time-histories of the motion of both the projectile object, u_1 (t), and that of the target object, u_2 (t), can hence be solved. The impact response behaviour of the two colliding objects can be resolved into five stages as illustrated schematically in Figure 2.4. The differential movement of the two objects, u_1 (t) - u_2 (t), is indicative of the stress-strain conditions developed at contact between the two objects whereas u_2 (t) on its own is indicative of the deflection of the structural element in support of the target.
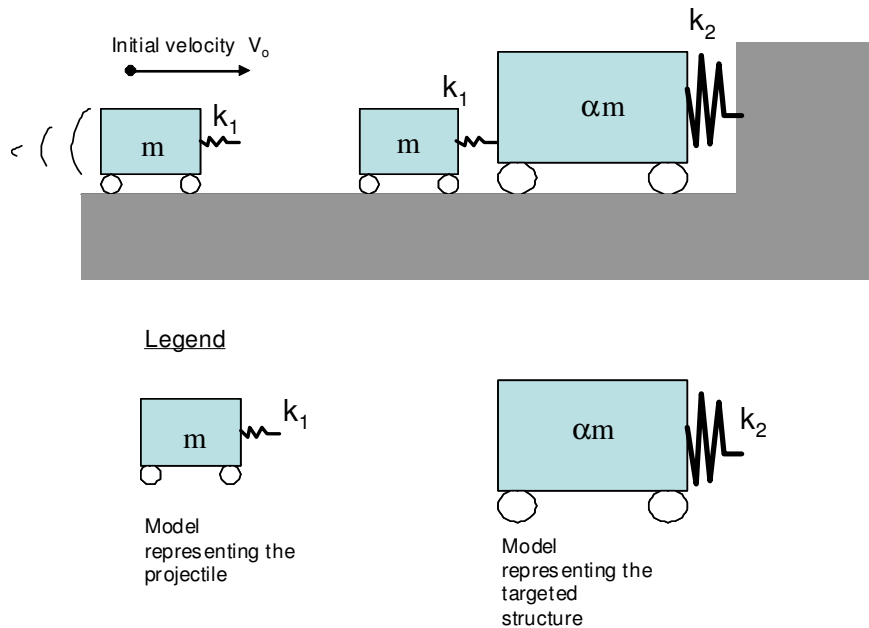
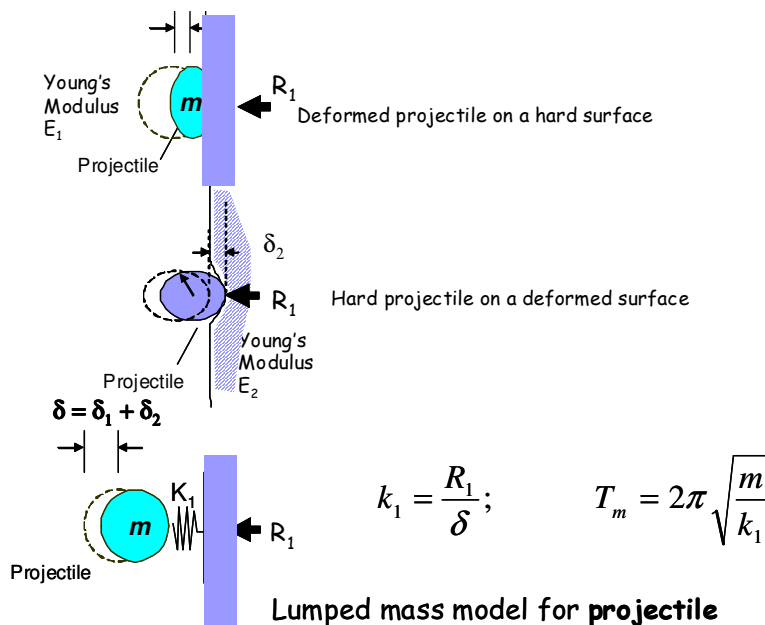Figure 2.1 *Two-degree-of-freedom* system model for impact analysis

$$k_1 = \frac{R_1}{\delta}; \qquad T_m = 2\pi\sqrt{\frac{m}{k_1}}$$
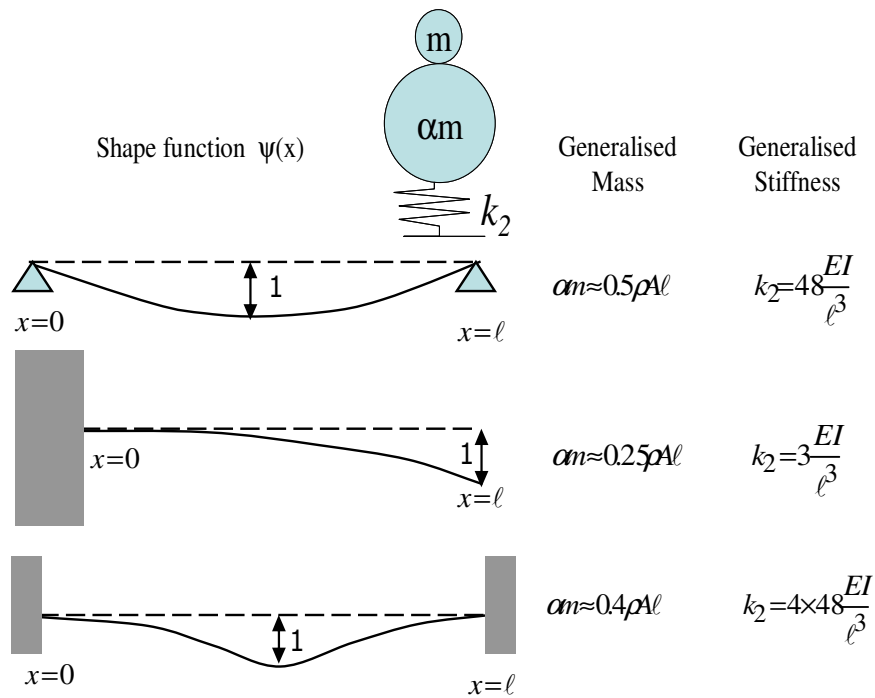
Figure 2.2 Frontal spring stiffness

Figure 2.3 Generalised mass and stiffness values for beams

Shape function $\psi(x)$ | Generalised Mass | Generalised Stiffness

$\alpha m \approx 0.5 \rho A \ell$ $k_2 = 48 \dfrac{EI}{\ell^3}$

$\alpha m \approx 0.25 \rho A \ell$ $k_2 = 3 \dfrac{EI}{\ell^3}$

$\alpha m \approx 0.4 \rho A \ell$ $k_2 = 4 \times 48 \dfrac{EI}{\ell^3}$

(a) Object approaching

(b) Initial contact made
time $t = \tau$

(c) Object-target interaction
time $\tau < t < t_1$

(d) Object about to rebound (detach)
time $t = t_1$

(e) Object rebounded
time $t > t_1$
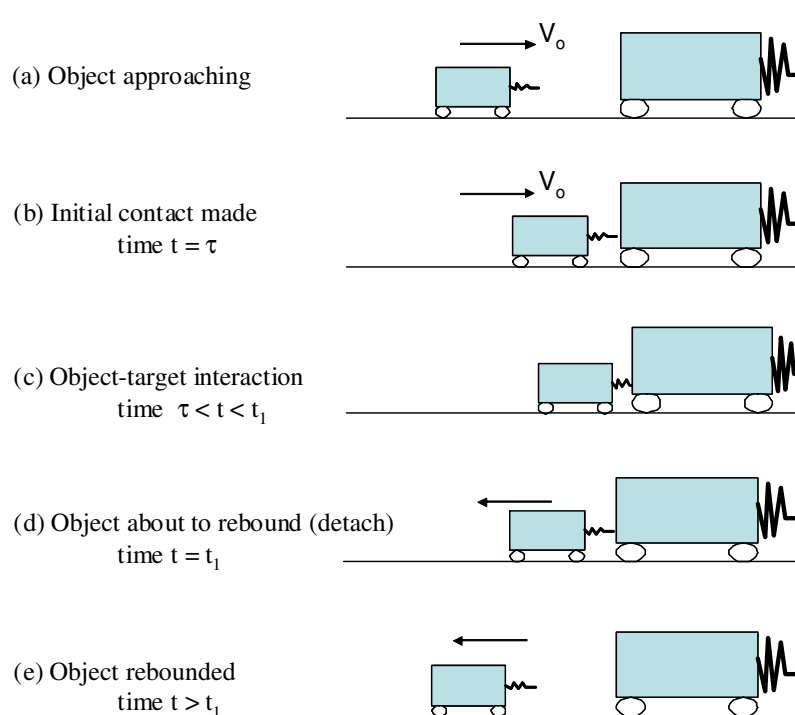
Figure 2.4 Stages of response to an impact

Alternatively, estimates of the force and displacement demand behaviour of the impact action can be obtained using a simplified analysis in which value of the frontal spring stiffness (k_1) is assumed to reach infinity. Momentum transfer between the *projectile* and the target is therefore instantaneous. Consequently, momentum transfer between the colliding objects is not subject to any interference by the rear spring (given that there is no time available to compress the spring and develop reaction forces during the course of momentum transfer). In

essence, the collision action can be analyzed as two free-bodies (lumped masses) colliding in space. Thus, the classical relationships based on *equal momentum* and *equal energy* principles may be employed.

If no energy is dissipated on collision between two free bodies, the *projectile* object will behave elastically and rebounds from the surface of the target. The realistic estimates of the displacement demand value on the target can be obtained as follow:

$$\Delta_0 = \beta \frac{mV_0}{\sqrt{k_2 m}}$$
(2.3)

Where

$$\beta = \sqrt{\frac{4\alpha}{(1 + \alpha)^2}} \ or \ \frac{2\sqrt{\alpha}}{1 + \alpha}$$
(2.4)

In practice, Eq. (2.3) will always provide conservative estimates given that some energy will always be dissipated on impact. In practice, for example, the *projectile* never rebounds to the same height where it is dropped. Nonetheless, this equation is able to provide an upper bound constraint to the impact induced displacement demand on the structural element.

If the *projectile* crumbles and dissipates energy it may not rebound but instead becomes attached to the target. The mass reduction factor (β) is obtained by Eq. (2.5).

$$\beta = \sqrt{\frac{1}{(1 + \alpha)}}$$
(2.5)

3. Experimental Investigation

The main objective of the experimental investigation was to quantify the mitigating effects of a *cushion* and a *dummy mass* in extenuating the displacement demand of impact actions on a lightweight structural element. The experiments involved dropping a steel sphere, the *projectile*, onto the mid-span position of a miniature timber specimen and recording the time-histories of the deflection. The concept of impact interface as introduced in [Chen and May, 2009] was adopted.

3.1. *Experimental Setup*

The timber beam specimen which spanned a clear distance of 1.35m was restrained from rotations in the vertical plane. The *projectile* which was a steel sphere weighing 534grams was dropped from heights of (i) 200 mm and (ii) 400 mm. The motion of the *projectile* in free fall was guided by a glass cylinder to impact on the beam at mid-span position as shown in Figure 3.1. The deflection time-histories of the beam at mid-span position and its maximum deflection value were recorded and analysed.
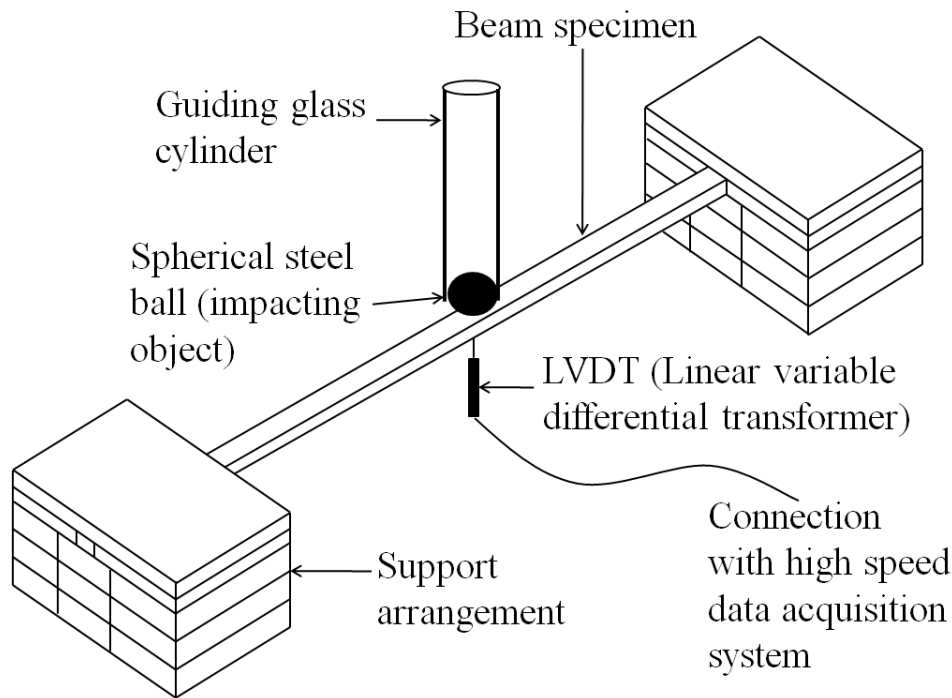
Figure 3.1 Experimentation setup

The experiments employed beam specimen made of softwood and of dimensions: 42 mm (width), 19 mm (thickness) and 1800 mm (length). The beam specimens were first statically loaded using the *four point bending method* as described in [Choi *et al*, 2007] to identify the load deflection relationship and hence obtain the value of K_2 (which is required for the *two-degree-of-freedom* system analysis as described in Section 2). Value of Young's Modulus (E) of beam specimen was then back calculated using the relationships summarized in Figure 2.3 and Table 3.1.

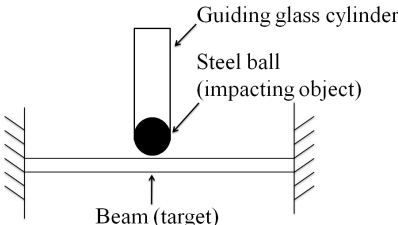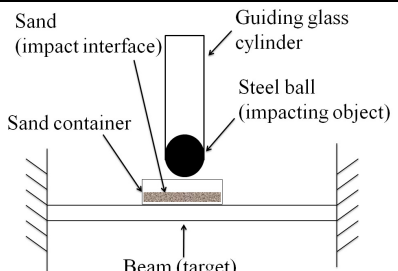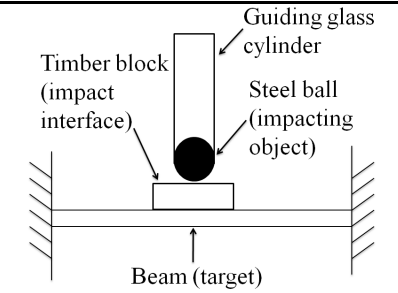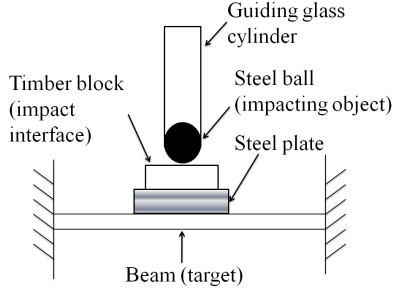Table 3.1 Parameters of impacting object and target beam

Object	Density (ρ) (kg/m^3)	Stiffness (K) (KN/m)	Young's Modulus (E) (GPa)	Effective Mass (αm) (grams)
Beam	513	16.8	8.8	236
Steel spherical ball	7774	-	200	534

The moisture contents of the specimens were inferred from information provided by the literature and was taken as 10% [Johnson, April 1986]. The value of the *Poisson's* ratio (ν) was accordingly taken as 0.26 [Trentacoste, 2005]. The steel sphere projectile was 51 mm in diameter; the value of Young's Modulus (E) was taken as 200 GPa and value of ν was taken as 0.3 [Patnaik and Hopkins, 2004].

3.2. *Impact scenarios*

The experimental investigation involved a total of eight impact scenarios as outlined in table 3.2. The projectile was dropped from a modest height of either (i) 200mm or (ii) 400mm in order that the dynamic tests were non-destructive.

Table 3.2 Impact scenarios (or cases)

Scenario or Case nos.	Impact interface	Height where *projectile* was dropped (mm)	Description of impact scenarios
1	None	200	
2	None	400	
3	Sand *cushion*	200	
4	Sand *cushion*	400	
5	Timber *cushion*	200	
6	Timber *cushion*	400	
7	Timber *cushion* with steel plate	200	
8	Timber *cushion* with steel plate	400	

Impact scenarios 01 and 02 featured timber specimen that was without any provision of interface between the *projectile* and the beam specimen. In other words, the *projectile* was made to strike the (unprotected) "bare beam". The length of beam within the clear span was only 0.8 times the total length. Given that the effective mass of a fixed-end beam (αm) is 0.4

times the total mass of the beam within the clear span (refer Figure 2.3). In comparison, the *projectile* had a much larger mass (*m*) of 534 grams. Thus, the value of the mass ratio (α) was considerably less than 1.0. As a result, little energy was expected to be dissipated on impact on the bare beam according to Eq. (2.4) in which elastic impact was assumed. Consequently, Eq. (1.1) and Eq. (2.4) provide similar estimates of the displacement demand value of the beam. A displacement value of 11 mm and 15 mm was estimated for a drop height of 200 mm (case 01) and 400 mm (case 02) respectively. These impact scenarios featuring no *interface* for protection were used as *control* experiment.

The rest of the experimental investigation involved the use of a range of materials as interface between the *projectile* and the beam specimen. Impact scenarios 03 and 04 featured the use of sand as *interface* (or *cushion*). A plastic container measuring 146 mm by 90 mm on plan was filled with a layer of sand which was 31 mm thick and of density equal to 1643 kg/m^3 giving a total mass of 657 grams. Impact scenarios 05 and 06 featured the use of a block of timber as *interface* (*cushion*) instead. The timber block was of dimensions: 105 mm (length) by 70 mm (width) by 35 mm (thick) and had mass of 134 grams. Finally, impact scenarios 07 and 08 incorporated a steel plate which was placed underneath the timber cushion in order that the conditions of the surface which was in contact with the *projectile* were kept the same as for impact scenarios 05 and 06. The steel plate was of dimensions: 100 mm (length) by 101 mm (width) by 25.5 mm (thick) and had mass of 1925 grams. Significantly, the mass of the combined interface comprising the steel plate and timber block combined was 2060 grams which was considerably heavier than interface used in the other impact scenarios.
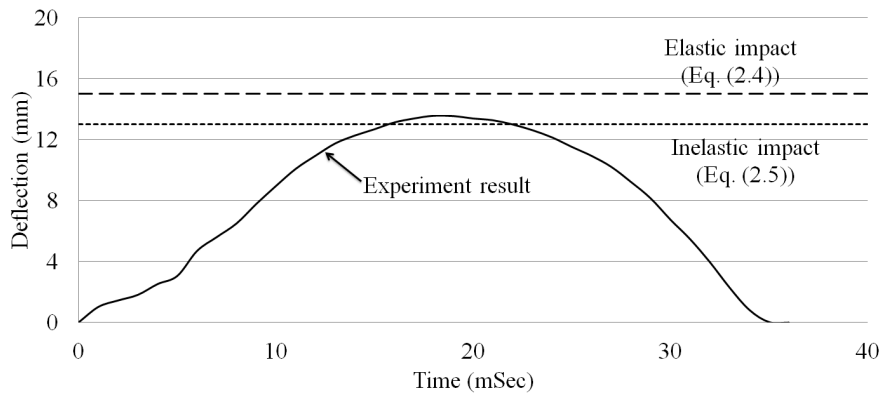
4. Results and Discussions

Results from the dynamic experiments involving the eight impact scenarios (01 – 08) are summarised in Figures 4.1 (a – h). This timber specimen had a total mass of approximately 750 grams which was translated to an effective mass (αm) of only 250 grams and a stiffness (K_2) value of 16.8 kN/m as shown in Table 3.1.
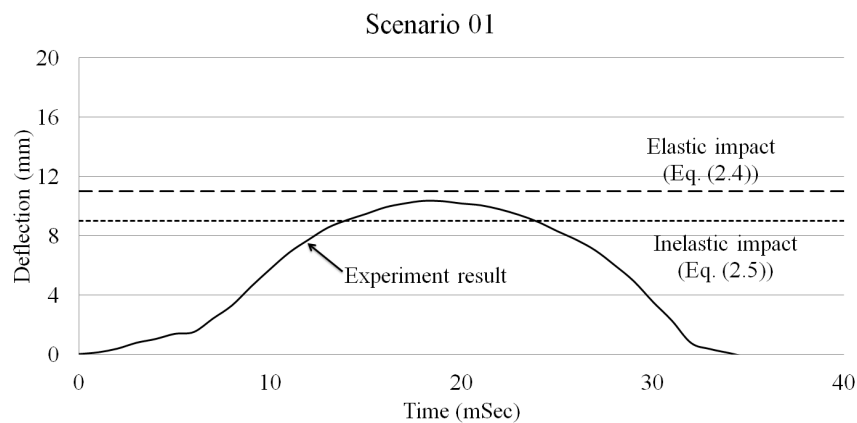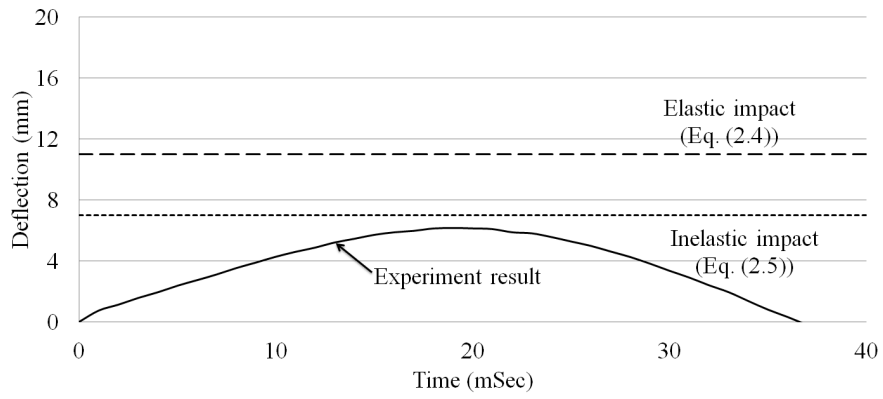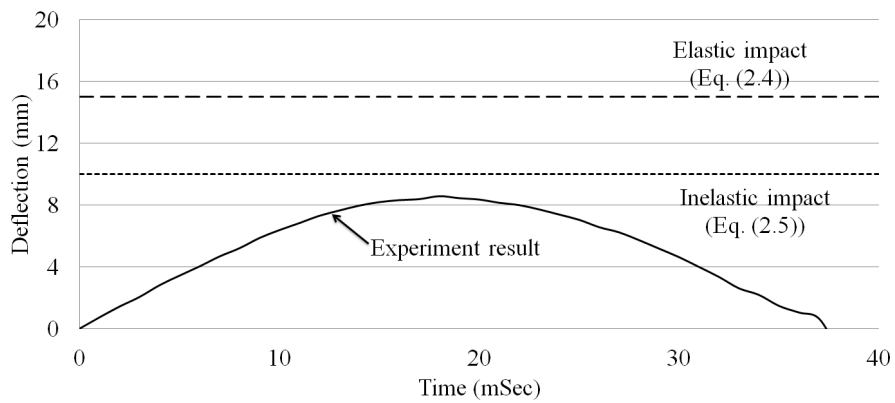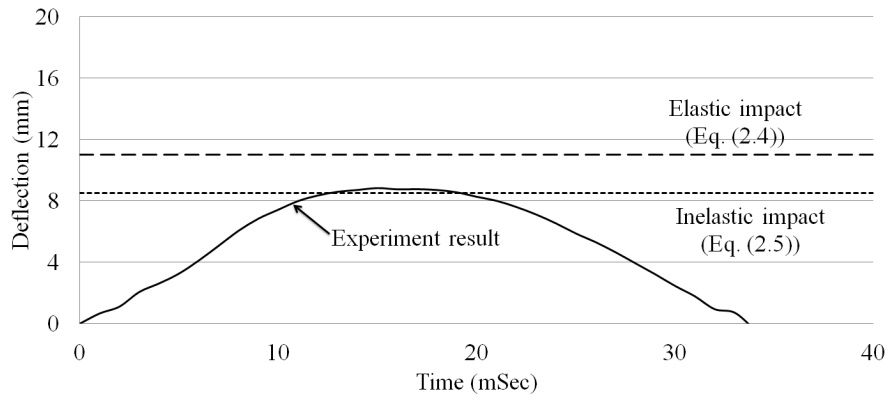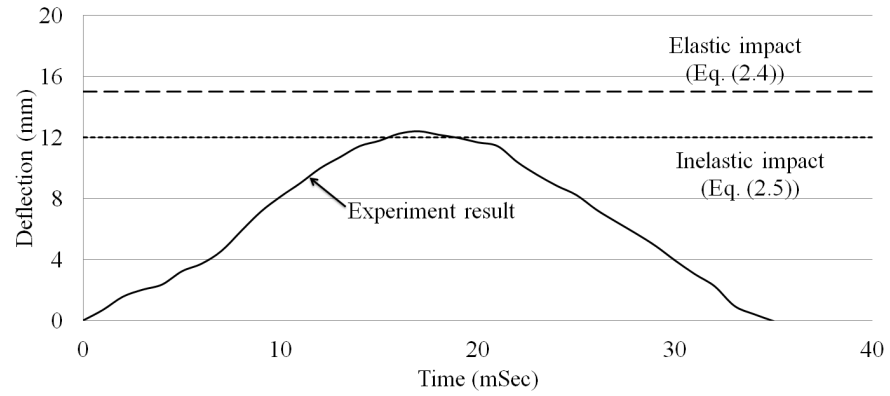

(a)

Scenario 02
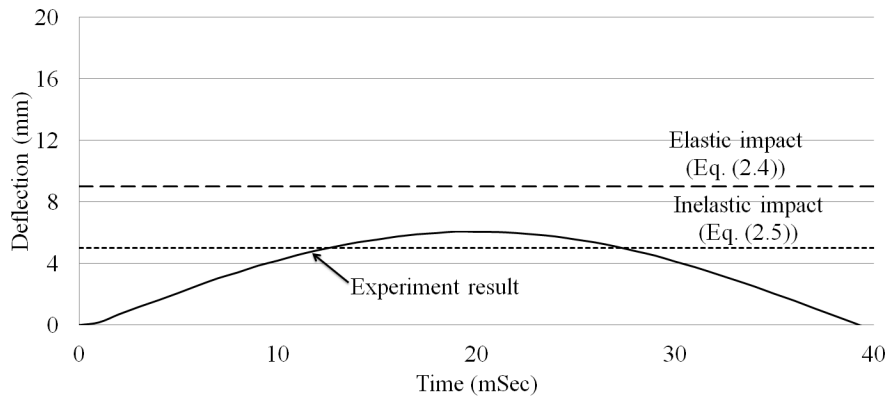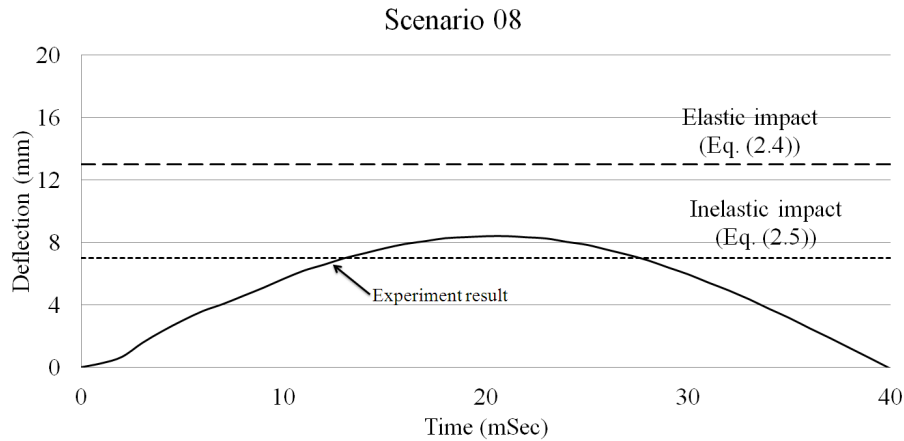
(b)

Scenario 03

(c)

Scenario 04

(d)

Scenario 05

(e)

Scenario 06

(f)

Scenario 07

(g)

Figure 4.1 Displacement time histories of beam specimen

The *control* experiments (impact scenarios 01 and 02) were characterised by very little energy dissipation. This was because of the lightweight nature of the timber specimen which was without the protection of its self-weight or that of an interface. For a drop height of 200 mm and 400 mm displacement demand values of 11 mm and 15 mm respectively were predicted by the use of Eq. (2.4) based on *elastic* impact, or by Eq. (1.1) consistent with current codes provisions. These predicted values were in reasonable agreement with the respective maximum deflection values of 11 mm and 14 mm which were actually recorded from the respective experiments (refer Figures 4.1 a – b). For both drop heights, the observed displacement values were constrained in between estimates defined by Eq. (2.4) and Eq. (2.5) for *elastic* and *inelastic* impact.

Further experimentations were conducted for impact scenarios 03 and 04 which featured the use of a pot of sand as *cushion* to dissipate energy on impact. The experiments recorded no rebounce of the *projectile* following contact with the sand *cushion*. It is shown in Figures 4.1 (c – d) that the maximum displacement values recorded from these impact scenarios were reduced by some 40 % - 50 % from the control experiments: from 11 mm to 6 mm (scenarios 01 to 03) and from 14 mm to 8 mm (scenarios 02 to 04). Interestingly, displacement estimates from hand calculations (Eq. (2.4) and Eq. (2.5)) for impact scenarios 03 and 04 were only slightly lower than estimates for scenarios 01 and 02. This was because the pot of sand only weighed 657 grams. Thus, what caused the reduction in the maximum displacement value of the impact had little to do with the small change in *mass* (inertia) of the set-up but was much attributed to energy dissipation within the sand *cushion* which prevented re-bounce of the *projectile*. This proposition is supported by the experimentally observed displacement values being exceeded by predictions from both Eq. (2.4) and Eq. (2.5) as shown in Figures 4.1 (c – d). The conservatism of values predicted by the hand calculations stemmed from the implicit assumption that the transfer of momentum was instantaneous. Thus, the sand cushion served to cause a slight delay in the momentum transfer as well as dissipating energy.

In scenarios 05 and 06, similar experiments were repeated whilst replacing the sand *cushion* by the much stiffer timber *cushion* (also of modest mass). As expected, the estimated displacement values from hand calculations (Eq. (2.4) and Eq. (2.5)) were not very different to those scenarios that had been tested earlier for the same drop height. Again, there was insignificant change in mass of the interface across the scenarios. However, the

experimentally observed displacement values as shown in Figures 4.1 (e – f) featured only a slight reduction from values associated with the *control* experiments, unlike scenarios 03 and 04.

Comparison of the observed displacement values across scenarios 01-03-05 and scenarios 02-04-06 reveals different degrees of reduction associated with the use of different "cushion" materials (sand and timber block) as medium for energy dissipation.

Table 4.1 Recorded Displacement Values

Scenarios	Descriptions of Interface	Drop Height of 200mm	Drop Height of 400mm
01 and 02	*Bare Beam* without Interface (control experiments)	11 mm (1.0)	14 mm (1.0)
03 and 04	*Sand Cushion* as Interface	6 mm (0.55)	8 mm (0.55)
05 and 06	*Timber Cushion* as Interface	9 mm (0.85)	12 mm (0.85)
07 and 08	*Timber Cushion* and *Steel Plate* as Interface	6 mm (0.55)	8 mm (0.55)

1. Values listed have been rounded-off to the nearest mm
2. Values in ( ) have been normalised w.r.t. observations from the control experiments

In scenarios 07 and 08, the same timber *cushion* (as used in scenarios 05 and 06) was deployed. Importantly, an additional steel plate was placed underneath the timber *cushion* to increase the mass of the combined interface to about 2000 grams. It is noted that the conditions of contact (between the *projectile* and timber *cushion*) were kept the same across scenarios 05 – 08. The important findings from this latest set of experiments were the 30% reduction in the observed displacement value from 9 mm to 6 mm (scenario 05 to 07) for a drop height of 200 mm and from 12 mm to 8 mm (scenario 06 to 08) for a drop height of 400 mm. Refer Figures 4.1 (g – h) in comparison with Figures 4.1 (e – f). This reduction in the displacement demand value was attributed totally to the protection by mass (inertia) phenomenon which was well captured by estimates from hand calculations (Eq. (2.4) and Eq. (2.5)).

Refer Table 4.1 for a listing of the experimentally recorded displacement values from specimen 01 (figures in brackets are values that have been normalised with respect to the displacement values recorded from the control experiments of scenarios 01 and 02).

In summary, the experimental investigation involving eight impact scenarios revealed two distinct phenomena: (i) *cushion* protection phenomenon and (ii) mass, or inertia, protection phenomenon. Some 40% - 50% reduction in the displacement demand value was seen when sand was used as *cushion* forming part of the impact interface (scenarios 03 and 04 in comparison with scenarios 01 and 02). In a separate set of experiments, a 30% of reduction was also seen when a steel block of significant mass was incorporated into the interface

whilst the cushion material in direct contact with the *projectile* was kept the same (scenarios 07 and 08 in comparison with scenarios 05 and 06).

5. Closing Remarks

Two important phenomena associated with the impact resistant behaviour of structural elements have been introduced: (i) protection by *cushion* and (ii) *protection by mass (inertia).* As demonstrated by laboratory experimentations involving miniature timber beam specimens, the impact induced displacement demand of the impact can be reduced considerably by placing a pot of sand as *cushion* interface between the *projectile* and the beam specimen. Significant reduction in the beam deflection can similarly be accomplished by simply placing an object of significant mass forming part of the interface at the position of contact. Simple expressions based on *equal momentum* and *equal energy* principles have been derived to provide estimates of the impact induced beam deflections taking into account both phenomena. Importantly, the calculated deflection values have been shown to be in good agreement with observations from experiments.

The concepts introduced herein also point to limitations in the use of parameters based on static conditions for assessing the potential performance of the beam, or column, element in an impact scenario. For example, a floor structure built of lightweight materials might not possess adequate impact resistant capacity even though its gravity load carrying capacity is comparable to that of a conventional floor structure. Thus, the phenomenon introduced herein has important implications in the design for robustness of lightweight structural elements. Meanwhile, the two phenomena introduced in the paper opens up new opportunities for design innovations for enhancing impact resistance.

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